PC 30 1.1 Horizontal & Vertical Translations

To determine the effects of *h* and *k* in *y = f(x - h) + k* on the graph of *y = f(x)* (Note: Sometimes the above equation *y = f(x - h) + k* is rewritten as *y - k = f(x - h)*)

Things to Review: Definition of a function, Function Notation, how to graph $y = x$, $y = x^2$, $y = |x|$ $y = \frac{1}{x}$

Investigation

Desmos APP (Phone) or desmos.com (Computer)

Step 1 – Click the  in the top left corner

Step 2 - Go to Transformations: Translating Graph

Step 3 - Change the Red Function to:

1) $f(x) = |x|$ This is your base function

A) What effect does the "k" value have on a function? (Move the slider in in row 5 or press play)

b) Determine the equation of an absolute function that has been translated 6 units down

c) What effect does the "h" value have on a function? (Move the slider in row 7 or press play)

d) Determine the equation of an absolute value function that has been translated 4 units up and 3 units to the left?

2) Complete Step 3 again using the following base functions: $f(x) = \frac{1}{x}$, $f(x) = x^2$

Transformation: A **transformation** of a function changes the equation and the location and or shape and or orientation (up/down) of the graph.

Translation:
A **translation** is a type of transformation that can move the graph of a function up, down, left or right but the shape and orientation of the graph stays the same.

- The variables used to describe translation are "h" and "k"
- If the original function is y = f(x), the translated function is y = f(x $\pm$ h) $\pm$ k

Mapping:
A mapping is the relationship between the points on the original graph and the points on the transformed graph. The new points are called IMAGE POINTS. The image points are often named with an apostrophe which is read as prime – A vs A'. We will be using mapping notation to help us graph transformed functions.

- Mapping Notation for Translations is $(x, y) \rightarrow (x \pm h, y \pm k)$ (we will be adding more information to this in the next couple of days!

Example #1:

Sketch the graph of $y = |x-5| + 3$ using mapping notation. Describe the transformations.

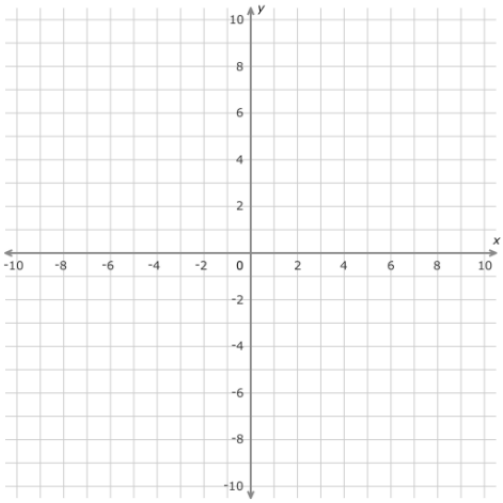
h = _____ k = _____ What do you think the BASE FUNCTION is? _____

Mapping Notation: (,) $\rightarrow$ (,)
DESCRIPTION:

BASE FUNCTION IMAGE POINTS FROM MAPPING

x	y
-3	
-1	
0	

_____	_____



NOTE: You can check the image points
Using the transformed function

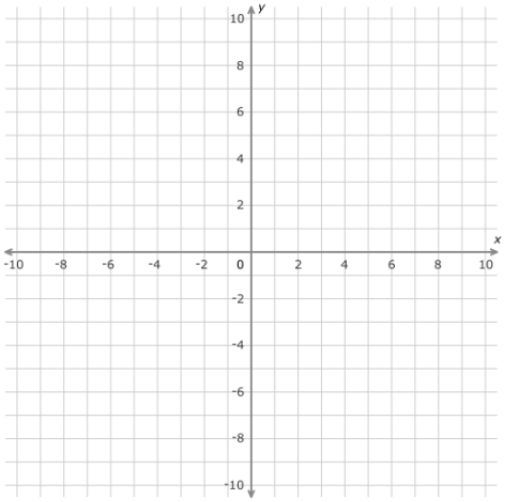
x	y

Example #2: Graph the function $y-2=(x+3)^2$ using mapping notation. Describe the transformations.

