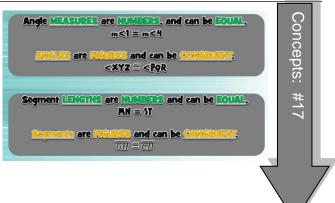
## PreAP FDN 20 2.1 PARALLEL LINES & 2.2 ANGLES IN PARALLEL LINES

**Congruent Vs Equal:** Congruence is a relationship of shapes and sizes, such as segments, triangles, and geometrical figures, while equality is a relationship of sizes, such as lengths, widths, and heights. Congruence deals with objects while equality deals with numbers. You don't say that two shapes are equal or two numbers are congruent.



#### **Supplementary Angles:**

#### **Complimentary Angles:**

#### **DO**: Draw 2 parallel lines.

"How do you know they are parallel?"

- Draw a line crossing both parallel lines (transversal)
- Label all angles 1-8
- "What angles are the same? What angles are supplementary?
- Make some conjectures about these angles

#### **Definitions:**

1) Transversal:

2) Vertically Opposite Angles

**THEOREM A:** In a pair of intersecting lines the vertically opposite angles are \_\_\_\_\_\_and will have \_\_\_\_\_\_measures

THEOREM B:	When a transversal intersects two parallel lines, Alternate Interior angles are	and will
have	measures	

#### 4) Exterior Angles:

**THEOREM C:** When a transversal intersects two parallel lines, Alternate Exterior angles are \_\_\_\_\_ have

and wi

5) Corresponding Angles:

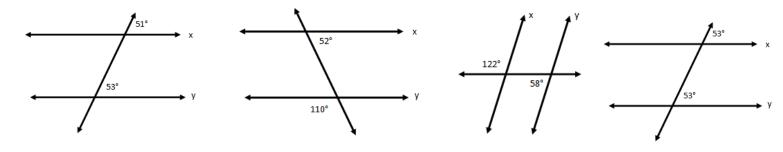
ſ	THEOREM D:	When a transversal intersects two parallel lines, Corresponding angles are	and will
	have		

- Converse of Theorem D:
  - When a transversal intersects a pair of lines creating equal corresponding angles , the two lines are parallel.
  - When a transversal intersects a pair of nonparallel lines the corresponding angles are not equal ( and vice versa)

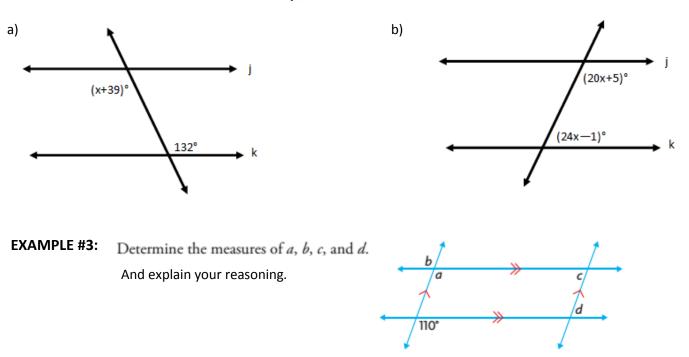
#### 6) Same-Side Interior Angles

**THEOREM D:** When a transversal intersects two parallel lines, the Same Side Interior Angles are \_\_\_ meaning that they add to \_\_\_\_\_\_

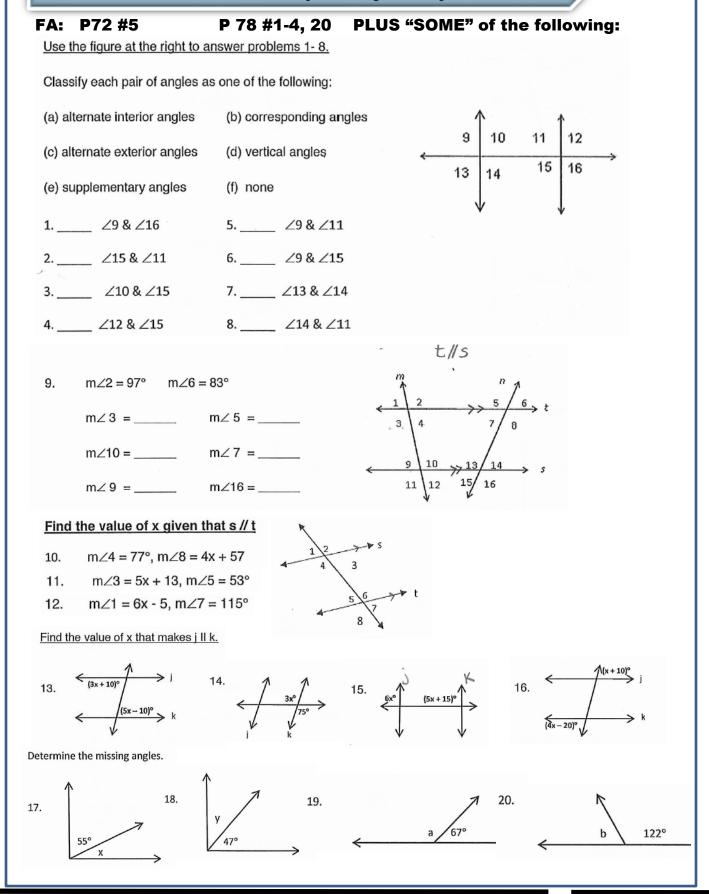
**EXAMPLE #1:** : Based on the given info are lines x and y parallel? Why or why not?



