FOUNDATIONS 20 **FM 20.9 QUADRATICS** - Day One Exercise – Graphing Quadratics

Today we will graph quadratics in the form $y = ax^2 + b x + c$ and learn about their properties.

DAY 1 NOTES: Properties and Characteristics of Quadratic Functions

https://www.youtube.com/watch?v=a2G40dBqZqg
http://plus.maths.org/content/101-uses-quadratic-equation-part-ii

Concepts: #5 , [6]

?: *QUADRATIC FUNCTIONS:*

EXAMPLE #1: Which of the following are quadratic?

a) $f(x) = 7x^2$

b) $f(x) = 5x + 8$

c) $y = 3x^2 - 5x$

d) $f(x) = 5x^3 - 1$

e) $f(x) = 3x^2 + 7x + 8$

f) $y = 4x^3 - 3x^2$

g) $y = (x - 6)^2$

h) $g(x) = 6(x + 3)^2 + 8$

EXAMPLE #2: Given quadratics in the form $y = ax^2 + bx + c$, find the values of a, b, and c.

a) $y = 7x^2 + 4x - 2$ a = _______, b = _______, c = _______

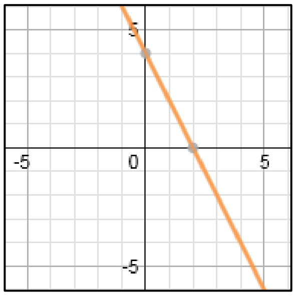
b) $y = \frac{1}{3}x^2 - 5x + 1$ a = _______, b = _______, c = _______

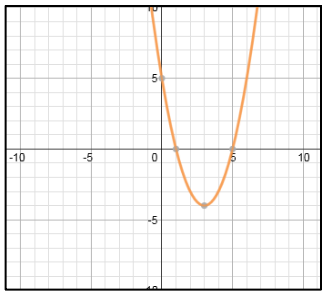
c) $y = x^2 + 3x$ a = _______, b = _______, c = _______

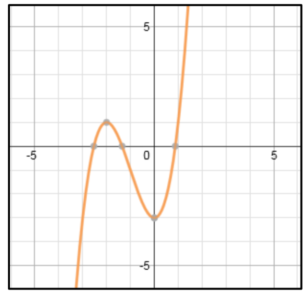
d) $f(x) = -\frac{2}{7}x^2 + 8$ a = _______, b = _______, c = _______

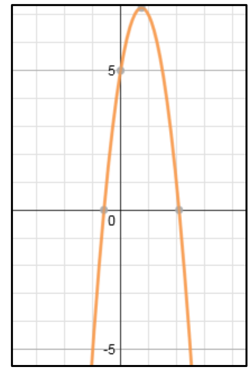
e) $y = -x^2$ a = _______, b = _______, c = _______

EXAMPLE #3: Which of the following are quadratic?









DAY 1 ACTIVITY: Properties and Characteristics of Quadratic Functions

PART 1:

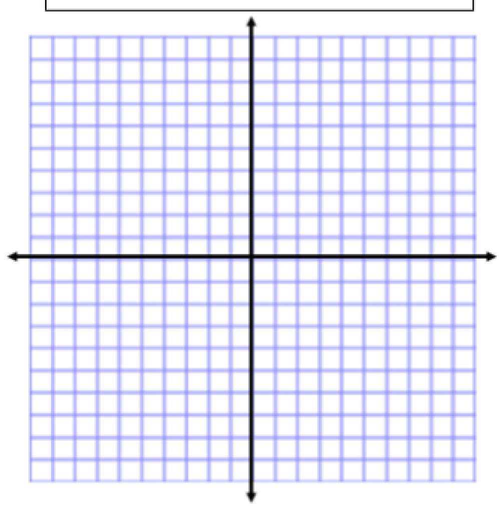
- Complete the table in column 1
- Graph the ordered pairs from the table onto the graph in column two. Join the points in a smooth curve
- Ignore column 3 for now........

Concepts: #5 , [6]

THE FUNCTION USING A TABLE	THE GRAPH	THE CHARACTERISTICS

x	$Y=x^2+x-8$	y
-4		
-3		
-2		
-1		
0		
1		
2		

a = ____ b = ____ c = _____



X Intercept:

Y Intercept:

Vertex:

Is the Vertex a Maximum or Minimum?

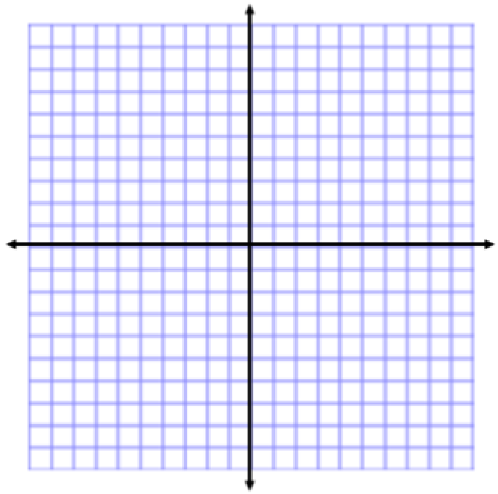
Axis of Symmetry:

Domain:

Range:

x	$y = -x^2 + 2x - 5$	y
-2		
-1		
0		
1		
2		
3		
4		

a = ____ b = ____ c = _____



X Intercept:

Y Intercept:

Vertex:

Is the Vertex a Maximum or Minimum?

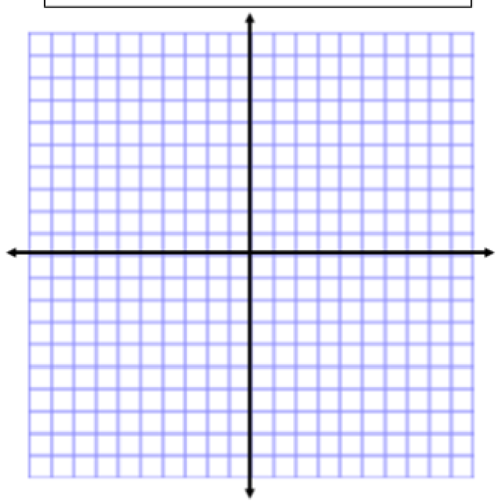
Axis of Symmetry:

Domain:

Range:

x	$y = 2x^2 + 12x + 10$	y
-6		
-5		
-4		
-3		
-2		
-1		
0		

a = ____ b = ____ c = _____



X Intercept:

Y Intercept:

Vertex:

Is the Vertex a Maximum or Minimum?

Axis of Symmetry:

Domain:

Range:

PART 2:

Q: What pattern do you see in the above graphs with respect to the value of "c" and a particular feature of each graph?

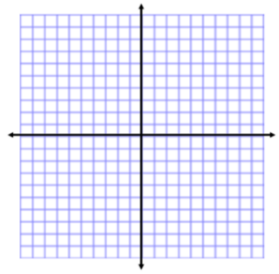
Assignment #1: FA: p 360 #1, 2, 4 & P 370 #7

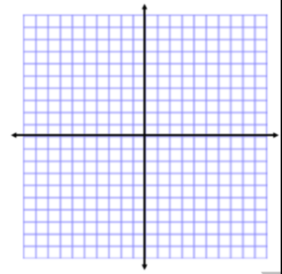
DAY 2 Notes.: Properties and Characteristics of Quadratic Functions Cont.