h = _____ k = _____ What do you think the BASE FUNCTION is? _____

Mapping Notation: (,) $\rightarrow$ (,)

DESCRIPTION:



BASE FUNCTION

x	y
-3	
-1	
0	

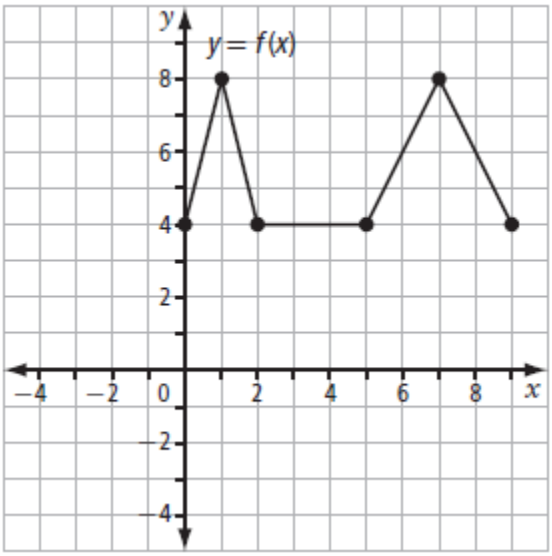
IMAGE POINTS FROM MAPPING

_____	_____

Example #3:

Graph the transformation of the following base function y = f(x) by using the transformation y = f(x – 1) – 3

NOTE: We are not given the equation, it is merely described as f(x). Our transformation equation describes what happens to f(x) without giving the original or the new equation.



a) Use mapping notation and an explanation to describe the transformation

b) Label and make a list of key points on f(x) and find the new image points for each of these key points.

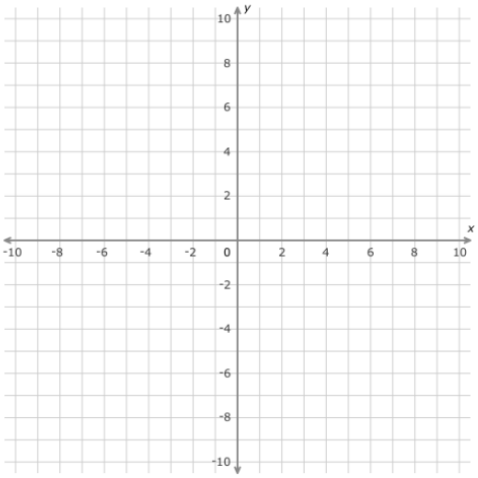
c) Sketch and label the transformed function in a different colour.

Example #4:

For $f(x) = |x|$, graph y + 6 = f(x – 4) and give the equation of the transformed function.

Steps:

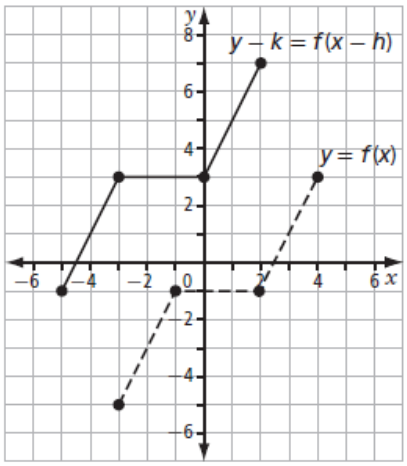
1. Rewrite the given equation in the form y = f(x – h) + k and describe the mapping.
2. Describe the transformation and state the mapping notation.
3. Sketch f(x) using a table of values
4. Using the graph of f(x) make a list of key points and find the image points.
5. Sketch the transformation in a different colour
6. Write the transformed equation.