EXAMPLE #2: : Find the value of x that makes j // k



2.1 & 2.2 ASSIGNMENT #1 (Concept #17)



*Foundations 20(Ms. Carignan)* FM20.4 Parallel Lines & Polygons (Ch 2) F

## PreAP FDN 20 REFERENCE SHEET: THEOREMS & POSTULATES FOR PROOF

1.	GIVEN
2.	<b>DEFINITION OF PERPENDICULAR LINES:</b> Two lines that meet or intersect to form right angles.
3.	<b>DEFINITION OF RIGHT ANGLE:</b> An angle with measure 90°.
4.	<b>DEFINITION OF ANGLE BISECTOR:</b> The bisector of $\angle ABC$ is a ray BD in the interior of $\angle ABC$ such that $\angle ABC = \angle DBC$ .
5.	<b>DEFINITION OF SEGMENT BISECTOR:</b> A line, segment, ray, or plane that intersects the segment at its midpoint.
6.	<b>DEFINITION OF PERPENDICULAR BISECTOR OF A</b> <b>SEGMENT:</b> A line, ray or segment that is perpendicular to the segment at its midpoint.
7.	<b>DEFINITION OF ALTITUDE OF A TRIANGLE:</b> The perpendicular segment from a vertex to the line containing the opposite side.
8.	<b>DEFINITION OF MEDIAN OF A TRIANGLE:</b> A segment from a vertex to the midpoint of the opposite side.
9.	<b>DEFINITION OF MIDPOINT OF A SEGMENT:</b> The point that divides the segment into two congruent segments.
10.	<b>DEFINITION OF COMPLEMENTARY ANGLES:</b> Two angles whose measure have the sum 90°.
11.	<b>DEFINITION OF SUPPLEMENTARY ANGLES:</b> Two angles whose measures of the sum 180°.
12.	If two adjacent angles form a straight line, they are supplementary.
13.	Supplements of congruent angles are congruent.
14.	DEFINITION OF BISECTOR OF A VERTEX ANGLE OF AN ISOCELES TRIANGLE: The bisector of the vertex angle of an isoceles triangle is perpendicular to the base at its midpoint.
15.	Right angles are congruent.
16.	If two lines form congruent adjacent angles, the angles are perpendicular.

17.	<b>SEGMENT ADDITION:</b> If B is between A and C, then $AB + BC = AC$
18.	ANGLE ADDITION: Two adjacent angles can be added to form a new angle whose measure is the sum of the two.
19.	The sum of the measures of the angles of a triangle is 180°.
20.	Vertically opposite angles are congruent.
21.	Each angle of an equiangular or equilateral triangle has a measure of $60^{\circ}$ .
22.	If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
23.	<b>ISOCELES TRIANGLE THEOREM:</b> If two angles of a triangle are congruent, then the sides opposite those angles are congruent.
24.	<b>DEFINITION OF A RIGHT TRIANGLE:</b> A right triangle is a triangle containing a right angle and two acute angles.
25.	ADDITION PROPERTY OF EQUALITY: If $a = b$ and $c = d$ then $a + c = b + d$
26.	SUBTRACTION PROPERTY OF EQUALITY: If $a = b$ and $c = d$ then $a - c = b - d$
27.	<b>MULTIPLICATION PROPERTY OF EQUALITY:</b> If a = b then ca = cb
28.	DIVISION PROPERTY OF EQUALITY: If $a = b$ and $c \neq 0$ then $\frac{a}{c} = \frac{b}{c}$
29.	<b>REFLEXIVE PROPERTY OF EQUALITY:</b> a = a (the size of an angle or segment is equal to itself)
30.	SYMMETRIC PROPERTY OF EQUALITY: If $a = b$ , then $b = a$ .
31.	<b>TRANSITIVE PROPERTY OF EQUALITY:</b> If $a = b$ and $b = c$ then $a = c$ .
32.`	<b>SUBSTITUTION PROPERTY OF EQUALITY:</b> If a = b, then either a or b may be substituted for the other in any equation or inequality.

