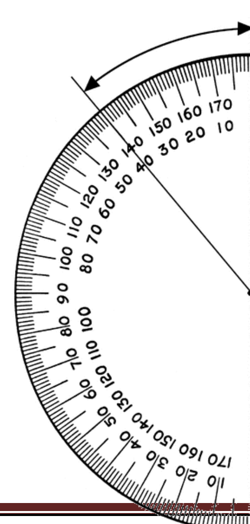
2.1 The Tangent Ratio

C 2.1 Concept: 14, 15

ACTIVITY:



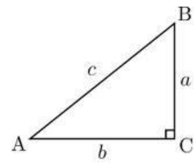
Trigonometry Table

A	SIN(A)	COS(A)	Tan(A)
0	0.0000	1.0000	0.0000
1	0.0175	0.9998	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
9	0.1564	0.9877	0.1584
10	0.1736	0.9848	0.1763
11	0.1908	0.9816	0.1944
12	0.2079	0.9781	0.2126
13	0.2250	0.9744	0.2309
14	0.2419	0.9703	0.2493
15	0.2588	0.9659	0.2679
16	0.2756	0.9613	0.2867
17	0.2924	0.9563	0.3057
18	0.3090	0.9511	0.3249
19	0.3256	0.9455	0.3443
20	0.3420	0.9397	0.3640
21	0.3584	0.9336	0.3839
22	0.3746	0.9272	0.4040
23	0.3907	0.9205	0.4245
24	0.4067	0.9135	0.4452
25	0.4226	0.9063	0.4663
26	0.4384	0.8988	0.4877
27	0.4540	0.8910	0.5095
28	0.4695	0.8829	0.5317
29	0.4848	0.8746	0.5543
30	0.5000	0.8660	0.5774
31	0.5150	0.8572	0.6009
32	0.5299	0.8480	0.6249
33	0.5446	0.8387	0.6494
34	0.5592	0.8290	0.6745
35	0.5736	0.8192	0.7002
36	0.5878	0.8090	0.7265
37	0.6018	0.7986	0.7536
38	0.6157	0.7880	0.7813
39	0.6293	0.7771	0.8098
40	0.6428	0.7660	0.8391
41	0.6561	0.7547	0.8693
42	0.6691	0.7431	0.9004
43	0.6820	0.7314	0.9325
44	0.6947	0.7193	0.9657
45	0.7071	0.7071	1.0000

A	SIN(A)	COS(A)	Tan(A)
45	0.7071	0.7071	1.0000
46	0.7193	0.6947	1.0355
47	0.7314	0.6820	1.0724
48	0.7431	0.6691	1.1106
49	0.7547	0.6561	1.1504
50	0.7660	0.6428	1.1918
51	0.7771	0.6293	1.2349
52	0.7880	0.6157	1.2799
53	0.7986	0.6018	1.3270
54	0.8090	0.5878	1.3764
55	0.8192	0.5736	1.4281
56	0.8290	0.5592	1.4826
57	0.8387	0.5446	1.5399
58	0.8480	0.5299	1.6003
59	0.8572	0.5150	1.6643
60	0.8660	0.5000	1.7321
61	0.8746	0.4848	1.8040
62	0.8829	0.4695	1.8807
63	0.8910	0.4540	1.9626
64	0.8988	0.4384	2.0503
65	0.9063	0.4226	2.1445
66	0.9135	0.4067	2.2460
67	0.9205	0.3907	2.3559
68	0.9272	0.3746	2.4751
69	0.9336	0.3584	2.6051
70	0.9397	0.3420	2.7475
71	0.9455	0.3256	2.9042
72	0.9511	0.3090	3.0777
73	0.9563	0.2924	3.2709
74	0.9613	0.2756	3.4874
75	0.9659	0.2588	3.7321
76	0.9703	0.2419	4.0108
77	0.9744	0.2250	4.3315
78	0.9781	0.2079	4.7046
79	0.9816	0.1908	5.1446
80	0.9848	0.1736	5.6713
81	0.9877	0.1564	6.3138
82	0.9903	0.1392	7.1154
83	0.9925	0.1219	8.1443
84	0.9945	0.1045	9.5144
85	0.9962	0.0872	11.4301
86	0.9976	0.0698	14.3007
87	0.9986	0.0523	19.0811
88	0.9994	0.0349	28.6363
89	0.9998	0.0175	57.2900
90	1.0000	0.0000	∞