Example #5:

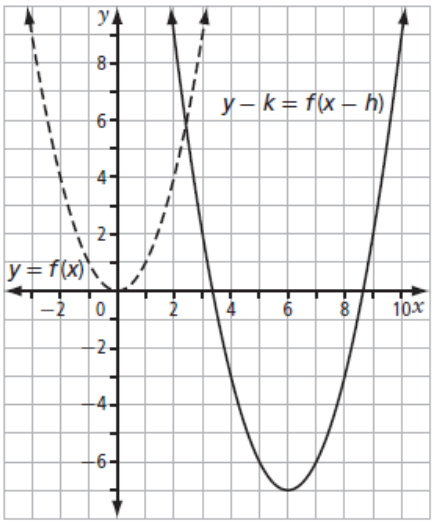
Describe the translation that has been applied to the graph of f(x) to obtain the graph of g(x). Determine the equation of the translated function in both of the following forms: y = f(x – h) + k and y – k = f(x – h)

a)



Note: Please answer a) in terms of f(x)

b)



Example #4:

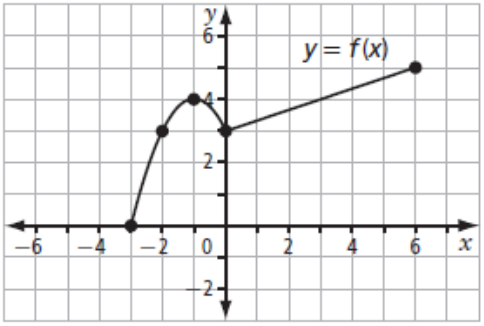
Given the graph of y = f(x), graph the following transformed functions. Use different colours and label.

a) f(x) = f(x) + 2

b) h(x) = f(x – 2)

c) s(x) = f(x + 4)

d) t(x) = f(x) - 2



SUMMARY OF VERTICAL AND HORIZONTAL TRANSLATIONS

Let c > 0. If the graph of y = f(x) is known, then:

1) h(x) = f(x) + c is a vertical shift upward of c units
2) h(x) = f(x) – c is a vertical shift downward of c units
3) h(x) = f(x – c) is a horizontal shift right of c units
4) h(x) = f(x + c) is a horizontal shift left of c units

REMEMBER:

- Foundational Assignments (FA) are to be done on numbered looseleaf or loose graph paper, fully labelled with the following information:

Page # YOUR NAME (First & Last) DATE Section # FA List of questions in FA

- You can have the Mid & Upper Level Assignment on the same or different page but you also must label the MLA as follows

Page # YOUR NAME (First & Last) DATE Section # MLA List of all questions in MLA

- At the beginning of class I will either as you to hand in your assignments or I will walk around the classroom. They must be fully labelled, stapled and ready to go when the bell rings. WORK MUST BE SHOWN FOR CREDIT! (NOTE: On days where there isn't a lot of time left to work I will often not do the homework check on that assignment until 2 days later). I will provide feedback on the FA assignments and do a Homework Check on the ULA assignments.
- It is VERY important that you always check your answers with the ones at the back of the book. Sometimes the instructions specify details about how the answer is to be given. Marks will be taken off if you do not state the answer as instructed on concept checks and comprehensive tests. Get in the habit of writing your answers correctly from the beginning!
- Doing only the Foundational Assignments (FA) can help you learn enough to possibly earn up to about 60% in this class. If you wish a higher mark you must also complete the Mid Level Assignments. Upper Level is for AP preparation.

1.1 ASSIGNMENT (Must use loose graph paper):

1.1 FA: P12 #1, 2ab, 3, 4ac, 5 (use both notations) 8, 11

1.1 MLA: P12 #6, 7, 9ab 10ab, 12

1.1 ULA: P12 #17, 18, 19, C2, C4

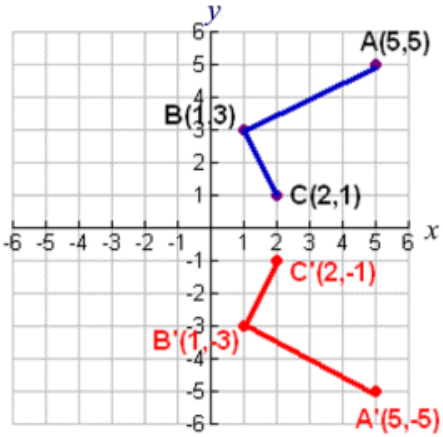
PC 30 — 1.2 Day 1: Horizontal & Vertical Translations

To determine the effects of reflections and vertical and horizontal stretches on the graphs of functions and their related equations.