33.	If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
34.	If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
35.	If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
36.	If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.
37.	SSS CONGRUENCE POSTULATE: If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
38.	<b>SAS CONGRUENCE POSTULATE:</b> If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
39.	ASA CONGRUENCE POSTULATE: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
40.	AAS CONGRUENCE POSTULATE: If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent.
41.	HL CONGRUENCE POSTULATE: If the hypotenuse and one leg of one right triangle are congruent to the corresponding parts of another right triangle, the two triangles are congruent.
42.	Corresponding parts of congruent triangles are congruent.

## PreAP FDN 20 2.1 & 2.2 ANGLE & PARALLEL LINE PROOFS

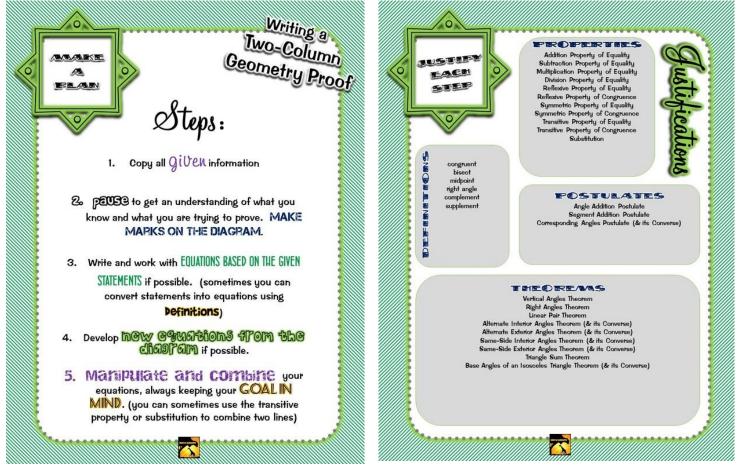
#### TWO COLUMN PROOFS REVIEW

Two column proofs are similar to proving that someone in court is telling the truth.

- A series of **STATEMENTS** are made that will go towards proving that either the mathematical concept or the person on trial is telling the truth.
- Each statement must be backed up with a **REASON** in the same way that each statement in court must have a corroborating witness.
  - Even the obvious facts must be presented in a statement and backed up with a reason.
- The last statement in a proof is always what is being proven.

If there is not a reason given to back up a statement, the truth of the statement is left in doubt. If the statements and reasons listed lead to a different conclusion than the one attempting to be proven, the original assumption of truth cannot be determined.

Every mathematical rule and theorem and postulate that is learned in school, has been proven to be true. Many of these were proven to be true using the two column proof format.



Concepts:

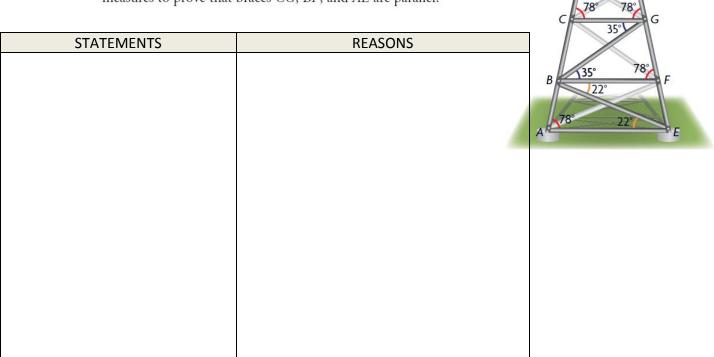
#18

**EXAMPLE #1:** Use a two column proof to deductively prove that alternate interior angles of parallel lines are equal.

STATEMENTS	REASONS

**EXAMPLE #2:** Use a two column proof to deductively prove that same side interior angles of parallel lines are supplementary.

STATEMENTS	REASONS



D

**EXAMPLE #3:** One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces CG, BF, and AE are parallel.

EXAMPLE #4: Use a two column proof to deductively prove that ST = TR

Given:  $QR \parallel ST$   $\angle QRS = \angle TRS$ Prove: ST = TR

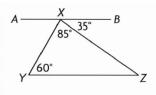
R S

	· ·
STATEMENTS	REASONS

#### EXAMPLE #5:

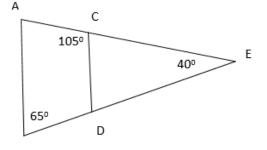
Joelle wrote this proof that  $AB \parallel YZ$ . Identify and correct her errors.

 $\angle AXY + \angle YXZ + \angle BXY = 180^{\circ}$ supplementary angles  $\angle AXY + 85^{\circ} + 35^{\circ} = 180^{\circ}$  $\angle AXY = 60^{\circ}$  $\angle AXY = \angle YXZ$ corresponding angles



Therefore,  $AB \parallel YZ$ .

