## PreAP FDN 20 2.1 PARALLEL LINES \& 2.2 ANGLES IN PARALLEL LINES

Congruent Vs Equal: Congruence is a relationship of shapes and sizes, such as segments, triangles, and geometrical figures, while equality is a relationship of sizes, such as lengths, widths, and heights. Congruence deals with objects while equality deals with numbers. You don't say that two shapes are equal or two numbers are congruent.


## Complimentary Angles:

D(1) Draw 2 parallel lines.
"How do you know they are parallel?"

- Draw a line crossing both parallel lines ( transversal)
- Label all angles 1-8
- "What angles are the same? What angles are supplementary?
- Make some conjectures about these angles


## Definitions:

1) Transversal:
2) Vertically Opposite Angles

THEOREM A: In a pair of intersecting lines the vertically opposite angles are $\qquad$ and will have measures

## 3) Interior Angles:

THEOREM B: When a transversal intersects two parallel lines, Alternate Interior angles are $\qquad$ and wil have $\qquad$ measures
4) Exterior Angles:

THEOREM C: When a transversal intersects two parallel lines, Alternate Exterior angles are $\qquad$ and wil have $\qquad$
5) Corresponding Angles:

THEOREM D: When a transversal intersects two parallel lines, Corresponding angles are $\qquad$ and will have $\qquad$

- Converse of Theorem D:
- When a transversal intersects a pair of lines creating equal corresponding angles, the two lines are parallel.
- When a transversal intersects a pair of nonparallel lines the corresponding angles are not equal (and vice versa)

THEOREM D: When a transversal intersects two parallel lines, the Same Side Interior Angles are $\qquad$ meaning that they add to $\qquad$

EXAMPLE \#1: : Based on the given info are lines $x$ and $y$ parallel? Why or why not?


EXAMPLE \#2: : Find the value of $x$ that makes $j / / k$
a)

b)


EXAMPLE \#3: $\quad$ Determine the measures of $a, b, c$, and $d$. And explain your reasoning.


## 2.1 \& 2.2 ASSIGNMENH :H1 (Concept \#17)

FA: P72 \#5

## P 78 \#1-4, 20 PLUS "SOME" of the following:

Use the figure at the right to answer problems 1-8.
Classify each pair of angles as one of the following:
(a) alternate interior angles
(b) corresponding angles
(c) alternate exterior angles
(d) vertical angles
(e) supplementary angles
(f) none

1. $\qquad$ $\angle 9 \& \angle 16$
2. $\qquad$ $\angle 9 \& \angle 11$
3. $\qquad$ $\angle 15$ \& $\angle 11$
4. $\qquad$ $\angle 9 \& \angle 15$
5. $\qquad$ $\angle 10 \& \angle 15$
6. $\qquad$ $\angle 13 \& \angle 14$
7. $\qquad$ $\angle 12 \& \angle 15$
8. $\qquad$ $\angle 14 \& \angle 11$

9. $\mathrm{m} \angle 2=97^{\circ} \mathrm{m} \angle 6=83^{\circ}$

$$
\begin{array}{ll}
\mathrm{m} \angle 3 & = \\
\mathrm{m} \angle 5 & = \\
\mathrm{m} \angle 10= & \mathrm{m} \angle 7= \\
\mathrm{m} \angle 9= & \mathrm{m} \angle 16=
\end{array}
$$



Find the value of $x$ given that $s / / t$
10. $\mathrm{m} \angle 4=77^{\circ}, \mathrm{m} \angle 8=4 \mathrm{x}+57$
11. $m \angle 3=5 x+13, m \angle 5=53^{\circ}$
12. $m \angle 1=6 x-5, m \angle 7=115^{\circ}$


Find the value of x that makes jll k .
13.

14.

15.

16.


Determine the missing angles.
17.

18.

19.