Concepts: **#5, 6**

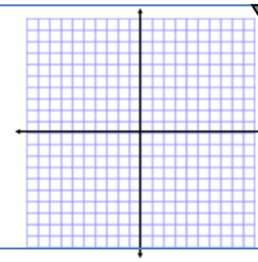
TERMS:

1. **VERTEX:**
   - An ordered pair (x, y) located at the top or bottom of the curve of a parabola
   - A vertex at the top of the parabola is called a MAXIMUM
   - A vertex at the bottom of the curve is called a MINIMUM

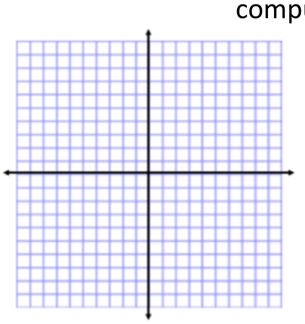


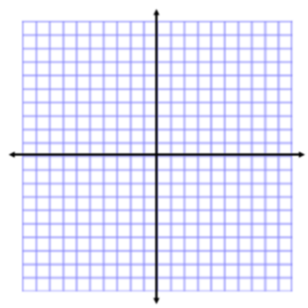


2. **AXIS OF SYMMETRY:**
   - An "imaginary" vertical line that goes through the vertex of a parabola and cuts it into two symmetrical halves
   - It is always written in equation form as x = The # that is the x coordinate of the vertex



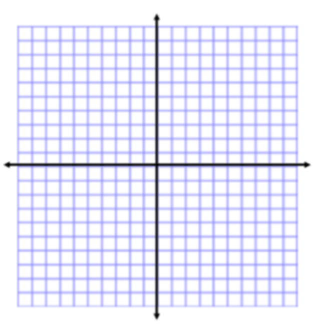
3. **DOMAIN:**
   - Describes the complete list of x values that the graph will cover/use when the entire graph is considered
   - Describes how far left and right the graph will spread
     - NOTE: When drawing a graph of a parabola that goes on forever, you must draw arrowheads. When given a textbook question or test question of a digitally drawn image of a parabola that extends to the edges of the graph you need to assume that there are arrowheads at the end (most computer programs will not allow them to be added on).

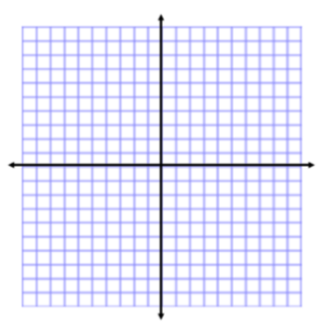




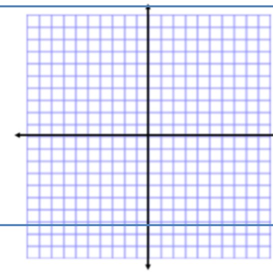
4. **RANGE:**
   - Describes the complete list of y values that the graph will cover/use when the entire graph is considered
   - Describes how down and up the graph will spread

   NOTE: Use the same arrowhead rule as described above in the domain.





5. Y INTERCEPT:

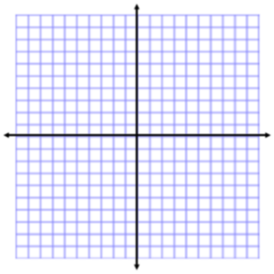
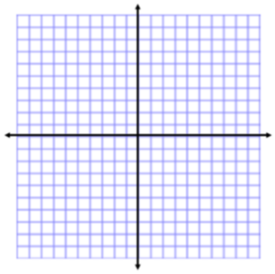
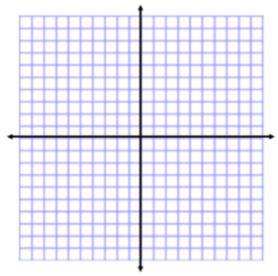


- Describes the coordinate where the graph crosses the y axis
- When a parabola is in standard form $y = ax^2 + bx + c$, the value of "c" will be the y intercept – the coordinates of the y intercept will then be (0, c)

6. X INTERCEPTS:

- Describes the coordinate(s) where the graph of the parabola crosses the x axis
- It is possible to have one, two or no x intercepts
- In this course, the following terms also mean the same thing as x intercepts (and I may use these words interchangeably)

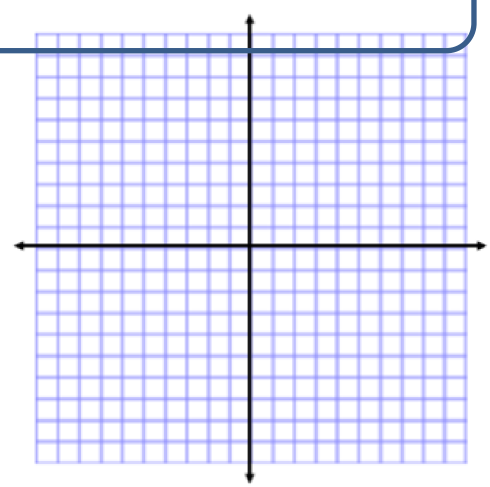
  X intercepts = Roots = Zeros = Solutions

EXAMPLE #1: Determine the vertex, the y intercept, the x intercept(s), the equation of the axis of symmetry, domain, range and sketch the following function: $y = -x^2+2x+8$

Method 1: Create a table of values, sketch the parabola and "read" the necessary information off of the graph. If you are not given values of x to use, choose a reasonable list and keep adding until your graph is a parabola in shape!

x	$y = -x^2+2x+8$	y	(x,y)

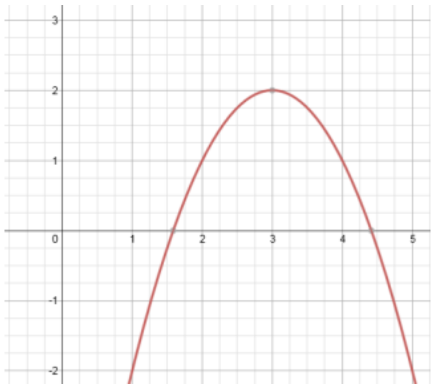


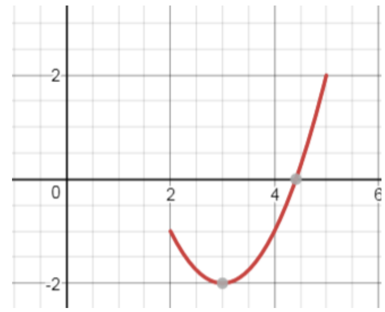
- VERTEX:
- Y Intercept:
- X Intercept(x):
- Axis of Symmetry:
- Domain:
- Range:

GO BACK TO THE NOTES FROM YESTERDAY AND FILL IN COLUMN THREE!