**EXAMPLE #6:** Prove that AB//CD.



D
D

STATEMENTS	REASONS

## 2.1 & 2.2 ASSIGNMENT #2 (Concept #18)

#### P79 #8,10, 12, 15, 16, 18 P 78 #1-4, 20 FA:

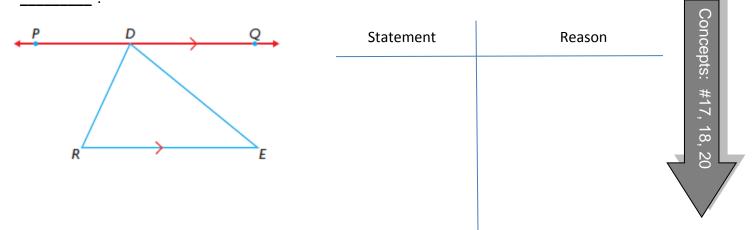
MLA: P79 #9, 17, 19,

Helpful hints for the assignment: An isosceles triangle has two sides of equal length and two equal angles.

If a line bisects an angles it divides the angle into two equal angles.

Recall: All interior angles of a triangle add up to \_\_\_\_\_

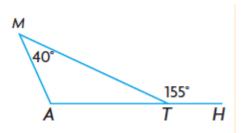
**EXAMPLE #1:** prove, deductively, that the sum of the measures of the interior angles of any triangle is



**Note: Exterior angles** are formed by **extending** a side of a polygon. For example, extend one side of this triangle to make an exterior angle:



**EXAMPLE #2:** In the diagram, angle MTH is an exterior angle of  $\Delta$ MAT. Determine the measures of the unknown angles in  $\Delta$ MAT.

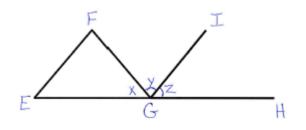


What relationship do you notice about angle AMT, angle MAT and the exterior angle MTH?

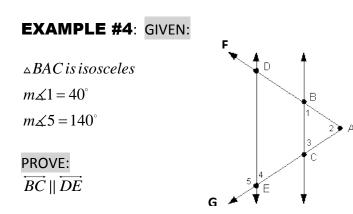
**EXAMPLE #3:** Prove deductively using a two column proof that an exterior angle of a triangle is equal to the sum of the two non- adjacent sides.

Statement Justification d c b

**EXAMPLE #4:** In  $\triangle EFG, GI$  bisects  $\measuredangle FGH$ a) If  $\measuredangle E = \measuredangle y$ , prove GI // EF.



b) Prove EF // GI, if  $\measuredangle F = \measuredangle z$ 



# FA: P90 #2, 3, 5, 7, 9, 19-12, 14, 15 PLUS the following proofs MLA: P90 #16, 18

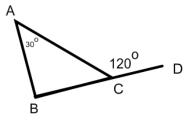
1. GIVEN: a $35^{\circ}$  b $120^{\circ}$  d

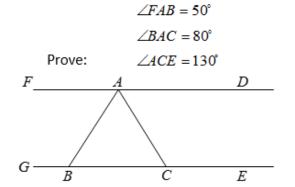
PROVE:  $\measuredangle a \cong 25^{\circ}$ 

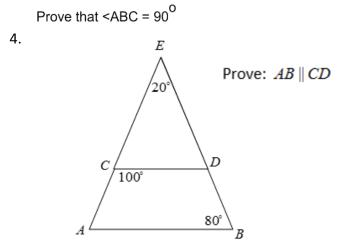
STATEMENTS	REASONS	
1.	1. Given	
	2. Given	
	3. Given	
$4.\measuredangle c \cong \_\_\_^{\circ}$	4.	
5.	5. Corresponding $\measuredangle$ 's of $\parallel$ lines are $\cong$	
6. $\measuredangle a \cong 25^{\circ}$	6.	

2. Given:







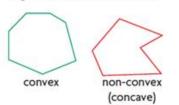


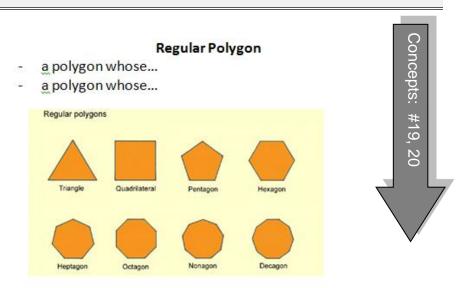
## PreAP FDN 20 2.4 ANGLE PROPERTIES IN POLYGONS

#### Types of Polygons:

#### convex polygon

A polygon in which each interior angle measures less than 180°.





What is the interior angle sum of **any triangle**? \_\_\_\_\_ (we proved this last section)

What will the 4 interior angles of any quadrilateral always add to? The 5 interior angles of a pentagon? Let's investigate:

Polygon	# of Sides	# of Triangles	Sum of Interior Angle Measures
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
$\bigcirc$			
Octagon			

Sum of the Interior Angles of a Convex	Polygon:
Given that n = the number of sides	Interior Angle Sum =
Measure of each Interior Angle of a REC	GULAR Convex Polygon:
Given that n = the number of sides of equal length	Each Interior Angle =

#### EXAMPLE #1:

a) What would be the sum of the measures of the interior angles of a regular dodecagon (12-sides)?