Online Video Lesson: : https://goo.gl/ocVG9J

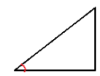
- In this Unit, we will study **Right Angled Triangles.** Right angled triangles are triangles which contain a right angle which measures 90˚ (the little "box" in the corner means that angle is a right angle).



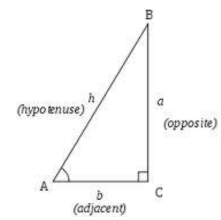
Skill: Labeling Right Angle Triangles

Quite often, the right triangle will be labeled using letters such as in the triangle above.

- Capital letters are used to label ___________
- Small letters are used to label _____________.
- We also need to be able to label the triangle with words and symbols.
  - We start by labelling the **ANGLE** we are using as a reference (NOT the right angle).We often put a small arc in that angle to label it, or we we greek letters such as ϴ (Theta)

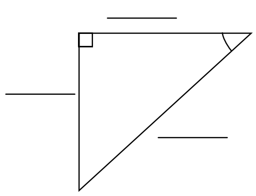


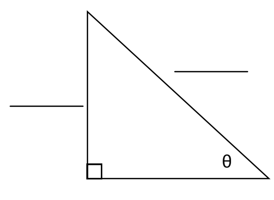
  - Next we label the **HYPOTENUSE:** This is the side of the triangle directly across from the right angle and is always the longest side in a right angle triangle).
  - Next we label the **OPPOSITE** side. This is the side opposite the angle we are using as a reference.
  - Lastly we label the **ADJACENT** side. This is the side beside the angle we are using as a reference (but not the hypotenuse)



- NOTE: Sometimes the Opposite and Adjacent sides are also called LEGS.

Example: Label the following triangles using the terms from above.





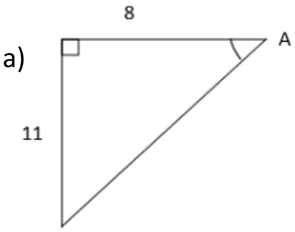
$$\text{The Tangent of Angle A} = \frac{\text{Length of the opposite side}}{\text{Length of the adjacent side}}$$

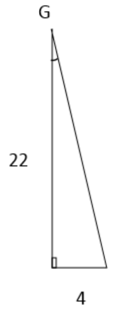
In simpler terms it usually looks like this: $\text{Tan A} = \dfrac{\text{Opposite}}{\text{Adjacent}}$

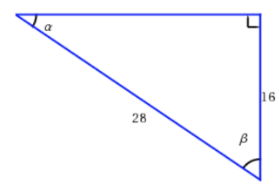
All of these decimal values have been stored in your scientific calculator. Let's check:

- First, make sure your calculator is in the correct "mode". It needs to say Deg, Degree or D at the top. If it says G, Grad, R or Rad, your calculator is in the incorrect mode and needs to fixed!
- Imagine drawing a triangle with a 60˚ angle in the corner. Imagine measuring opposite and the adjacent sides and dividing them using the tangent ratio. Do you think your decimal will be larger or smaller than 1? _________ Larger or smaller than 0.5?___________
- On your calculator press the following buttons: 60 and then tan. You should see 1.74205. If you don't see that, you may need to press the buttons in the order tan and then 60 for your calculator.
- We always round off this decimal from our calculator to 4 decimal places. This will be 1.7321 (Remember – if the first number we are "getting rid of" is 5 or bigger, we "bump" the one in front of it up one value)
- This means that every triangle in the world with a 60˚angle in the corner will have an opposite side divided by an adjacent side that will always turn into 1.7321.