20.




| 33. |  | If two angles of one triangle are congruent to two angles of <br> another triangle, then the third angles are congruent. |
| :--- | :--- | :--- |
| 34. |  | If two parallel lines are cut by a transversal, then the alternate <br> interior angles are congruent. |
| 35. |  | If two parallel lines are cut py a transversal, then the <br> corresponding angles are congruent. |
| 36. |  | If two parallel lines are cut by a transversal, then the same side <br> interior angles are supplementary. |
| 37. |  | SSS CONGRUENCE POSTULATE: If three sides of one <br> triangle are congruent to three sides of another triangle, then <br> the triangles are congruent. |
| 38. |  | SAS CONGRUENCE POSTULATE: If two sides and the <br> included angle of one triangle are congruent to two sides and <br> the included angle of another triangle, then the two triangles <br> are congruent. |
| 39. |  | ASA CONGRUENCE POSTULATE: If two angles and the <br> included side of one triangle are congruent to two angles and <br> the included side of another triangle, then the triangles are <br> congruent. |
| 40. |  | AAS CONGRUENCE POSTULATE: If two angles and a <br> non-included side of one triangle are congruent to two angles <br> and a non-included side of another triangle, then the two <br> triangles are congruent. |
| 42. |  | HL CONGRUENCE POSTULATE: If the hypotenuse and <br> one leg of one right triangle are congruent to the corresponding <br> parts of another right triangle, the two triangles are congruent. |

## PreAP FDN 20 2.1 \& 2.2 ANGLE \& PARALLEL LINE PROOFS

## TWO COLUMN PROOFS REVIEW

Two column proofs are similar to proving that someone in court is telling the truth.

- A series of STATEMENTS are made that will go towards proving that either the mathematical concept or the person on trial is telling the truth.

- The last statement in a proof is always what is being proven.
- Each statement must be backed up with a REASON in the same way that each statement in court must have a corroborating witness.
- Even the obvious facts must be presented in a statement and backed up with a reason.

If there is not a reason given to back up a statement, the truth of the statement is left in doubt. If the statements and reasons listed lead to a different conclusion than the one attempting to be proven, the original assumption of truth cannot be determined.

Every mathematical rule and theorem and postulate that is learned in school, has been proven to be true. Many of these were proven to be true using the two column proof format.


EXAMPLE \#1: Use a two column proof to deductively prove that alternate interior angles of parallel lines are equal.

| STATEMENTS | REASONS |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

EXAMPLE \#2: Use a two column proof to deductively prove that same side interior angles of parallel lines are supplementary.

| STATEMENTS |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

EXAMPLE \#3: One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces $C G, B F$, and $A E$ are parallel.

| STATEMENTS | REASONS |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

EXAMPLE \#4: Use a two column proof to deductively prove that ST = TR
Given: $Q R \| S T$

$$
\angle Q R S=\angle T R S
$$

Prove: $S T=T R$


| STATEMENTS |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## EXAMPLE \#5:

Joelle wrote this proof that $A B \| Y Z$. Identify and correct her errors.

$$
\begin{aligned}
\angle A X Y+\angle Y X Z+\angle B X Y & =180^{\circ} \quad \text { supplementary angles } \\
\angle A X Y+85^{\circ}+35^{\circ} & =180^{\circ} \\
\angle A X Y & =60^{\circ} \\
\angle A X Y & =\angle Y X Z \quad \text { corresponding angles }
\end{aligned}
$$



Therefore, $A B \| Y Z$.

EXAMPLE \#6: Prove that $A B / / C D$.


B

| STATEMENTS |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## 2.1 \& 2.2 ASSJCNMENJ : 2 (Concept \# 18)

## FA: $\quad$ P79 \#8,10, 12, 15, 16, 18

P 78 \#1-4, 20
MLA: P79 \#9, 17, 19,
Helpful hints for the assignment: An isosceles triangle has two sides of equal length and two equal angles.
If a line bisects an angles it divides the angle into two equal angles.


## PreAP FDN 20 2.3 ANGLE PROPERTIES IN TRIANGLES \& PROOFS

Recall: All interior angles of a triangle add up to $\qquad$ .
EXAMPLE \#1: prove, deductively, that the sum of the measures of the interior angles of any triangle is
$\qquad$ $\stackrel{\circ}{\circ}$


| Statement | Reason |
| :--- | :--- |
|  |  |



Note: Exterior angles are formed by extending a side of a polygon. For example, extend one side of this triangle to make an exterior angle:


EXAMPLE \#2: In the diagram, angle MTH is an exterior angle of $\triangle$ MAT. Determine the measures of the unknown angles in $\triangle M A T$.