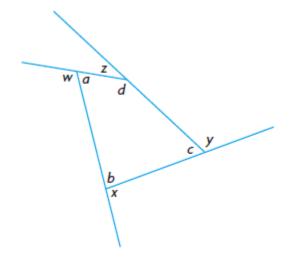
b) Determine the measure of each interior angle of a regular dodecagon?



### Measure of each EXTERIOR Angle of a REGULAR Convex Polygon:

Given that n = the number of sides of equal length Each Interior Angle =

**EXAMPLE #2**: Deductively prove that the sum of the exterior angles of any polygon will be 360<sup>0</sup>



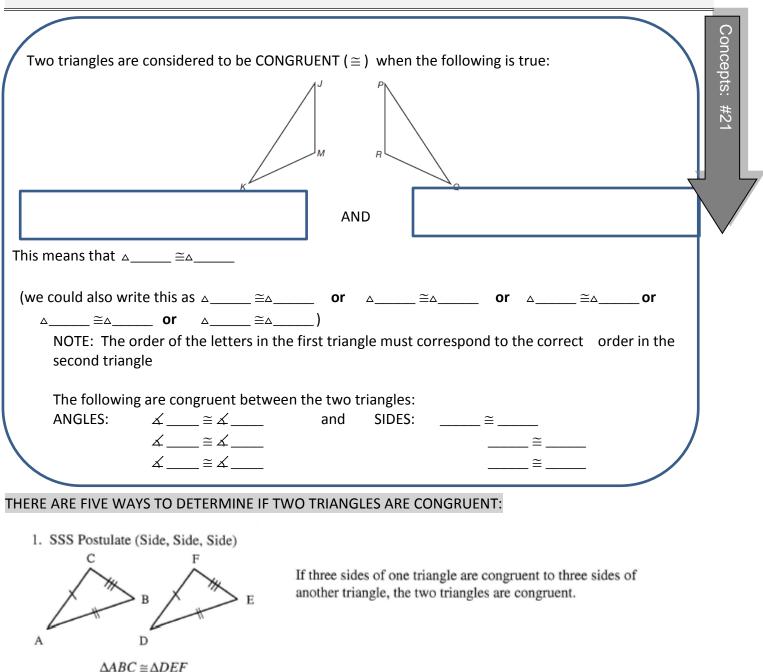
**EXAMPLE #3:** : Bob is tiling his floor. He uses regular hexagons and regular triangles. The side length of a triangle is equal to the side length of a hexagonal tile. Can he tile the floor without leaving any gaps between tiles? (Concept 20)

**EXAMPLE #3:** Kieran drew a 14 sided convex polygon. One of the interior angle measures 155<sup>0</sup>, Is it a regular polygon?

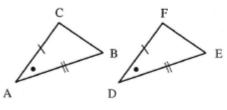
## 2.4 ASSIGNMENT (Concept #19, 20)

FA: P99 #1, 2, 37, 10, 16, 18
MLA: P99 #13
ULA: P99 #20, 21

## PreAP FDN 20 2.5 CONGRUENT TRIANGLES (Not in Textbook)

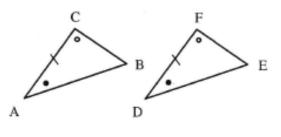


2. SAS Postulate (Side, Angle, Side)



 $\Delta ABC \cong \Delta DEF$ 

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the two triangles are congruent. 3. ASA Postulate (Angle, Side, Angle)

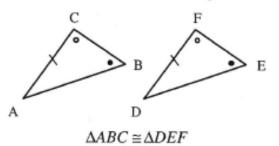


 $\Delta ABC \cong \Delta DEF$ 

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the two triangles are congruent.

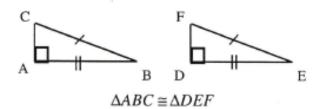
Note: The side must be in the middle.

AAS Postulate (Angle, Angle, Side)



If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, the two triangles are congruent.

5. HL Postulate (Hypotenuse, Leg)



If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, the two triangles are congruent.