Things to Review: Domain and range (both set notation and interval notation)

EXPLORATION ACTIVITY FOR 1.2: Reflections in the X and Y Axis

1. The points A,B and C are **reflected in the *x*-axis**, thus creating points A',B' and C'. Analyze the ordered pairs B and B'. How are they related?



If an original ordered pair is (x, y), what will the new ordered pair be when the graph is reflected is reflected in the x axis?

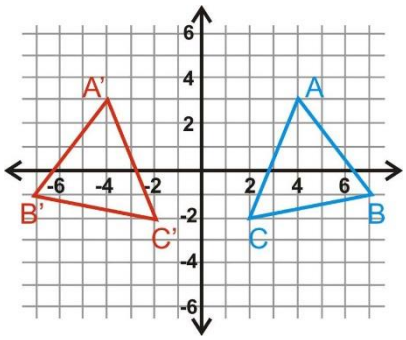
What do we change in the ordered pair of a function when we reflect it about the x axis? What would this look like in MAPPING NOTATION?

If the original equation is an unknown y = f(x), what would the new reflected equation be called?

When an equation y = f(x) is reflected/flipped about the x axis, the new equation will look like _____

Ex: Given $y = x^2$, it's reflection in the x axis will be _____

2. The points A,B and C are **reflected in the *y*-axis**, thus creating points A',B' and C'. Analyze the ordered pairs B and B'. How are they related?



If an original ordered pair is (x, y), what will the new ordered pair be when the graph is reflected is reflected in the y axis?

What do we change in the ordered pair of a function when we reflect it about the y axis? What would this look like in MAPPING NOTATION?

If the original equation is an unknown y = f(x), what would the new reflected equation be called?

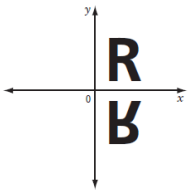
When an equation y = f(x) is reflected/flipped about the x axis, the new equation will look like _____

Ex: Given y = x + 2, it's reflection in the x axis will be _____

REFLECTION: A reflection of a graph creates a mirror image of the graph of a function across a line of reflection. Reflections, like translations, do not change the shape of the graph but may change the orientation. Any points where the function crosses the line of reflection do not move are called **INVARIANT POINTS.**

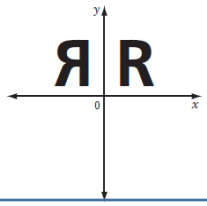
- When the output (outside) of the function y = f(x) is multiplied by -1, the result, y = -f(x) is a vertical reflection of the graph in the x axis

Vertical reflection:
- $y = -f(x)$
- $(x, y) \rightarrow (x, -y)$
- line of reflection: x-axis
- also known as a reflection in the x-axis



- When the input (inside) of the function = f(x) is multiplied by -1, the result, y = f(-x) is a horizontal reflection on the graph in the y axis

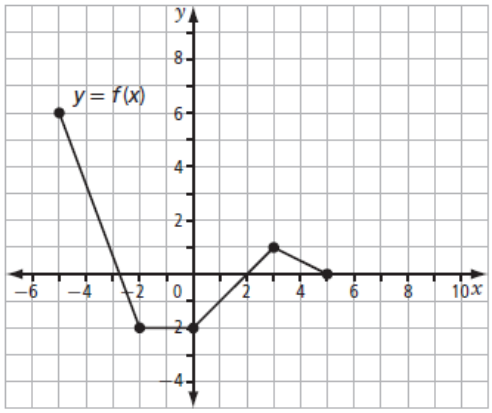
Horizontal reflection:
- $y = f(-x)$
- $(x, y) \rightarrow (-x, y)$
- line of reflection: y-axis
- also known as a reflection in the y-axis



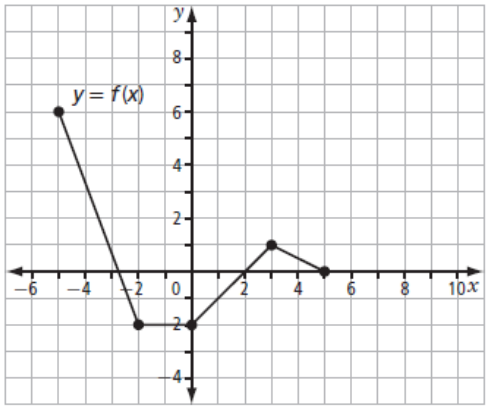
Example #1:

Given $y = f(x)$, graph the indicated transformation on the same set of axes. Give the mapping notation and describe the transformation. Identify any invariant points.

a) $y = f(-x)$



b) $y = -f(x)$



Example #2:
Complete the following table by finding the equations of the reflected equations.

f(x)	-f(x)	f(-x)
f(x) = 4x		
f(x) = x^2 - 7		
f(x) = $\lvert 5x \rvert$		
f(x) = $\frac{1}{6x}$		

Example #3:
Find the value of each:

x	f(x) = $\lvert x+4 \rvert$	g(x) = -f(x)	h(x) = f(-x)
5	f(5) = $\lvert (5)+4 \rvert$ = 9	g(5) = -f(5) =	h(5) = f(-5) =
3			
22			

Example #4:
Rewrite each following domain and range in either interval notation or set notation

SET NOTATION	INTERVAL NOTATION
$\{x \mid -13 \le x \le -5, x \in R\}$	
$\{y \mid y \le 23, y \in R\}$	
$\{y \mid y \in R\}$	
	D: [-3, 6]
	R: $(12, \infty)$
	R: (-7, 16]

1.2A ASSIGNMENT (Must use loose graph paper):

1.2A FA: P28 #1, 3, 4
1.2 MLA: P28 #15ab
1.2 ULA: P28 #14b, 15ab,

PC 30 — 1.2 Day 2: Horizontal & Vertical Translations

To determine the effects of reflections and vertical and horizontal stretches on the graphs of functions and their related equations.

EXPLORATION ACTIVITY A FOR 1.2: Stretches in the X and Y Axis

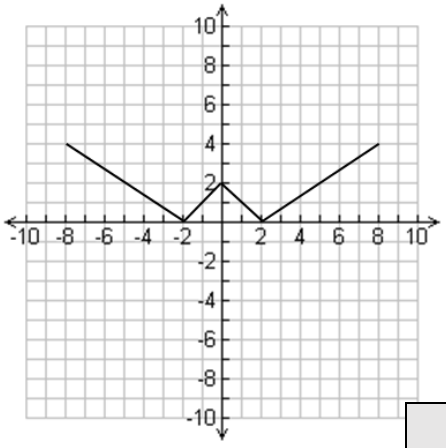
Given a function y = f(x), explore what happens when we consider y = a f(x)

1. Go to Desmos.com and enter $y=ax^2$ as your first equation.
   - It should ask you if you want to add a slider $y = ax^2$ add slider: a . Click on the "a" to add a slider for "a"
   - What happens to the graph vertically as you slide the value of "a" to the right of 1 (a is getting larger)? _____
   - What happens to the graph vertically as you slide the value of "a" between 0 and 1 (a is getting smaller)?_____
   - What happens to the graph vertically as you slide the value of "a"smaller than 1 (a is getting smaller)?_____
2. In general, how would you describe the vertical change to the original graph as the value of "a" is adjusted between 0 and infinity?
   - Is there another way we could describe the change?

3. Do you think this stretch changes the x or the y values if you were to compare the original table of values to each new table of values?

Example #1:

An original function y = f(x), has the following graph. First, use the graph to fill in the first two columns to find the original ordered pairs. Then, fill in each of the additional three tables to find three different transformed equations. Graph each on the same graph using a different colour.