What relationship do you notice about angle AMT, angle MAT and the exterior angle MTH?

EXAMPLE \#3: Prove deductively using a two column proof that an exterior angle of a triangle is equal to the sum of the two non- adjacent sides.

EXAMPLE \#4: In $\triangle E F G, G I$ bisects $\measuredangle F G H$
a) If $\measuredangle E=\measuredangle y$, prove GI // EF.

b) Prove EF // GI, if $\measuredangle F=\measuredangle z$

EXAMPLE \#4: GIVEN:
$\triangle B A C$ is isosceles
$m \measuredangle 1=40^{\circ}$
$m \measuredangle 5=140^{\circ}$

PROVE:
$\overleftrightarrow{B C} \| \overleftrightarrow{D E}$


## FA: <br> P90 \#2, 3, 5, 7, 9, 19-12, 14, 15 PLUS the following proofs <br> MLA:

1. 



PROVE: $\measuredangle a \cong 25^{\circ}$

| STATEMENTS | REASONS |
| :--- | :--- |
| 1. | 1. Given |
|  | 2. Given |
|  | 3. Given |
| $4 . \measuredangle c \cong$ | 4. |
| 5. | 5. Corresponding $\measuredangle^{\prime} '$ s of $\\|$ lines are $\cong$ |
| $6 . ~$ |  |
| 6 $\cong 25^{\circ}$ | 6. |

2. 

Given:


Prove that $\angle A B C=90^{\circ}$
3.

Given: $F D \| G E$

4.


## PreAP FDN 20 2.4 ANGLE PROPERTIES IN POLYGONS

## Types of Polygons:

convex polygon
A polygon in which each interior angle measures less than $180^{\circ}$.

convex


Regular Polygon

- a polygon whose...
- a polygon whose...


What is the interior angle sum of any triangle? $\qquad$ (we proved this last section)
What will the 4 interior angles of any quadrilateral always add to? The 5 interior angles of a pentagon? Let's investigate:

| Polygon | \# of Sides | \# of Triangles | Sum of Interior Angle Measures |
| :--- | :--- | :--- | :--- |
| Triangle |  |  |  |
| Quadrilateral |  |  |  |
| Pentagon |  |  |  |

## Sum of the Interior Angles of a Convex Polygon:

Given that $\mathrm{n}=$ the number of sides Interior Angle Sum =

## Measure of each Interior Angle of a REGULAR Convex Polygon:

Given that $\mathrm{n}=$ the number of sides of equal length Each Interior Angle $=$

## EXAMPLE \#1:

a) What would be the sum of the measures of the interior angles of a regular dodecagon (12-sides)?
b) Determine the measure of each interior angle of a regular dodecagon?

## Sum of the EXTERIOR ANGLES of a Convex Polygon:

Given that $\mathrm{n}=$ the number of sides EXTERIOR Angle Sum =

## Measure of each EXTERIOR Angle of a REGULAR Convex Polygon:

Given that $\mathrm{n}=$ the number of sides of equal length Each Interior Angle $=$

EXAMPLE \#2: : Deductively prove that the sum of the exterior angles of any polygon will be $360^{\circ}$


EXAMPLE \#3: : Bob is tiling his floor. He uses regular hexagons and regular triangles. The side length of a triangle is equal to the side length of a hexagonal tile. Can he tile the floor without leaving any gaps between tiles?( Concept 20)

EXAMPLE \#3: Kieran drew a 14 sided convex polygon. One of the interior angle measures $155^{\circ}$, Is it a regular polygon?