Example: Find the Tangent Ratio for the indicated angle(s) in the following Triangles. Leave your answer in EXACT FORM.

a) 

b) 

c) 

Note: The above question will often be asked another way : Determine tan A and tan G

Things to Remember from Last Year:

- All three Angles in a Triangle Add to 180˚.
- The Pythagorean Theorem:

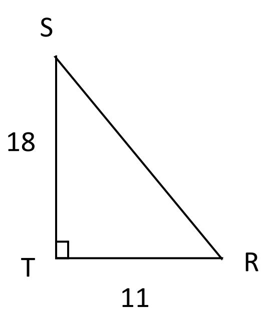
Example: If tanA = 0.5418, determine the measure of $\measuredangle$ to the nearest degree.

- Our calculator has the decimal value for every size angle and its tangent. This question is asking us to look at the question from the opposite direction – we are given the decimal that came from dividing the opposite and the adjacent sides. What size was the angle in the triangle?

  Use your Grade 9 and 10 skills to "Solve for A" in the question: tanA = 0.5418

- We are going to have to use the SHIFT or the 2nd button in our calculator along with tan to find this answer. You are either going to press the buttons 0.5418 then SHIFT/2nd then Tan or you are going to press SHIFT/2nd then Tan then 0.5418.
- The calculator shows us _________________ which is the size of the angle with decimals to be very exact. Our question asks us to round to the nearest degree. Our answer will be ________.

Example: Determine the measures of $\measuredangle$ R and $\measuredangle$ S to the nearest tenth of a degree.



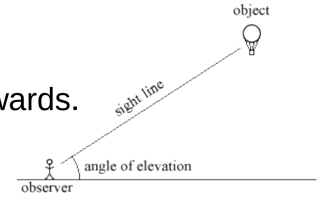
STEPS:

1. Decide which angle to use as your "Starting Point".
2. Label your angle, your hypotenuse, your opposite and your adjacent
3. Write down the formula $\tan A = \frac{Opp}{Adj}$
4. Fill in the formula with information from your triangle. The "A" will become the letter you chose as your starting point. The Opp will be the length of the opposite side and the Adj will be the length of the adjacent side.
5. Divide your fraction. Leave your answer to 4 decimal places.
6. Use the SHIFT/2nd procedure to find your angle.
7. Our answer is to tenths which means round to one decimal!
8. To find the other angle, subtract the angle you found and the right angle from 180˚

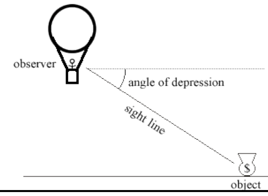
Other Terms to Learn:

Acute Angle: An angle whose size is between 0˚ and 90˚

Angle of Inclination (Also known as Angle of Elevation):
The angle measured between a horizontal line and a line angling upwards.



Angle of Depression:
The angle measured between a horizontal line and a line angling downwards.



Example: Ms. C is standing in the courtyard which is 32 feet away from the school and directly across from her classroom. She looks up to the second floor and sees the windows of her classroom which are 16 feet high (and is annoyed when she sees her students hanging out of the windows instead of doing their work). What is the angle of inclination that Ms. C is looking up at? Leave your answer to the nearest hundredth. Please disregard Ms. C's height (she's pretty short anyway).

STEPS:
1. Draw a diagram. Be sure that your angle of elevation contains a horizontal line _____ and a line slanting upwards. Put all given numerical information on the diagram.
2. Label your angle, your hypotenuse, your opposite and your adjacent
3. Write down the formula $\tan A = \frac{Opp}{Adj}$
4. Fill in the formula with information from your triangle. Divide your fraction. Leave your answer to 4 decimal places.
5. Use the SHIFT/2nd procedure to find your angle.
6. Our answer is to hundredths which means round to two decimals!

CALCULATORS ARE ALLOWED!!! Label this assignment properly!