В

В

B

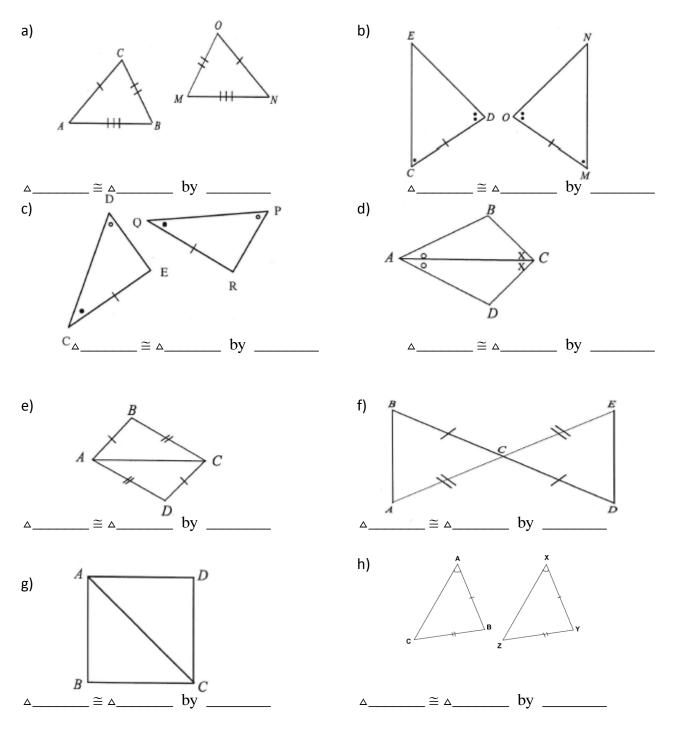
**Note:** Angle, Side, Side is not enough information to conclude that the triangles are congruent. As two different triangles can be made is an angle and is opposing sides are congruent.

### EXAMPLE #1:

List the 6 congruencies if  $\Delta BIS \cong \Delta CUT$ .

### EXAMPLE #2:

Determine if the following sets of triangles would be congruent using the above five reasons: SSS, ASA, SAS, AAS or HL. State the triangle congruency if there is one.



## 2.5 ASSIGNMENT (Concept #21)

Suppose $\Delta BIG \cong \Delta CAT.$ a)  $\angle G \cong ?$ d)  $\overline{BI} \cong ?$ b)  $m \angle A = ?$ e)  $\Delta IGB \cong ?$ 

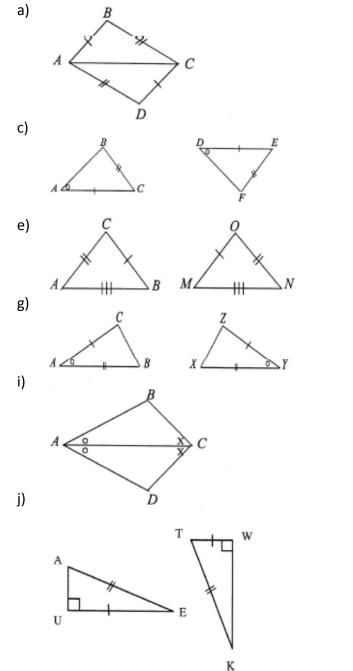
f)  $\Delta CTA \cong ?$ 

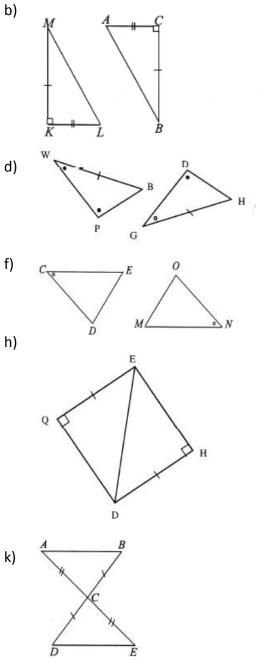
1. a) Complete the following chart.

c)  $\overline{AT} \cong ?$ 

Answer the following given  $\triangle ABC \cong \triangle DEF$ .

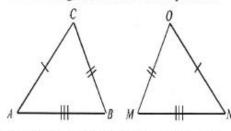
- b) a) Name the three pairs of corresponding vertices.
  - b) Name the three pairs of corresponding sides.
  - c) Is it correct to say  $\triangle BAC \cong \triangle EFD$ ?
  - d) Is it correct to say  $\triangle CAB \cong \triangle FDE$ ?
- 2. Determine if the following sets of triangles would be congruent using the above five reasons: SSS, ASA, SAS, AAS or HL. State the triangle congruency if there is one.





*Foundations 20(Ms. Carignan)* FM20.4 Parallel Lines & Polygons (Ch 2)

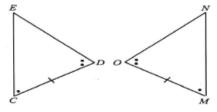
3. a) Use the diagram to answer the question.



- Which angle in  $\Delta MON$  is equal to  $\angle A^2$ . c) Use the diagram to answer the question.

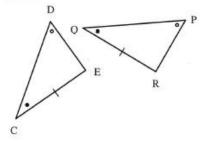
Which angle in  $\triangle CDE$  is the same size as  $\angle B$ ?

Use the diagram to answer the question.
 b) E



Which side in  $\triangle CED$  is the same length as  $\overline{MN}$ ?

d) Which angle in  $\Delta PQR$  is the same measure as  $\angle E$ ?