### 2.4 ASS (ণNMENJ (Concept \#19, 20)

FA: $\quad$ P99 \#1, 2, 37, 10, 16, 18
MLA: P99 \#13
ULA: P99 \#20, 21

## PreAP FDN 20 2.5 CONGRUENT TRIANGLES (Not in Textbook)

Two triangles are considered to be CONGRUENT ( $\cong$ ) when the following is true:


This means that $\Delta$ $\qquad$ $\cong \Delta$ $\qquad$
(we could also write this as $\Delta$ $\qquad$ $\cong \Delta$ $\qquad$ or
$\triangle$ $\qquad$ $\cong \Delta$ $\qquad$ or $\Delta$ $\qquad$ $\cong \Delta$ $\qquad$ or
$\triangle$ $\qquad$ $\cong \Delta$ $\qquad$ or $\qquad$ $\cong \Delta$ $\qquad$ )
NOTE: The order of the letters in the first triangle must correspond to the correct order in the second triangle

The following are congruent between the two triangles:
ANGLES:
 $\cong \measuredangle$ $\qquad$ and SIDES: $\qquad$ $\cong$
$\qquad$
$\measuredangle$ $\qquad$ $\cong \measuredangle$ $\qquad$
$\qquad$ $\cong$ $\qquad$
$\qquad$
$\cong$ $\qquad$

THERE ARE FIVE WAYS TO DETERMINE IF TWO TRIANGLES ARE CONGRUENT:

1. SSS Postulate (Side, Side, Side)

2. SAS Postulate (Side, Angle, Side)


If three sides of one triangle are congruent to three sides of another triangle, the two triangles are congruent.

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the two triangles are congruent.
3. ASA Postulate (Angle, Side, Angle)

$\triangle A B C \cong \triangle D E F$
4. AAS Postulate (Angle, Angle, Side)


If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the two triangles are congruent.

## Note: The side must be in the middle.

If two angles and a non-included side of one triangle ;re congruent to two angles and a non-included side of another triangle, the two triangles are congruent.
5. HL Postulate (Hypotenuse, Leg)


If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, the two triangles are congruent.

## EXAMPLE \#2:

Determine if the following sets of triangles would be congruent using the above five reasons: SSS, ASA, SAS, AAS or HL. State the triangle congruency if there is one.
a)

$\Delta$ $\qquad$ $\cong{ }_{\mathrm{D}}$ $\qquad$ by $\qquad$
c)

b)
$\qquad$
$\qquad$ $\cong \Delta$
by $\qquad$
d)

$\Delta$ $\qquad$ $\cong \Delta$ $\qquad$ by $\qquad$

$\qquad$ $\cong \Delta$ $\qquad$ by $\qquad$
f)

$\Delta$ $\qquad$ $\cong \Delta$ $\qquad$ by $\qquad$
h)

$\qquad$ $\cong \Delta$ $\qquad$ by $\qquad$


### 2.5 ASSJCNMENU (Concept \#21)

Answer the following given $\triangle A B C \cong \triangle D E F$.

1. a) Complete the following chart.

| Suppose | $\triangle B I G \cong \triangle C A T$. |
| :--- | :--- |
| a) $\angle G \cong ?$ | d) $\overline{B I} \cong ?$ |
| b) $m \angle A=?$ | e) $\triangle I G B \cong ?$ |
| c) $\overline{A T} \cong ?$ | f) $\triangle C T A \cong ?$ |

b)
a) Name the three pairs of corresponding vertices.
b) Name the three pairs of corresponding sides.
c) Is it correct to say $\triangle B A C \cong \triangle E F D$ ?
d) Is it correct to say $\triangle C A B \cong \triangle F D E$ ?
2. Determine if the following sets of triangles would be congruent using the above five reasons: SSS, ASA, SAS, AAS or HL. State the triangle congruency if there is one.
a)

c)

e)

g)

i)

b)

f)

h)

3. a) Use the diagram to answer the question.


Which anole in $\triangle M O N$ ic anmal to / $\Delta$ ?
c)


Which angle in $\triangle C D E$ is the same size as $\angle B$ ?
b)


Which side in $\triangle C E D$ is the same length as $\overline{M N}$ ?
d) Which angle in $\triangle P Q R$ is the same measure as $\angle E$ ?