EXAMPLE #2: State whether the parabola has a maximum or minimum value. State its value. State the domain and range.

a) b)





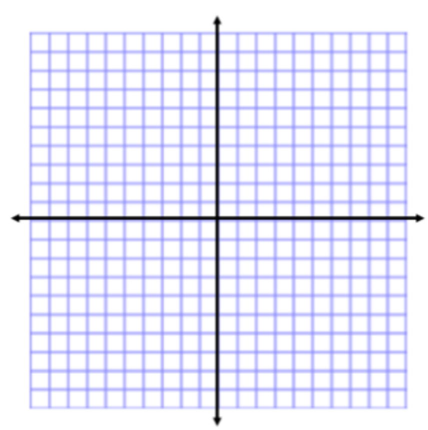
EXAMPLE #3: Predict whether the parabola's will have a maximum or minimum?

a) $y = -5x^2+8x+3$ b) $y = 7x^2+2x+5$

THE VALUE OF "a": **Given a parabola in the form of $y = ax^2 + bx + c$**

- If the value of "a" is positive, the parabola opens up and the vertex will be at the bottom and will be a MINIMUM
- If the value of "a" is negative, the parabola opens down, the vertex will be at the top and will be a MAXIMUM

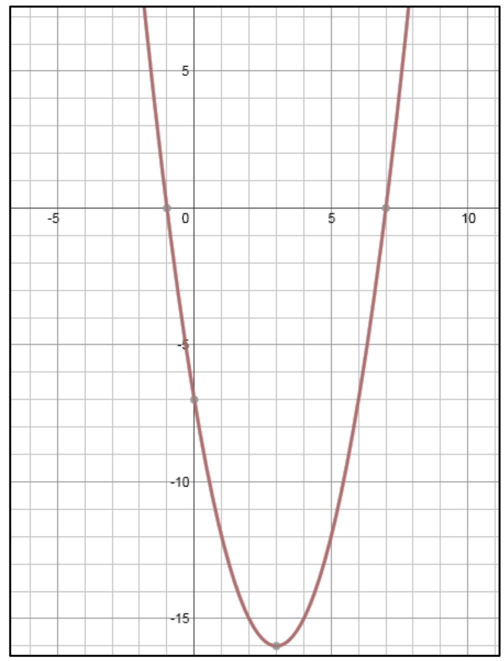
EXAMPLE #4: The points (4, 3) and (8, 3) lie on the same parabola. Sketch the points and predict the equation of the axis of symmetry.



Q: Can you think of a method to determine the axis of symmetry without graphing?

EXAMPLE #5:

Given the following graph, identify the equation of the axis of symmetry, the coordinates of the vertex, if there is a maximum or minimum value and what that value is, the domain, the range, the x intercept(s) and the y intercept.



axis of symmetry:

vertex:

maximum or minimum:

Value of max or min:

x intercept(s):

y intercept:

domain:

- If it had arrowheads:
- If it stops at the edge of the graph:

range:

- If it had arrowheads:
- If it stops at the edge of the graph:

Assignment #2: FA P 368 #1-6, 8-10, 11ab(create a suitable table of values)

DAY 3 NOTES : Quadratics Algebra Review

REVIEW OF EXPANDING BINOMIALS (FOIL/BINOMIAL BOB)

Concepts: #7

EXAMPLE #1: Expand and simplify the following quadratic functions so they are in the form $Y=ax^2+bx+c$.

a) $y = (x - 3)(x + 5)$

b) $y = (x - 4)^2 + 7$

c) $y = -3(x + 2)^2 - 12$

d) $f(x) = (2r + 5t)^2$

EXAMPLE #2: Which of the above quadratics open up and which open down? How do you know?

EXAMPLE #3: Find the y intercept of the following: $y = -2(x- 1)^2$

Review of Factoring using GCF, Inspection and Window Method

y = (x – 4)(x + 3) ⟷ **y = x²- x - 12**

FACTORED FORM

EXPANDED FORM

Note:

- **Factoring quadratics is the opposite of expanding(FOIL'ing/Binomial Bob). Factoring adds the brackets while FOIL/BB removes the brackets**

EXAMPLE #4: Greatest Common Factor

FACTOR THE FOLLOWING:

a) $y = x^2 + 2x$

b) $f(x) = -2x^2 - 8x$

c) $y = 6x^2 - 9x + 30$

d) $f(x) = -15x^2y^6 - 10x^3y^9 + 15x^{11}$

Trinomials Where a = 1

EXAMPLE #5: **F**actor the following:

a) $y = x^2 - 10x + 16$

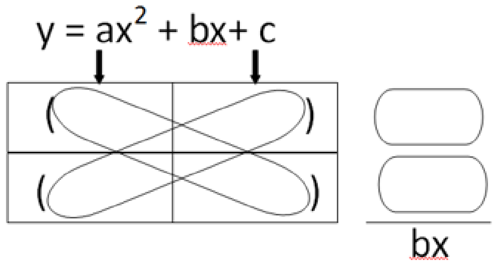
b) $y = x^2 + 2x - 8$

c) $y = x^2 + 3x - 10$

d) $f(x) = x^2 + 13x + 40$

Trinomials Where a ≠1 (You can use the following method, decomposition or trial and error)

FACTORING WHEN "A" IS A NUMBER OTHER THAN 1:



ANSWER WILL BE:

y = (   )(   )

EXAMPLE #6: Factor the following

a) $y = 2x^2 - 10x - 12$

b) $f(x) = -x^2 + 12x - 35$

c) $g(x) = 2x^2 + x - 3$

d) $y = 3x^2 - 20x - 7$

e) $y = 8x^2 - 18x - 5$

f) $y = 5x^2 + 7x - 6$

Difference of Squares

What do we need in order for a question to be a difference of squares?

-
-
-

EXAMPLE #7: Factor the following

a) $y = x^2 - 4$

b) $f(x) = 4x^2 - 25$

Putting it all Together

FACTORING FLOWCHART:

1. **ALWAYS CHECK FOR GCF FIRST, factor it out if there is one**
2. **Do you have ax^2+bx+c where $a = 1$? Do the "easy factoring"**
3. **Do you have ax^2+bx+c where a is NOT 1? Use the "Window" method of factoring**
4. **Do you have $ax^2- b$ where "a" and "b" are perfect squares? Use difference of squares**

EXAMPLE #8: Factor the following

a) $h(x) = 2x^2 - 10x - 12$

b) $y = -x^2 + 12x - 35$

c) $y = 2x^2 + 18x + 28$

d) $f(x) = 4x^2 - 100$

e) $y = -9x^2 + 48x + 36$

Assignment #3

1. Factor the following questions:

a)	$y = 15x^2 - 65x + 20$	b)	$g(x) = 18x^2 + 15x - 18$
c)	$y = 12x^2 - 52x - 40$	d)	$f(x) = 24x^2 - 2x - 70$
e)	$y = 4x^2 + 4x - 48$	f)	$y = -5x^2 + 40x - 35$
g)	$h(x) = -3m^2 - 18m - 24$	h)	$f(x) = 10x^2 + 80x + 120$
i)	$y = 7x^2 - 35x + 42$	j)	$y = 18x^2 - 2$
k)	$f(x) = 16x^2 - 1$	l)	$g(x) = -x^2 + 1$
m)	$y = 16x^2 - 81$	n)	$h(x) = 2 - 8x^2$