#### Answers:

1a) Complete the following chart.

Suppose $\Delta BIG \cong \Delta CAT.$ a)  $\angle G \cong ? \ \angle T$ d)  $\overline{BI} \cong ? \overline{CA}$ b)  $m \angle A = ? \ \angle I$ e)  $\Delta IGB \cong \triangle ATC$ c)  $\overline{AT} \cong ? \overline{IG}$ f)  $\Delta CTA \cong \triangle BGI$ 

b) a)  $\measuredangle A \cong \measuredangle D, \measuredangle B \cong \measuredangle E, \measuredangle C \cong \measuredangle F$ 

b)  $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$ c) NO d) Yes

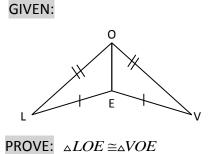
2) a) SSS;  $\triangle ABC \cong \triangle CDA$  b) SAS;  $\triangle MKL \cong \triangle BCA$  c) Not enough info to determine if they are congruent d) AAS;  $\triangle WPB \cong GDH$  e) SSS;  $\triangle ABC \cong \triangle NMO$  f) Not enough info g) SAS;  $\triangle ABC \cong YXZ$ h) HL;  $\triangle EQD \cong \triangle DHE$  i) ASA;  $\triangle ABC \cong \triangle ADC$  j) HL;  $\triangle AUE \cong \triangle TWK$  k) SAS;  $\triangle ACB \cong \triangle ECD$ 

3.) a) 
$$\angle N$$
 b)  $\overline{CE}$  c)  $\angle D$  d)  $\angle R$ 

## PreAP FDN 20 2.6 PROVING TRIANGLES CONGRUENT (Not in Textbook)

#### Refer to your reference sheet on reasons to use in a two column proof

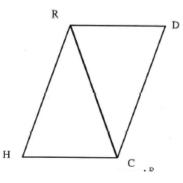
#### EXAMPLE #1:



STATEMENTS	REASONS	
		$\left  \right\rangle$

**EXAMPLE #2**: Prove the following in a formal, two column proof.

Given:  $\overline{RD} \cong \overline{HC}$  $\overline{HR} \cong \overline{DC}$ Prove:  $\angle D \cong \angle II$ 



Concept: #21

STATEMENTS	REASONS

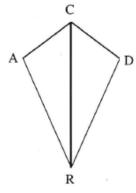
#### EXAMPLE #3:

:

Prove the following using a formal proof

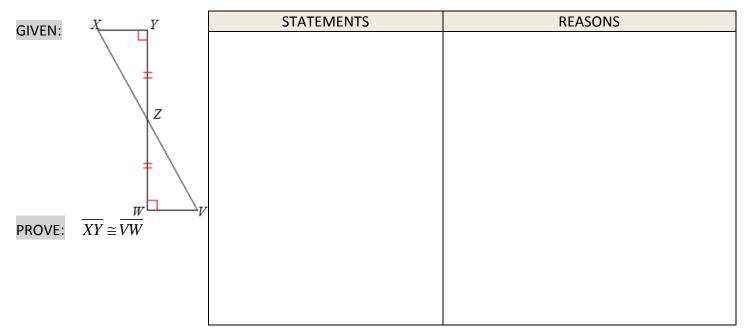
Given:  $\overline{AC} \cong \overline{DC}$  $\overline{RC}$  bisects  $\angle ACD$ 

Prove:  $\angle CAR \cong \angle CDR$ 



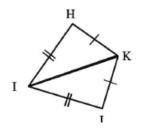
STATEMENTS	REASONS

#### EXAMPLE #4:



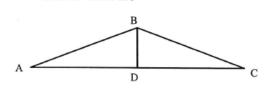
## 2.6 ASSIGNMENT (Concept #21)

1. Given:  $\overline{HK} \cong \overline{KJ}$  $\overline{HI} \cong \overline{IJ}$ Prove:  $\Delta HIK \cong \Delta JIK$ 



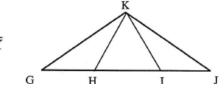
STATEMENTS	REASONS
1. $\overline{HK} \cong \overline{KJ}$	1.
2.	2. Given
3. $\overline{IK} \cong \overline{IK}$	3.
4. $\Delta HIK \cong \Delta JIK$	4.

2. Given: D is the midpoint of  $\overline{AC}$  $\overline{AB} \cong \overline{BC}$ Prove:  $\angle A \cong \angle C$ 



STATEMENTS	REASONS
<b>1.</b> D is the midpoint of $\overline{AC}$	1.
2.	2. Given
<b>3.</b> $\overline{AD} \cong \overline{DC}$	3.
4.	4. Reflexive
5. $\triangle ABD \cong \triangle CBD$	5.
$6. \angle A \cong \angle C$	6.