## Answers:

## 1a) Complete the following chart.

b) a) $\measuredangle A \cong \measuredangle D, \measuredangle B \cong \measuredangle E, \measuredangle C \cong \measuredangle F$

| Suppose | $\Delta B I G \cong \triangle C A T$. |
| :--- | :--- |
| a) $\angle G \cong ? \angle T$ | d) $\overline{B I} \cong$ ? $\overline{C A}$ |
| b) $m \angle A=? \angle I$ | e) $\triangle I G B \cong \triangle A T C$ |
| c) $\overline{A T} \cong ? \overline{I G}$ | f) $\triangle C T A \cong \triangle B G I$ |

b) $\overline{A B} \cong \overline{D E}, \overline{A C} \cong \overline{D F}, \overline{B C} \cong \overline{E F}$
c) NO d) Yes
2) a) SSS ; $\triangle A B C \cong \triangle C D A$ b) $\mathrm{SAS} ; \triangle M K L \cong \triangle B C A \quad$ c) Not enough info to determine if they are congruent d) AAS ; $\triangle W P B \cong G D H$
e) SSS; $\triangle A B C \cong \triangle N M O$ f) Not enough info
g) SAS; $\triangle A B C \cong Y X Z$
h) $\mathrm{HL} ; \triangle E Q D \cong \triangle D H E$
i) ASA ; $\triangle A B C \cong \triangle A D C$ j) HL ; $\triangle A U E \cong \triangle T W K$
k) SAS; $\triangle A C B \cong \triangle E C D$
3.) a) $\angle N$
b) $\overline{C E}$
c) $\angle D$
d) $\angle R$

## PreAP FDN 20 2.6 PROVING TRIANGLES CONGRUENT (Not in Textbook)

Refer to your reference sheet on reasons to use in a two column proof

## EXAMPLE \#1:

GIVEN:


PROVE: $\triangle L O E \cong \triangle V O E$

| STATEMENTS |  |
| :---: | :---: |
|  |  |
|  |  |

EXAMPLE \#2: Prove the following in a formal, two column proof.
Given: $\overline{R D} \cong \overline{H C}$

$$
\overline{H R} \cong \overline{D C}
$$

Prove: $\angle D \cong \angle I I$
H


| STATEMENTS | REASONS |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## EXAMPLE \#3:

Prove the following using a formal proof

$$
\begin{aligned}
\text { Given: } & \overline{A C} \cong \overline{D C} \\
& \overline{R C} \text { bisects } \angle A C D
\end{aligned}
$$

Prove: $\angle C A R \cong \angle C D R$


| STATEMENTS | REASONS |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## EXAMPLE \#4:

| GIVEN: | STATEMENTS | REASONS |
| :--- | :--- | :--- | :--- |
|  |  |  |

### 2.6 ASSJGNWINN (Concept \#2 1)

1. Given: $\overline{H K} \cong \overline{K J}$

$$
\overline{H I} \cong \overline{I J}
$$

Prove: $\Delta H I K \cong \Delta J I K$


| STATEMENTS | REASONS |
| :--- | :--- |
| 1. $\overline{H K} \cong \overline{K J}$ | 1. |
| 2. | 2. Given |
| 3. $\overline{I K} \cong \overline{I K}$ | 3. |
| 4. $\Delta H I K \cong \Delta I I K$ | 4. |

2. Given: $\mathbf{D}$ is the midpoint of $\overline{\mathrm{AC}}$

$$
\overline{A B} \cong \overline{B C}
$$

Prove: $\angle A \cong \angle C$


| STATEMENTS | REASONS |
| :--- | :--- |
| 1. $\mathbf{D}$ is the midpoint of $\overline{\text { AC }}$ | 1. |
| 2. | 2. Given |
| 3. $\overline{A D} \cong \overline{D C}$ | 3. |
| 4. | 4. Reflexive |
| 5. $\triangle A B D \cong \triangle C B D$ | 5. |
| 6. $\angle A \cong \angle C$ | 6. |

3. Given: $\angle K H I \cong \angle K I H$

$$
\overline{G H} \cong \overline{I J}, \overline{G K} \cong \overline{J K}
$$

Prove: $\Delta G K H \cong \Delta J K I$


| STATEMENTS | REASONS |
| :--- | :--- |
| 1. | 1. Given |
| 2. | 2. Given |
| 3. | 3. Given |
| 4. $\overline{K H} \cong \overline{K I}$ | 4. |
| 5. $\triangle G K H \cong \Delta I K I$ | 5. |