2. Factor fully . Use the strategy that you prefer.

a)	$9k + 6$	b)	$3x^2 – 6x^4$	c)	$-3c^2 – 13c^4 – 12c^3$
d)	$x^2 + 12x – 28$	e)	$y^2 – 2y – 48$	f)	$8a^2 + 18a – 5$
g)	$15a^2 – 65a + 20$	h)	$s^2 + 11s + 30$	i)	$2x^2 + 14x + 6$
j)	$3x^2 + 15x – 42$	k)	$15a^3 – 3a^2b – 6ab^2$	l)	$w^2 + 10w – 24$
m)	$3c^2d – 10cd – 2d$	n)	$f^2 + 17f + 16$	o)	$4t^2 + 9t – 28$
p)	$h^2 – 25j^2$	q)	$6x^2 – 17xy + 5y^2$	r)	$28a^2 – 7a^3$
s)	$25t^2 + 20tu + 4u^2$	t)	$3x^2 – 3x – 60$	u)	$18m^2 – 2n^2$

DAY 4 NOTES : Factored Form of a Quadratic Equation

$y = (x – 4)(x + 3)$ ⟷ **$y = x^2 - x - 12$**

FACTORED FORM

EXPANDED FORM

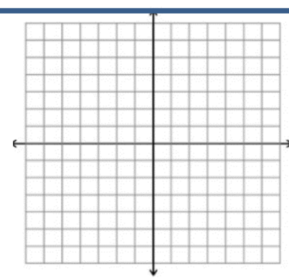
Concepts: #7

- **A quadratic in factored form is used to find the x intercepts (solve each bracket for 0)**
- **A quadratic in EXPANDED form is used to find the y intercept**

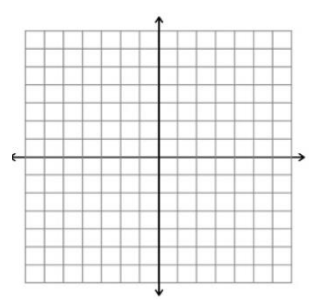
EXAMPLE #1: Find the x intercepts of the following by factoring using THE GREATEST COMMON FACTOR. Sketch the graph.

Step 1: An x-intercept is when y = 0, so set the function equal to 0.
Step 2: Factor out a GCF if there is one
Step 3: Solve for the x-intercepts.
Step 4: Find the vertex (algebraically) and the y intercept.
Step 5: Graph by sketching all the above points.

a) $y = x^2 – 3x$

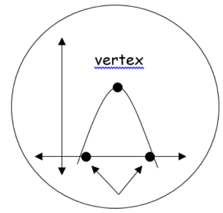


b) $f(x) = -2x^2 - 8x$

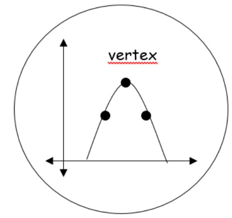


NOTE:

- If we want to sketch the graph of a quadratic function without using a table of values, we need to know <u>two points and the vertex</u>. Finding the x intercepts is usually the best two points to find. It is also helpful to have the y intercept as well (and pretty easy to find as well) .The following graphs SHOULD have arrowheads on them!



2 x-intercepts



2 symmetrical points

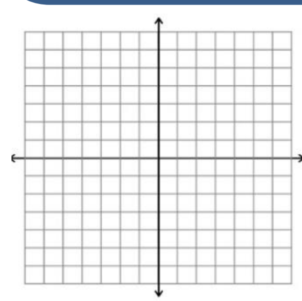
- Two symmetrical points we often use are the **x-intercepts**, which are also called the **<u>zeroes of the function</u>**, the **<u>roots of the function</u>** or the **<u>solutions of the function</u>**.

- Before you find the x intercepts/zero's/solutions/root's of any Quadratic function, ensure that you replace the y with zero and then ensure that all other terms are on one side and zero on the other BEFORE YOU FACTOR

EXAMPLE #3: Find the zero's of the following and sketch.

$y = x^2 - 10x + 16$

<u>Step 1:</u> An x-intercept/root/solution/zero is when y = 0, so set the function equal to 0.
<u>Step 2:</u> Factor your trinomial into two sets of brackets using your preferred method
<u>Step 3:</u> Solve for the x-intercepts.
<u>Step 4:</u> Find the vertex (algebraically) and the y intercept.
<u>Step 5:</u> Graph by sketching all the above points.



EXAMPLE #4:

Find the roots of the following quadratic functions:

a) $y = x^2 + 2x - 8$

b) $g(x) = 2x^2 - 10x - 12$

c) $y = -x^2 + 12x - 35$ d) $f(x) = 4x^2 - 25$

EXAMPLE #5:

Given the following quadratics that have already been factored and graphed, determine how you find the x intercepts from the factors.

a) $y = (2x - 1)(2x + 5)$ b) $y = -2(2x - 7)(4x - 1)$

What is the difference between the answers for the following questions:

a) Find the x intercept of $y = x^2 + 5x + 4$ b) Solve $0 = x^2 + 5x + 4$

Would it make sense to switch these questions so that they looked like this instead:

a) Solve $y = x^2 + 5x + 4$ b) Find the x intercepts of $x^2 + 5x + 4 = 0$

RULE: A question will ask you to "solve" if ________________________________

A question will ask you to find the x intercepts/the zero's/the roots if

____

EXAMPLE #6:

Solve the following. Verify your solutions in part "a".

a) $6x^2 - 13x - 5 = 0$ b) $-13x - 20 = 2x^2$

Assignment #4 FA: P 405 # 1-5 plus the following highlighted questions

1. **Factor each of the following quadratic equations. State the x-intercepts.**

a) $y = x^2 + 4x + 3$
b) $y = x^2 + 7x + 10$
c) $y = x^2 - 12x + 20$
d) $y = x^2 - 10x + 24$
e) $y = x^2 + 4x - 12$
f) $y = x^2 - 4x - 32$
g) $y = 5x^2 + 8x - 4$
h) $f(x) = 6x^2 - 20x + 6$
g) $y = -6x^2 - 11x + 10$

2. **Factor each of the following quadratic equations. State the x-intercepts.**

a) $y = x^2 - 25$
b) $y = x^2 - 16$
c) $y = x^2 - 64$
d) $y = x^2 - 100$
e) $y = 9x^2 - 100$
f) $y = 4x^2 - 9$

3. **Factor each of the following quadratic equations. State the x-intercepts.**

a) $y = x^2 + 8x$
b) $y = x^2 - 10x$
c) $y = 2x^2 + 10x$
d) $y = -3x^2 - 9x$
e) $y = -7x^2 + 49x$
**f) $y = 2x^2 + 6x + 4$

4. Using the x intercepts you have already found for the above questions, find the vertex, the y intercept and sketch the above quadratic functions.