Original Function	
x	y = f(x)

First transformation Y = g(x) = 2f(x)	
new x	new y = g(x)=2f(x)

Second transformation y = h(x) = ½ f(x)	
new x	new y = h(x)= ½ f(x)

Third transformation y = m(x) = -3f(x)	
new x	new y = m(x)=-3 f(x)

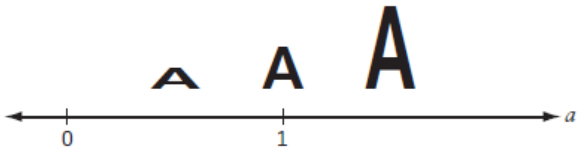
VERTICAL STRETCH RULE (Parameter "a")

A stretch, unlike a translation or reflection, changes the shape of a graph by stretching it left/right or up down, but does not change its orientation (whether it opens up or down)

When the STRETCH NUMBER IS OUTSIDE THE BRACKET & IN FRONT OF f(x)

- When the output (outside) of a function y = f(x) is multiplied by a constant *a,* the result y = af(x) is a vertical stretch of the graph about the x axis by a factor of $|a|$. If a<0 then the negative sign with "*a*" causes an additional reflection of the graph about the x axis in addition to the stretch.
- A vertical stretch makes a function shorter (compression) or taller (expansion) because the stretch multiplies or divides each *y*-coordinate by a constant factor while leaving the *x*-coordinate unchanged (it is because the x coordinates remain unchanged that we say it is a stretch **ABOUT THE X AXIS**)

Vertical stretch by a factor of $|a|$:
- $y = a\,f(x)$ or $\frac{1}{a}y = f(x)$
- $(x, y) \rightarrow (x, ay)$
- shorter: $0 < |a| < 1$
- taller: $|a| > 1$



EXPLORATION ACTIVITY B FOR 1.2: Stretches in the Y Axis

Given an original function y = f(x), explore what happens when we look at y = f(bx)

4. Go to Desmos.com and enter $y=(bx)^2$ as your second equation.
   - It should ask you if you want to add a slider. Click on the "b" to add a slider for "b"

NOTE: The value of "b" will be used to view the graph as stretching horizontally rather than vertically

- What happens to the graph horizontally as you slide the value of "b" to the right of 1 (b is getting larger)? _____
- What happens to the graph horizontally as you slide the value of "b" between 0 and 1 (b is getting smaller)?_____
- What happens to the graph horizontally as you slide the value of "b"smaller than 0 (b is negative)?_____

5. In general, how would you describe the change to the original graph as the value of "b" is adjusted between 0 and infinity?

6. Did the value of "b" behave horizontally in the same way that "a" behaved vertically?

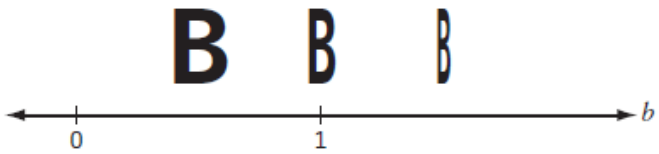
7. Do you think this stretch changes the x or the y values if you were to compare the original table of values to each new table of values?

HORIZONTAL STRETCH RULE

When the STRETCH NUMBER IS INSIDE THE BRACKET of f(x)

- When the input (inside) of a function y = f(x) is multiplied by a constant *b,* the result y = f(bx) is a horizontal stretch of the graph about the y axis by a factor of $\left|\frac{1}{b}\right|$ (NOTE: Because we are dealing with a number INSIDE the bracket we do the opposite operation of that number). If b<0 then that negative sign with "b" causes an additional reflection of the graph about the y axis in addition to the stretch.
- A horizontal stretch makes a function narrower (compression) or wider (expansion) because the stretch multiplies or divides each *x*-coordinate by a constant factor while leaving the *y*-coordinate unchanged (it is because the y coordinates remain unchanged that we say it is a stretch **ABOUT THE Y AXIS**)
-

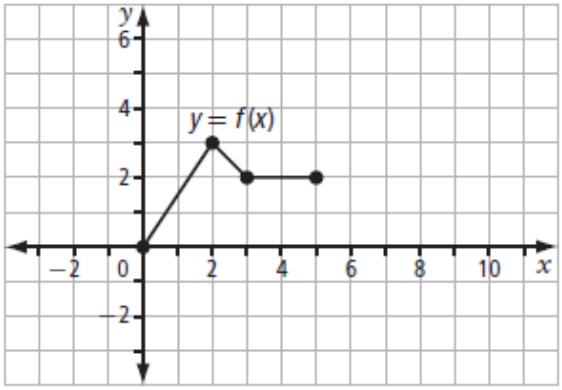
Horizontal stretch by a factor of $\frac{1}{|b|}$:
- $y = f(bx)$
- $(x, y) \rightarrow \left(\frac{1}{b}x, y\right)$
- wider: $0 < |b| < 1$
- narrower: $|b| > 1$