2.1 *FA (Foundational Assignment) (C14, 15)
P75 3ac, 4ab, 5ab, 8a, 10ac, 14, 17 PLUS THE PRACTICE QUESTIONS BELOW:

2.1 MLA (Mid Level Assignment) (C14, 15)
P75 #11a, 7, 12, 13, 15, 16 one of 19

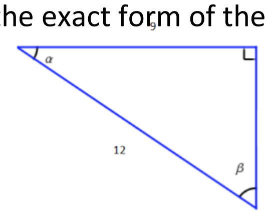
2.1 ULA (Upper Level Assignment)
P75 #21 - 23

C 2.1 Assignment for Concepts #14, 15

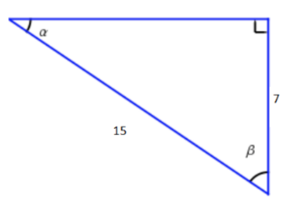
Practice Questions:

Find the exact form of the tanα:

1. a)



b)



2. From the top of a fire tower, a forest ranger sees his partner on the ground at an angle of depression of 40º. If the tower is 45 feet in height, how far is the partner from the base of the tower, to the *nearest tenth of a foot*?

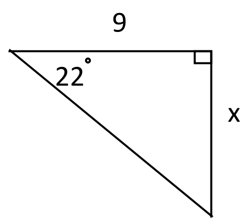
2.2 Using The Tangent Ratio to Calculate Lengths

C 2.2 Concept: #16

Online Video Lesson: : https://goo.gl/ocVG9J

In this section, we will continue to use right angled triangles and the Tangent ratio. This time we will use the Tangent ratio to find unknown sides in the triangle instead of finding the unknown angle.

Example #1: Find the length of x to the nearest tenth.



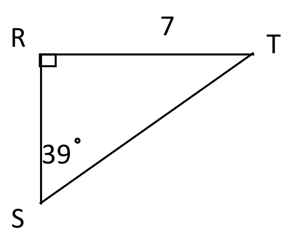
STEPS:

1. Label all of your given information using: angle, hypotenuse, opposite, adjacent.
2. Write down the formula

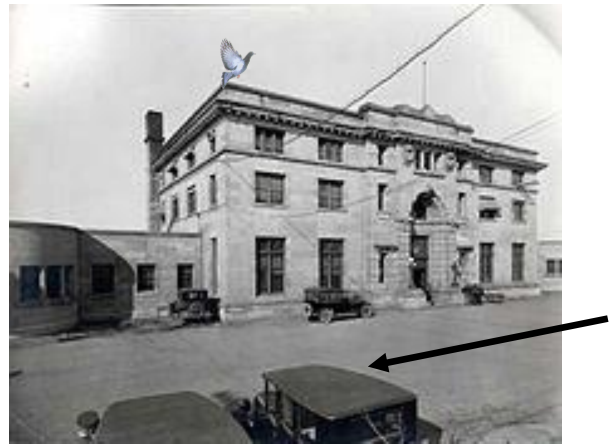
$$\tan A = \frac{Opp}{Adj}$$

3. Fill in the formula with information from your triangle.
4. Use your "Tan" button to find the Tan of the angle. Do not use your shift or 2^{nd} key when you know the number beside the word Tan! Write down the number from the calculator to 4 decimals.
5. Put the decimal on the left side over 1.
6. Cross multiply to find the answer to your unknown side!

Example #2: Find the length of RS to the nearest hundredth. (Note: Sides that are named with two capital letters like RS refer to the side between the angles at R and S)



Example: The following picture shows what the current “Casino Regina” looked like when it was the Union Train Station in Regina in 1911. Let’s assume that the building is 21 m tall. There is a person sitting in the Model T car that is pointed out with the arrow. The person looks up at an angle of elevation of 65˚ and sees a Pigeon sitting on the roof. Using trigonometry, determine how far the car is from the building.



b) If the person in the car wanted to know how far they were from the pigeon along “his line of sight” how could you find that out? Calculate that distance.

CALCULATORS ARE ALLOWED – be sure it is in degree mode!!! Label this assignment properly!