1a, e, g

2a, e

3a,d

DAY 5 NOTES : WRITING QUADRATIC EQUATIONS IN STANDARD FORM USING THE INTERCEPTS

Concepts: #8

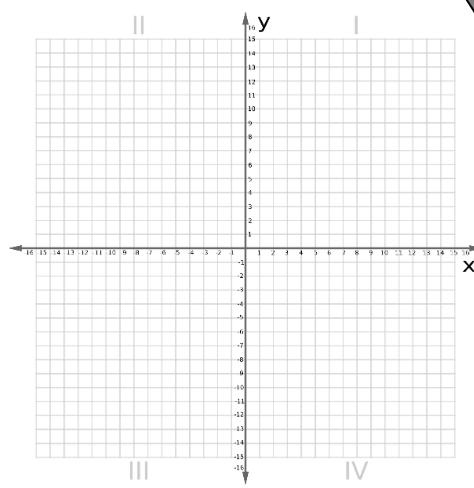
EXAMPLE #1: Given the quadratic $y = x^2 - 6x + 5$, find the x intercepts.

EXAMPLE #2: Given the quadratic $y = (x + 3)(2x - 5)$, find the following:

a) x intercepts
b) equation of the axis of symmetry
c) vertex

d) the y intercept

e) sketch

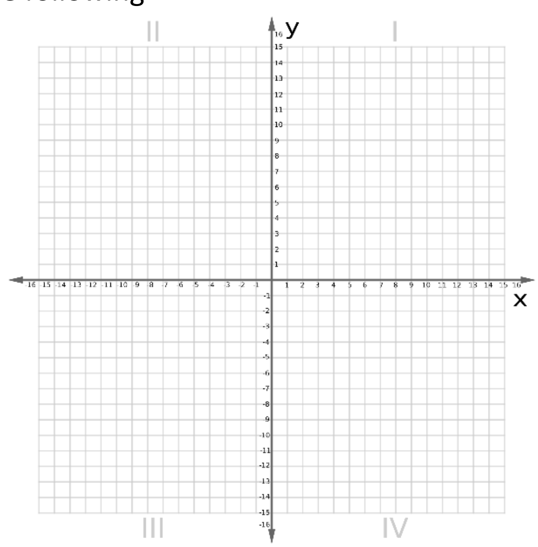


EXAMPLE #3: Given the quadratic $y = 3(x + 3)(2x - 5)$, find the following

a) x intercepts
b) equation of the axis of symmetry
c) vertex

d) the y intercept

e) sketch



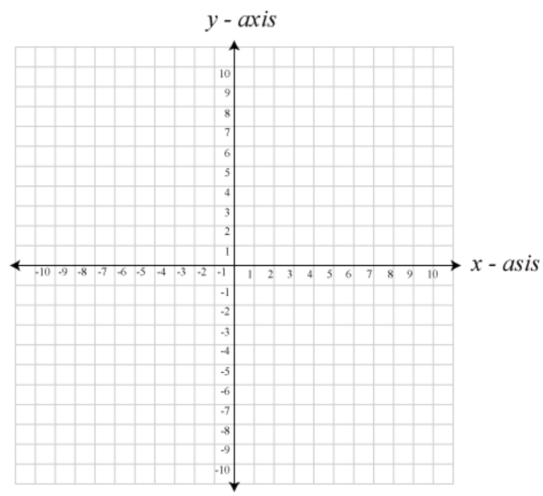
QUESTION: Does the "3" in front of the brackets in this example change anything about the value of the x intercepts, the equation of the axis of symmetry, the vertex, the y intercept or the sketch of the very similar quadratic given to you in example 2? Describe this change:

RULE: When a quadratic in the form $y = ax^2 + bx + c$ is factored and written in the form $Y = a(qx - r)(sx - t)$, then the following is true:

- The x intercepts are at x = $\boxed{}$ and x = $\boxed{}$
- The axis of symmetry will always be ______________________________ of the above two x intercepts
- The y intercept is the value of _____ of the equation in standard form. To find it infactored form you use: _______________
- The value of a affects the following characteristics of a parabola by:
  - Direction of opening:
  - Vertical Stretch affecting width:
  - X intercepts:
  - Axis of Symmetry:
  - Y intercept:
  - Vertex

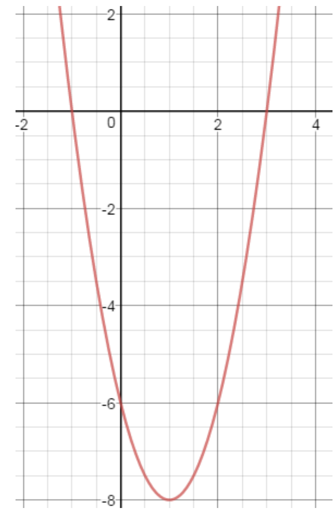
EXAMPLE #4: A quadratic has x intercepts of x = 8 and x = -2. Find an equation that would represent this quadratic.

- Draw a quick sketch of this parabola. Is there only one sketch possible?
- What value of "a" does your equation have?
- Can you write another quadratic equation with a different value of "a" that will satisfy the original conditions in the above example?
- How many quadratic equations can you write that would satisfy the original conditions?
- Can you write **THE** quadratic equation that satisfies the original conditions AND has a y intercept of -80?? ☺



- Can you write **THE** quadratic equation that satisfies the original conditions AND has a y intercept of 4?? Let's see who's awake in the class.......☺

EXAMPLE #5: Find **THE** equation to the following parabola.



EXAMPLE #6: Find **AN** equation of a parabola that has roots at $-\frac{4}{3}$ and 5.

EXAMPLE #7: Find **THE** equation of the parabola in standard form that has roots at $-\frac{4}{3}$ and 5 and passes through the point (-1, 30)

Assignment #5
FA: P 391 # 1, 2, 3, 4abde, 11ab, 13 P 398 #8-11
MLA: P391 #6, 7, 11cd, 12,

DAY 6 NOTES : THE QUADRATIC FORMULA

What we know so far:

Concepts: #9

Review:

- **SOLVING** a quadratic equation means to find the x intercepts of it. We also call this finding the roots, the zero's or the solutions.
- We can graph the quadratic to find the x intercepts
- We can factor the quadratic equation to find the x intercepts

PROBLEMS WITH THESE METHODS:

- Graphing by hand can be time consuming – we don't always know which values of x to start with in the table
- Graphing using the calculator can be fussy and a graphing calculator is not always available
- Some quadratics won't factor

NEW METHOD TO FINDING THE X INTERCEPTS:

Given a quadratic of the form y = ax^2+bx+c, the x intercepts/zero(s)/root(s)/Solution(s) can be found by using the following formula which is known as the **QUADRATIC FORMULA:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

http://www.youtube.com/watch?v=79F2QxpjBz0 (P.Gt.W) http://www.youtube.com/watch?v=hcFCttlIiec (B)

EXAMPLE #1:

Given the quadratic equation, solve y = $3x^2-5x-4$ using the quadratic formula.

a = ______, b = _______, c = _______

EXAMPLE #2:

Given $4x^2 - 3 = 7x$, find the zero's using the quadratic formula. (next year change to $9x^2 = 4 - 6x$ and add a

Question b) $x^2 - 5x = 14$)

EXAMPLE #3:

Find the roots for the quadratic $y = x^2+9x+23$ using the quadratic formula.