Example #2:

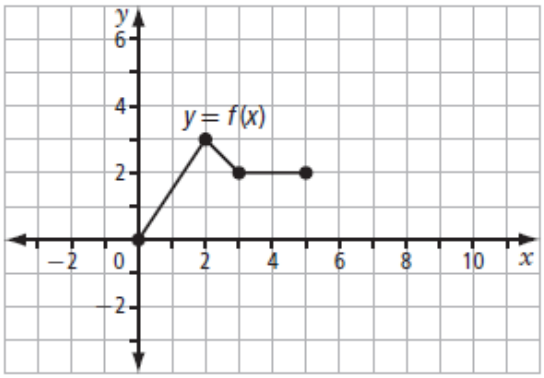
Write an equation representing each of the following transformations of y = f(x). State the domain and range of both the original function(use interval notation) and the mapping transformation. Graph each transformation.

a) vertical stretch by a factor of 2



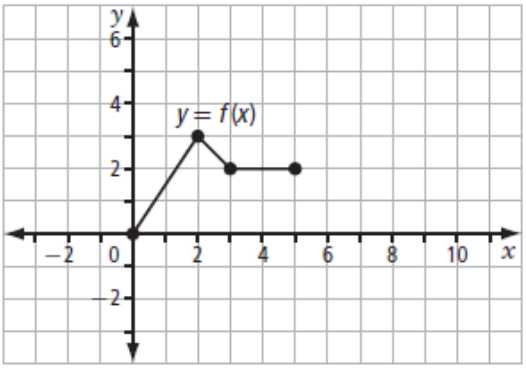
Equation of transformed function:

b) reflection in the x-axis and horizontal stretch by a factor of 2



Equation of transformed function:

c) reflection in the y-axis and horizontal stretch by a factor of $\frac{1}{2}$



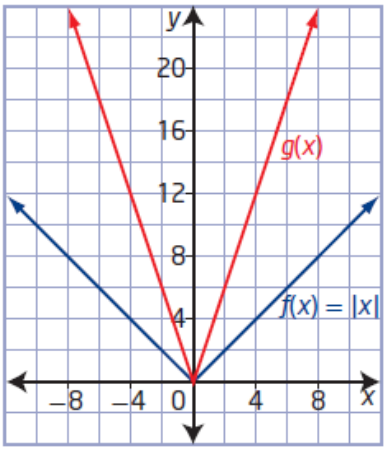
Equation of transformed function:

Recall that $y = f(bx)$ results in a horizontal stretch of $\frac{1}{|b|}$.

Example #4:

The graph of the function $y = f(x)$ has been transformed by either a stretch or a reflection. Write the equation of the transformed graph, $g(x)$.

a)



Summary:

Vertical (affecting the y-values): $y = af(x)$	Horizontal (affecting the x-values: $y = f(bx)$
$a < 0$ (reflection across x-axis)	$b < 0$ (reflection across y-axis)
$\lvert a \rvert > 1$ is a vertical expansion	$\lvert b \rvert > 1$ is a horizontal compression
$\lvert a \rvert < 1$ is a vertical compression	$\lvert b \rvert < 1$ is a horizontal expansion

1.2 ASSIGNMENT (Must use loose graph paper):

1.2 FA: **P28 #2ab, 5, 6, 7ac, 9**
1.2 MLA: **P28 #7bd, 8, 10, 11, 12, 15cd**
1.2 ULA: **P28 #14, C1, C2, C3, C4**

1.3 Combining Transformations

To sketch the graph of a function that has undergone more than one transformation in the form of *y = af(b(x – h)) + k.* To write the equation of a function that has undergone translations, reflections and stretches.