2.2 *FA (Foundational Assignment) (C16)
P82 #3ac, 4ab, 5b, 6, 7, 9a, 11

2.2 MLA (Mid Level Assignment) (C16)
P82 #10, 12

2.2 ULA (Upper Level Assignment)
P82 # 13-16

C 2.2 Assignment for Concepts #16

2.4 & 2.5 The Sine and Cosine Ratio

C 2.4 & 2.5 Concepts: #14, 15, 16

Online Video Lessons: **https://goo.gl/dZcBEi** & **https://goo.gl/7IhmLb**

In this section, we will continue to use right angled triangles and learn two new ratio's: the Sine Ratio and the Cosine Ratio. We will use these ratio's to find unknown angle and side measures. We use these formula's in much the same way we used the Tangent Ratio.

$$\text{The Cosine of Angle A} = \frac{\text{Length of the adjacent side}}{\text{Length of the hypotenuse}}$$

In simpler terms it usually looks like this:

$$\text{Cos A} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{The Sine of Angle A} = \frac{\text{Length of the opposite side}}{\text{Length of the hypotenuse}}$$

In simpler terms it usually looks like this:

$$\text{Sin A} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

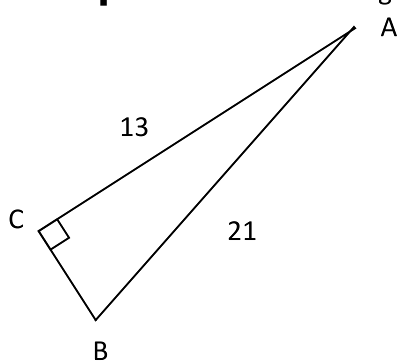
Example #1: Determine the sine and cosine of each angle to the nearest hundredth. (Note: when you are asked to find the sin or cos or tan of an angle it is asking you to use the sin or cos or tan button along with the angle in your calculator – do NOT use your Shift/2nd button!)

a) 62˚ b) 22˚

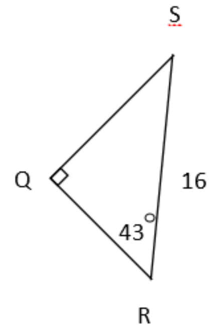
Example #2: Determine the measure of each $\measuredangle$T to the nearest degree. (Note: When you are asked to find the size of the angle using sin or cos or tan – you will ALWAYS use the Shift/2nd button on your calculator!)

a) sin T = 0.7834 b) cos T = 0.1279 c) tan T = 1.5834

Example #3: Find the length of the unknown angles using either sin or cos.



Example #4: Find the length of QS to the nearest tenth.



b) Find QR.

Example #5: While she was teaching at Balfour, Ms.C's Calculus class was standing outside of the school for the Halloween class picture. Ms. C looked up at an angle of elevation of 67˚ and saw a UFO approaching the school. If the UFO is at a height of 231 m, how far is the UFO away from Ms. C along her line of sight?

Example #7: A person leaves the front steps of the parliament buildings in Regina, walks 35 m North, turns to the right and walks East for a while and then stops. The angle between his original path going north and the line that now connects where he stopped and the front steps of the parliament building is 50˚. How far is he now from the front steps of the parliament building?

Example #7: What is the sin90? What is the cos90?
http://zonalandeducation.com/mmts/trigonometryRealms/introduction/rightTriangle/trigRightTriangle.html

CALCULATORS ARE ALLOWED – be sure it is in degree mode!!! Label this assignment properly!

2.4/2.5 *FA (Foundational Assignment)(C14-16)
P95 #5c, 6ac, 7b, 8b, 9a, 10ab, 13, 14
P101 #3a, 4c, 5ac, 6, 8, 9
PLUS THE PRACTICE QUESTIONS BELOW

2.4/2.5 MLA (Mid Level Assignment) (C14-16)
P95 #11, 12, 14,15,17
P 101 # 7, 10, 11, 12, 13, 14

2.4/2.5 ULA (Upper Level Assignment) (C14-16)
P 101 #13,14

C 2.4 & 2.5: Assignment for Concepts #14, 15, 16

Practice Questions:

1. If a man is just about to ski down a steep mountain. He estimates the angle of depression from where he is now to the flag at the bottom of the course to be 24°. He knows that he is 800 feet higher than the base of the course. How long is the path that he will ski? (Round to the nearest foot)
2. Ms. C is 5'2" tall. She hears a loud bang and looks down in horror to see a calculator hit the floor. If the calculator is 320cm away from her eyes once it is on the floor, at what angle of depression is she is looking down at?