What does this tell you about the graph of the corresponding quadratic? Graph it to see….

EXAMPLE #3:

Solve the quadratic equation $2x^2 - 5x + 1 = 0$. State your solution as an EXACT ANSWER and to the nearest tenth. (next year change to $9x^2 + 6x - 4 = 0$ and add b) $5x^2 = 2$)

Add simplifying questions: a) $x = \frac{-4 \pm \sqrt{8}}{10}$ b) $x = \frac{3 \pm \sqrt{8}}{10}$ c) $x = \frac{14 \pm 3\sqrt{12}}{2}$

Assignment #6
FA: The following questions 1, 2, 3 & P428 1cd, 2bc
MLA: The following questions #3 & P428 #13

1) Use the quadratic formula to solve each of the quadratic equations in exact form and to the nearest tenth.

a. $x^2 - 2x - 2 = 0$ b. $x^2 - 4x - 3 = 0$ c. $x^2 + 6x + 7 = 0$

d. $x^2 + 1 = 4x$ e. $x^2 - 4x + 2 = 0$ f. $2x^2 = 8x - 5$

g. $2x^2 + 3x - 4 = 0$ h. $2x^2 - 2x - 3 = 0$ i. $6x^2 - 8x = 0$

2) Determine x-intercepts and the vertex for each of the following quadratic functions:

a) $y = -x^2 + 6x - 5$ b) $y = 2x^2 - 5x + 3$ c) $y = \frac{1}{3}x^2 - 2x + 3$

3) Use the quadratic formula to solve each of the following quadratic equations.

a) $\frac{1}{2}x^2 + 11x + 12 = 0$ b) $x^2 - 2\sqrt{2}x + 2 = 0$ c) $\sqrt{3}x^2 - 7x = -2\sqrt{3}$

DAY 7 NOTES : Applications of Quadratics in Standard Form

Today we learn how quadratic equations and their characteristics apply in real life situations.

Concepts: #10

EXAMPLE #1:

Ms. C takes her class to visit Cape Enrage. While standing on the cliff, she looks down and notices one of her students standing in the water below. Since Math is the solution to all problems, she tosses him a graphing calculator to SAVE HIS LIFE. The path or the parabola is given by the function:
$y = -4.9x^2 + 8x + 43$ where y is the height of the calculator and x is the time the calculator spends in the air.

a) What important points can we find that will help us graph the parabola? Find these points!

b) Graph the function. What part of the graph do we use?

c) How high is the cliff?

d) Is the calculator ever 27 m above the water?

e) When does the calculator hit the water?

f) EXTENSION: When is the calculator 45 m above the water?

g) Is the student alive? Who do you think it was?

EXAMPLE #2:

Ms. C takes her class to Mars for a class trip. Same kid as on the Cape Enrage trip tries to escape the spacecraft. Luckily – Ms. C is there to save the day! She tosses him a graphing calculator from the window of the Enterprise. The path it follows is given by the equation $h(t) = -1.89t^2+5t+7.5$. H is the height in metres of the calculator and t is the time in seconds.

a) Does this parabola open up or down?

b) What is the y intercept of this parabola?

c) What are the x intercept(s) of this parabola?

d) What is the vertex of this parabola?

e) Sketch

f) Ms. C wants to know how long it will take the Calculator to land. What point will this represent on the graph?

g) How high did the calculator go?

h) How high is the calculator at 5 seconds?

h) EXTENSION: When is the calculator 10 m above Mars?

EXAMPLE #3:

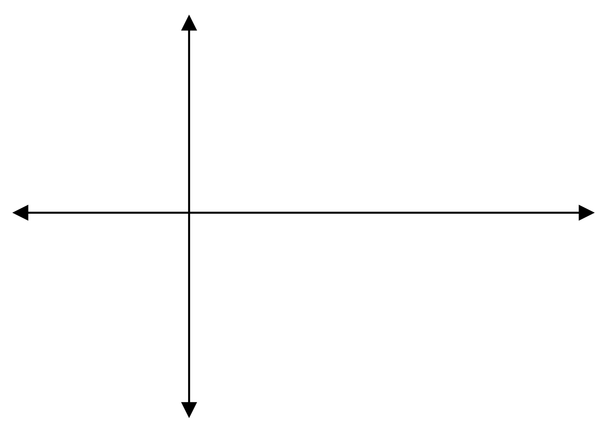
A ball is thrown into the air and follows the path given by $h(t) = -5t^2 + 20t + 1$, where *h* represents height and *t* represents time in seconds.

a) Determine ***the initial height*** of the ball.

b) Determine the ***vertex***.

c) Determine ***two symmetrical points*** for graphing purposes.

d) Sketch the path of the ball. (Label three key coordinates and the axes.)



e) What is the ball's ***maximum height***? ______

f) How long does it take for the ball to reach its ***maximum height***? ______

g) What is the height of the ball after 3 seconds?

EXAMPLE #4:

You are on the moon and hit a golf ball. The height of the ball h(t) over t seconds is given by the equation $h(t) = -0.81t^2 + 5t$

a) When is the ball on the ground?

b) What is the maximum height? When does this happen?

c) What is the domain and range?

d) How high is the calculator at 2 seconds?

DAY 7 USING THE GRAPHING CALCULATOR : Applications of Quadratics in Standard Form

Answer the following questions using the method indicated – only graphing calculator, only algebra or using both methods. Show answers to the nearest thousandth.

1. GRAPHING CALCULATOR

The profits of Mr. Unlucky's company can be represented by the equation $y = -3x^2 + 18x - 4$, where y is the amount of profit in hundreds of thousands of dollars and x is the number of years of operation. He realizes his company is on the downturn and wishes to sell before he ends up in debt.

- When will Unlucky's business show the maximum profit?
- What is the maximum profit?
- At what time will it be too late to sell his business? (When will he start losing money?)

2. GRAPHING CALCULATOR

Ex 9. Jocelyn and Kelly built rockets from assembly kits and are going to launch them at the same time to see whose rocket flies higher. If Jocelyn's rocket's height, in feet, can be described by the equation $J(x) = -16x^2 + 180x$ while Kelly's is represented by $K(x) = 16x^2 + 240x$.

- Who wins the rocket race? (What is the max height for both rockets?)
- After how many seconds does each rocket land?
- To the nearest tenth of a second, what was the difference in time for the two different rockets to reach their respective max heights?

3. GRAPHING CALCULATOR & ALGEBRAICALLY

A ball rolls down a slope and travels a distance $d = 6t + \frac{t^2}{2}$ feet in t seconds. Find when the distance is 17 feet.