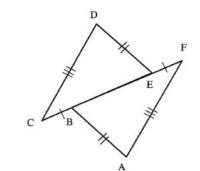
3. Given:  $\angle KHI \cong \angle KIH$  $\overline{GH} \cong \overline{IJ}, \overline{GK} \cong \overline{JK}$ Prove:  $\triangle GKH \cong \triangle JKI$ 



STATEMENTS	REASONS
1.	1. Given
2.	2. Given
3.	3. Given
4. $\overline{KH} \cong \overline{KI}$	4.
5. $\Delta GKH \cong \Delta JKI$	5.

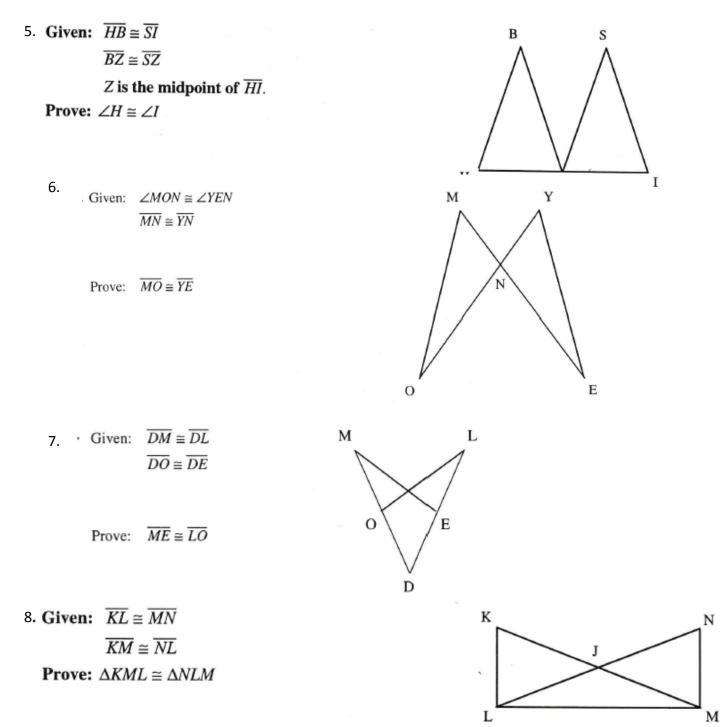
4. Given:  $\overline{CD} \cong \overline{AF}$  $\overline{DE} \cong \overline{BA}, \overline{CB} \cong \overline{EF}$ Prove:  $\angle D \cong \angle A$ 

4.

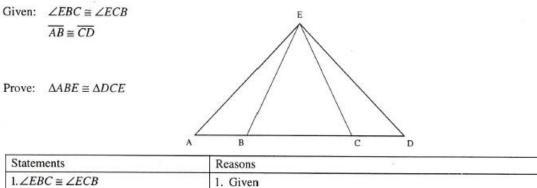


STATEMENTS	REASONS
1. $\overline{\mathbf{CD}} \cong \overline{\mathbf{AF}}$	1.
2. $\overline{\text{DE}} \cong \overline{\text{BA}}$	2.
3. $\overline{\mathbf{CB}} \cong \overline{\mathbf{EF}}$	3.
4.	4. Reflexive
5.CB + BE = CE	5.
6.	6. Segment Addition Postulate
7. $\overline{\mathbf{CE}} \cong \overline{\mathbf{BF}}$	7.
8.	8. SSS
9.	9. Corresponding parts of congruent triangles are congruent.

Use a two column proof for the following.



#### Extra QUESTIONS IF NEEDED

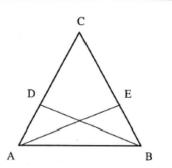


$1.2EDC \equiv 2ECD$	1. Given
2.	2. Given
3. $\overline{BE} \equiv \overline{EC}$	3.
4.	4. If two adjacent angles form a straight line, they are supplementary.
5. $\angle ECB$ and $\angle ECD$ are supplementary	5.
6	6. Supplements of congruent angles are congruent.
$7.\Delta ABE \cong \Delta DCE$	7.

Given:  $\overline{AD} \cong \overline{BE}$ 

Prove:  $\angle AEC \cong \angle BDC$ 

 $\overline{DC} \cong \overline{EC}$ 



Statements	Reasons
1.	1. Given
$2.\overline{DC} \cong \overline{EC}$	2.
3.	3. Reflexive Property of Equality
4.AD + DC = AC	4.
5.	5. Segment Addition.
$6.AD + DC \cong BE + EC$	6.
7.	7. Substitution Property of Equality
$8.\Delta ACE \cong \Delta BCD$	8.
9.	9. Corresponding parts of congruent triangle are congruent.

Given:  $\Delta PQS$  is isoceles

 $\overline{PR}$  is the perpendicular bisector of  $\overline{QS}$ 

Prove:  $\Delta QPR \cong \Delta SPR$ 