2.6 & 2.7 Using Trig Ratios to Solve Right Triangles

Online Video Lessons: https://goo.gl/grwf6s

In this section, we will use Sin, Cos and Tan to Solve Right Triangles. Solving a right triangle means that at the end of each question you will know the length of all three sides and the size of all three angles. We can use the following to help us solve the triangles:

$$\text{Sin}A = \frac{\text{opp}}{\text{hyp}}, \quad \text{Cos}\,A = \frac{\text{adj}}{\text{hyp}}, \quad \text{Tan}\,A = \frac{\text{opp}}{\text{adj}}, \quad a^2 + b^2 = c^2, \quad \measuredangle A + \measuredangle B + \measuredangle C = 180°$$

Example #1: Solve the following triangles. Find side lengths to the nearest tenth and angles to the nearest degree.

a)

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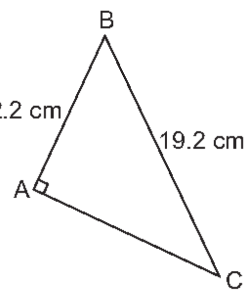
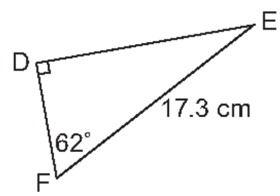
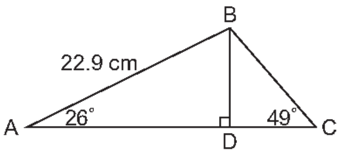


b)

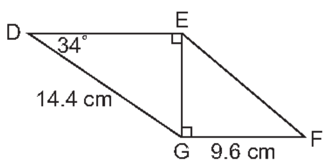


Example #2: Find the length of BC.



Example #3:

. Find the measure of ∠F to the nearest degree.



Example #4:

Jason is lying on the ground midway between two trees, 100 m apart.
The angles of elevation of the tops of the trees are 13° and 18°. How much taller is one tree than the other? Give the answer to the nearest tenth of a metre.

Example #5:

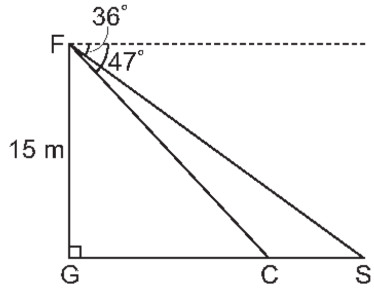
The angle of elevation of the top of a tree, T, is 27°. From the same point on the ground, the angle of elevation of a hawk, H, flying directly above the tree is 43°. The tree is 12.7 m tall. How high is the hawk above the ground? Give your answer to the nearest tenth of a metre.

Example #6:

From a small plane, V, the angle of depression of a sailboat is 21°.
The angle of depression of a ferry on the other side of the plane is 52°.
The plane is flying at an altitude of 1650 m.
How far apart are the boats, to the nearest metre?

Example #7:

This diagram shows a falcon, F, on a tree, with a squirrel, S, and a chipmunk, C, on the ground. From the falcon, the angles of depression of the animals are 36° and 47°. How far apart are the animals on the ground to the nearest tenth of a metre?



CALCULATORS ARE ALLOWED – be sure it is in degree mode!!! Label this assignment properly!

2.6/2.7 *FA (Foundational Assignment) (C15, 16, 17)
P111 #6ac, 7, 8, 9
P118 #3c, 4c, 5ac, 8, 9, 11, 13

2.6/2.7 MLA (Mid Level Assignment) (C15, 16, 17)
P111 #10, 11, 12, 13, 14
P118 #6, 15, 16

2.6/2.7 ULA (Upper Level Assignment) (C15, 16, 17)
P111 #15, 16
P118 # 17-21

C 2.6 & 2.7: Assignment for Concepts #15, 16, 17