4. GRAPHING CALCULATOR

The number of horsepower needed to overcome a wind drag on a certain automobile is given by $N(s) = 0.005s^2 + 0.007s - 0.031$, where s is the speed of the car in miles per hour. How much horsepower is needed to overcome the wind drag on this car if it is traveling 50 miles per hour? At what speed will the car need to use 200 horsepower to overcome the wind drag?

5. GRAPHING CALCULATOR & ALGEBRAICALLY

The number of board feet in a 16 foot long tree is approximated by the model $F(d) = 0.77d^2 - 1.32d - 9.31$ where F is the number of feet and d is the diameter of the log. How many board feet are in a log with diameter 12 inches? What is the diameter that will produce the minimum number of board feet?

6. GRAPHING CALCULATOR

For the years of 1983 to 1990, the number of mountain bike owners *m* (in millions) in the US can be approximated by the model $m = 0.337t^2 - 2.265t + 3.962,\ 3 \le t \le 10$ where $t = 3$ represents 1983. In which year did 2.5 million people own mountain bikes? In what year was the number of mountain bike owners at a minimum?

7. GRAPHING CALCULATOR

The path of a high diver is given by $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 10$ where y is the height in feet above the water and x is the horizontal distance from the end of the diving board in feet. What is the maximum height of the diver and how far out from the end of the diving board is the diver when he hits the water?

QUADRATICS Assignment #7

TEXTBOOK PORTION: P 371: #12-15

P 394 #17, 19 or 20

P 407 #13, 14 (BY FACTORING)

P 428 #7, 8, 9, 10

P 380 (USING THE GRAPHING CALCULATOR) #7, 9

DAY 8 NOTES : Vertex Form of a Quadratic Function

Concepts: #11, 12

https://teacher.desmos.com/marbleslides-parabolas
https://student.desmos.com

- REVIEW: A quadratic in **STANDARD FORM** looks like

A Quadratic in **VERTEX FORM**:

- There are two different v ersions of what vertex form can look like (depending on the textbook). Our textbook uses the following:
  $y = a(x - h)^2 + k$ Other textbooks use this version: **$y = a(x - p)^2 + q$**
- The value of "a" affects the graph of the parabola in the following way(s):

- The value of h (or p) affects the graph of the parabola in the following way(s):

- The value of k (or q) affects the graph of the parabola in the following way(s):

EXAMPLE #1: Given the function $f(x) = -2(x - 1)^2 + 8$, determine the following:

a) Vertex

b) Equation of the axis of symmetry

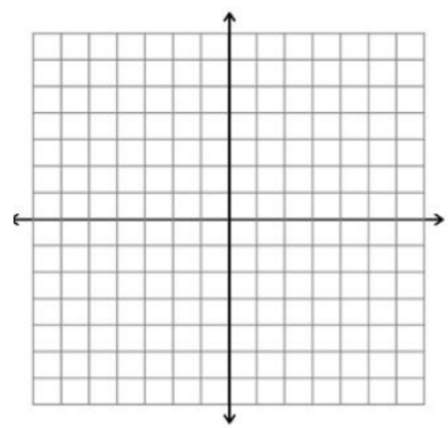
c) Direction of Opening and Max/Min

c) The y intercept

d) The x intercept(s)

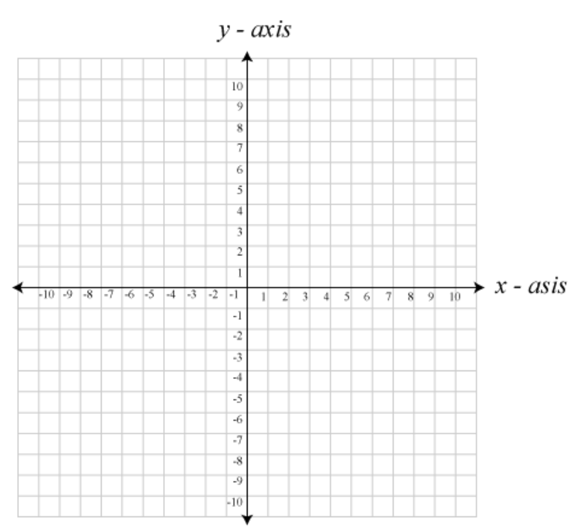
EXAMPLE #2: Sketch the function $y = 3(x + 2)^2 - 1$ using all the appropriate points (this will always be the vertex, y intercept and it's mirror point, the x intercept(s)). State the domain and range.

NOTE: you should always show how you got your points and what your points are as ordered pairs in addition to graphing them!



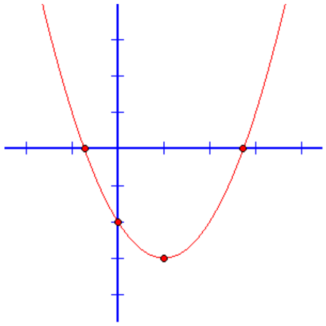
EXAMPLE #3: Complete the equation of the following parabola by filling in the square so that it doesn't have any x intercepts. Graph the parabola and state the domain and range.

$Y = -(x - 5)^2 \boxed{}\ 3$

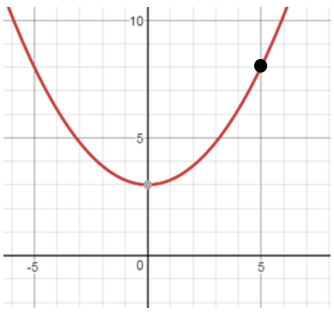


EXAMPLE #3: Find the equation of the following parabolas in vertex form.

a)



b)



EXAMPLE #4:

Given the equation $y = x^2 + 4$. If the graph is shifted down 2 units, which equation describes the new graph?

a. $y = x^2 + 6$
b. $y = x^2 + 2$
c. $y = (x - 2)^2 + 2$
d. $y = (x + 2)^2 + 4$
e. $y = (x - 2)^2 + 4$

EXAMPLE #5:

Given $y = -2(x - 3)^2$. If the function is shifted 8 units to the right and 3 units up, write an equation that describes the new function.

EXAMPLE #6: A toy rocket is shot up in the air from a hill. Its height in meters above ground, *h*, is recorded after *t* seconds. The path the rocket follows is given by the following equation:

$h(t) = -4(t-6)^2 + 149$

a) Write the function in $y = ax^2 + bx + c$ form.

b) What is the initial height of the rocket?

c) Sketch the path of the rocket. (Label your sketch)

d) When will the rocket reach its maximum height?

e) What is the maximum height of the rocket?

f) How long does the toy rocket remain in the air for?

EXAMPLE #7:

A cable that hangs between two telephone poles makes a parabola shape that has the equation $y = 12(x-5)^2 - 18$, where x and y are measured in feet. If the cable is attached to both poles at a height of 54 feet and the lowest point of the cable is 36 feet above the ground, how far away are the poles from each another?