4. Given: $\overline{C D} \cong \overline{A F}$

$$
\overline{D E} \cong \overline{B A}, \overline{C B} \cong \overline{E F}
$$

Prove: $\angle D \cong \angle A$

4.

| STATEMENTS | REASONS |
| :--- | :--- |
| 1. $\overline{\mathrm{CD}} \cong \overline{\mathrm{AF}}$ | 1. |
| 2. $\overline{\mathrm{DE}} \cong \overline{\mathrm{BA}}$ | 2. |
| 3. $\overline{\mathrm{CB}} \cong \overline{\mathrm{EF}}$ | 3. |
| 4. | 4. Reflexive |
| 5. $\mathrm{CB}+\mathrm{BE}=\mathrm{CE}$ | 5. |
| 6. | 6. Segment Addition Postulate |
| 7. $\overline{\mathrm{CE}} \cong \overline{\mathrm{BF}}$ | 7. |
| 8. | 8. SSS |
| 9. | 9.Corresponding parts of congruent |
|  | triangles are congruent. |

Use a two column proof for the following.
5. Given: $\overline{H B} \cong \overline{S I}$
$\overline{B Z} \cong \overline{S Z}$
$Z$ is the midpoint of $\overline{H I}$.
Prove: $\angle H \cong \angle I$
6.

$$
\begin{aligned}
\text { Given: } & \angle M O N \cong \angle Y E N \\
& \overline{M N} \cong \overline{Y N}
\end{aligned}
$$

Prove: $\overline{M O} \cong \overline{Y E}$

7. Given: $\overline{D M} \equiv \overline{D L}$

$$
\overline{D O} \cong \overline{D E}
$$

Prove: $\overline{M E} \cong \overline{L O}$

8. Given: $\overline{K L} \cong \overline{M N}$

$$
\overline{K M} \cong \overline{N L}
$$

Prove: $\triangle K M L \cong \triangle N L M$


## Extra QUESTIONS IF NEEDED

Given: $\begin{aligned} & \angle E B C \cong \angle E C B \\ & \overline{A B} \cong \overline{C D}\end{aligned}$


| Statements | Reasons |
| :--- | :--- |
| $1 . \angle E B C \cong \angle E C B$ | 1. Given |
| 2. | 2. Given |
| $3 . \overline{B E} \cong \overline{E C}$ | 3. |
| 4. | 4. If two adjacent angles form a straight line, they are supplementary. |
| $5 . \angle E C B$ and $\angle E C D$ are supplementary | 5. |
| 6. | 6. Supplements of congruent angles are congruent. |
| $7 . \triangle A B E \cong \triangle D C E$ | 7. |

Given: $\overline{A D} \cong \overline{B E}$
$\overline{D C} \cong \overline{E C}$

Prove: $\angle A E C \cong \angle B D C$


| Statements | Reasons |
| :--- | :--- |
| 1. |  |
| 2. $\overline{D C} \cong \overline{E C}$ | 1. Given |
| 3. | 2. |
| $4 . A D+D C=A C$ | 3. Reflexive Property of Equality |
| 5. | 4. |
| $6 . A D+D C \cong B E+E C$ | 5. Segment Addition. |
| 6. |  |
| 7. | 7. Substitution Property of Equality |
| $8 . \triangle A C E \cong \triangle B C D$ | 8. |
| 9. | 9. Corresponding parts of congruent triangle are congruent. |

Given: $\triangle P Q S$ is isoceles
$\overline{P R}$ is the perpendicular bisector of $\overline{Q S}$

Prove: $\quad \triangle Q P R \cong \triangle S P R$


Given: $\overline{S R}$ bisects $\angle A R O$ and $\angle A W O$

Prove: $\overline{A R} \cong \overline{O R}$


Given: $\overline{B E} \cong \overline{D G}$
$\overline{A C} \equiv \overline{F C}$
$\angle C G D \cong \angle C B E$

Prove: $\quad \overline{A D} \cong \overline{F E}$