QUADRATICS Assignment #8

P 417 #1-5, 7-15 At least one of 17-19 And the following:

1. Given $y - 1 = 2(x + 1)^2$. If the equation is shifted left 5 units, which equation describes the new graph?
   a. $y - 6 = 2(x - 4)^2$
   b. $y - 1 = 2(x - 4)^2$
   c. $y - 1 = 2(x + 6)^2$
   d. $y - 1 = 2(x + 5)^2$
   e. $y - 1 = 2(x - 6)^2$
2. If the given function $y = (x - 1)^2 + 3$ is shifted up 3 units and left 4 units, which equation describes the new graph?
   a. $y = (x - 4)^2 + 3$
   b. $y - 3 = (x + 4)^2$
   c. $y - 7 = (x + 2)^2$
   d. $y = (x - 5)^2$
   e. $y = (x + 3)^2 + 6$
3. If the given function $y = (x + 3)^2 + 4$ is shifted down 5 units, which equation describes the new function?
   a. $y = (x + 3)^2 + 9$
   b. $y = (x + 3)^2 - 1$
   c. $y = (x + 8)^2 + 4$
   d. $y = (x - 2)^2 + 4$
   e. $y = - 5(x + 3)^2 + 4$

4. Given $y = x^2$. If the function is shifted 4 units to the left, write an equation that describes the new function.

ADDITIONAL QUESTIONS:

1. Expand the following quadratic functions into $y = ax^2 + bx + c$.

   a) $y = (x+1)^2 - 6$ b) $y = 3(x-2)^2 + 4$ c) $y = -2(x-3)^2 + 1$

2. A toy rocket is shot up in the air from a hill. Its height in meters above ground, *h*, is recorded after *t* seconds. The path the rocket follows is given by the following equation: $h(t) = -4(t-6)^2 + 149$

   a) Write the function in $y = ax^2 + bx + c$ form.
   b) What is the initial height of the rocket?
   c) Sketch the path of the rocket. (Label your sketch)
   d) When will the rocket reach its maximum height?
   e) What is the maximum height of the rocket?
   f) How long does the toy rocket remain in the air for?

3. An airplane flies at an altitude given by $y = -x^2 - x + 5006$, where x is the time in seconds before and after reaching the airfield, and y is the altitude in metres. At what times will the plane reach an altitude of 5000 m to allow two skydivers to jump out?

4. The daily revenue *y* dollars at an outdoor ice rink is given by the equation $y = -10x^2 - 400x + 2000$, where *x* represents the temperature in degrees Celsius.

   a) At **what temperature** will the daily revenue reach its maximum?
   b) Which temperatures will result in **no daily revenue**?

5. Jack owns a burger stand at a football stadium. He is looking at what his price should be this season (on his burger and fry combo) in order to maximize profit. The relationship between the profit(p) you make and the price you charge(c) is given by:

   $$p(c) = -10c^2 + 220c - 200$$

   ***Let the x-axis represent the charge and the y-axis represent the profit.

   a) Calculate the x and y intercept(s) and the vertex. Do a rough sketch.
   b) State the domain and range.
   c) What price should he charge for the burger and fry combo in order to maximize his profit each game?
   d) How much profit would he lose if he decided to charge $15 instead?
   e) Why is the y-intercept a negative? What does it mean?

Assignment: P 345 1ab, 2ab, 3, 4ac, 6ab, 8

DAY 3 SOLUTIONS :

1.

a)	$y = 5(x-4)(3x-1)$	b)	$g(x) = 3(2x+3)(3x-2)$	c)	$y = 4(3x+2)(x-5)$
d)	$f(x) = 2(3x+5)(4x-7)$	e)	$y = 4(x+4)(x-3)$	f)	$y = -5(x-7)(x-1)$
g)	$h(x) = -3(x+4)(x+2)$	h)	$f(x) = 10(x+6)(x+2)$	i)	$y = 7(x-3)(x-2)$
j)	$y = 2(3x-1)(3x+1)$	k)	$f(x) = (4x-1)(4x+1)$		

l) $g(x) = -(x-1)(x+1)$ or $g(x) = (1-x)(1+x)$ m) $y = (4x-9)(4x+9)$

n) $h(x) = 2(1-2x)(1+2x)$

2.

a)	$3(3K+2)$	b)	$3x^2(1-2x^2)$	c)	$-c^2(3+13c^2+12c)$
d)	$(x+14)(x-2)$	e)	$(y-8)(y+6)$	f)	$(4a-1)(2a+5)$
g)	$5(3a-1)(a-4)$	h)	$(s+5)(s+6)$	i)	$2(x^2+7x+3)$
j)	$3(x+7)(x-2)$	k)	$3a(5a^2-ab-2b^2)$	l)	$(w+12)(w-2)$
m)	$d(3c^2-10c-2)$	n)	$(f+16)(f+1)$	o)	$(4t-7)(t+4)$
p)	$(h-5j)(h+5j)$	q)	$(3x-y)(2x-5y)$	r)	$7a^2(4-a)$
s)	$(5t+2u)^2$	t)	$3(x-5)(x+4)$	u)	$2(3m-n)(3m+n)$

DAY 6 SOLUTIONS

#1 (all decimal answers are approximate)

a) x = 2.7 x = -0.7 b) x = 4.6 and x = -0.6 c) x = -1.6 and x = -4.4 d) x = 3.7 and x = 0.3
e) x = 3.4 and x = 0.6 f) x = 3.2 and x = 0.8 g) x = 0.9 and x = -2.4 h) x = 1.8 and x = -0.8
i) x = 0 and x = 4/3

#2

a) x = 1 and x = 5; vertex: (3, 4) b) x = 1.5 and x = 1; vertex: (1.25, -0.125) c) x = 3 and x = 3; vertex: (3, 0) hmm… ☺

#3 (all decimal answers are approximate) – NOTE: These answers are not written to the three decimal AP standard

a) x = -20.8 and x = -1.2 OR $x = -11 \pm \sqrt{97}$

b) x = 1.4 OR $x = \sqrt{2}$

c) a) x = 3.5 and x = 0.6 OR $x = 2\sqrt{3} \;\; and \;\; \frac{\sqrt{3}}{3}$

VIDEOS THAT MAY SUPPORT YOUR LEARNING

DAY 1: Exploring Quadratic Relations

https://goo.gl/AKHxhS
https://goo.gl/chCXZp

DAY 2: Properties of Graphs of Quadratic Functions

https://goo.gl/XgJgbY
https://goo.gl/FFvAMr

DAY 3: Review of Factoring

https://goo.gl/B53bR4
https://goo.gl/nHqD5u
https://goo.gl/hgsS9v
https://goo.gl/MnDbGP
https://goo.gl/s6z77x
https://goo.gl/TRDG7p
https://goo.gl/EuzL8w

DAY 4: Solving Quadratics by Factoring

https://goo.gl/eAnnAe
https://goo.gl/eAnnAe

DAY 5:
DAY 6: Solving Quadratics Using the Quadratic Formula

https://goo.gl/yRDzUp
https://goo.gl/FqwVeE

DAY 7:
DAY 8: Vertex Form of Quadratic Functions

https://goo.gl/XAc2nu
https://goo.gl/SxAbdF
https://goo.gl/AGNdWS