- Multiple transformations can be applied to a function using the general transformation model:
  y = af(b(x – h) + k → Please remember that the *a, b, h* and *k* variables are called parameters and will be replaced with numbers but the *y, f* and *x* values remain as variables
- Because the values for a and k are OUTSIDE the function, they affect the y value in the mapping diagram and we use the original operation in our mapping diagram
- Because the values for b and h are INSIDE the function, they affect the x value in the mapping diagram and we use the opposite operation for b and h in the mapping diagram
- We will have to take BEDMAS into account when we transfer between the general transformation equation and a mapping diagram. Because *a* and *b* are being multiplied/divided, those operations must be performed before *h* and *k*, as *h* and *k* are being added/subtracted. Therefore we perform stretches and reflections BEFORE translations

MAPPING GUIDE: $(x, y) \rightarrow \left(\frac{1}{b}x + h,\ ay + k\right)$

Example #1: Describe the transformations from y = f(x)

$g(x) = -\frac{1}{2} f(3x - 30) + 4$

WRITE THIS TRANSFORMATION IN MAPPING NOTATION

- a = b = h = k =

Describe all changes:

-
-
-
-

Example #2: Given that (3, 5) is a point on the function y = f(x):

i) Write the mapping notation ii) Find the new point

a) y = 3f(x) – 2

b) $y = f\left(-\frac{1}{3}(x+2)\right)$

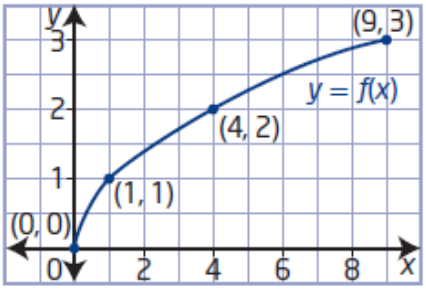
c) y = -2f(3x) – 5

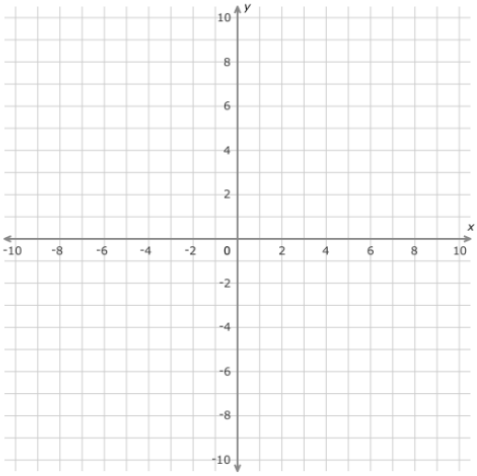
d) y + 3 = -f(x – 6)

e) y = f(3x + 6) + 2

f) $y = f\left(\frac{1}{2}x - 2\right)$

Example #3: Given y = f(x)





a) Sketch the graph of g(x) = 3f(2x)

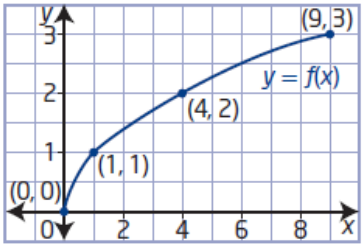
→ Identify the values of *h, k, a, b*

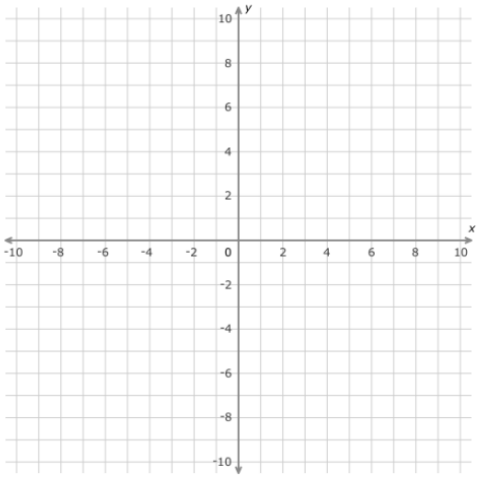
→ Describe the transformation in words

→ Write the mapping notation

→ Create a table of values for f(x) and g(x) using the mapping notation

b) Using the same original f(x)





Sketch the graph of g(x) = f(3x + 6)

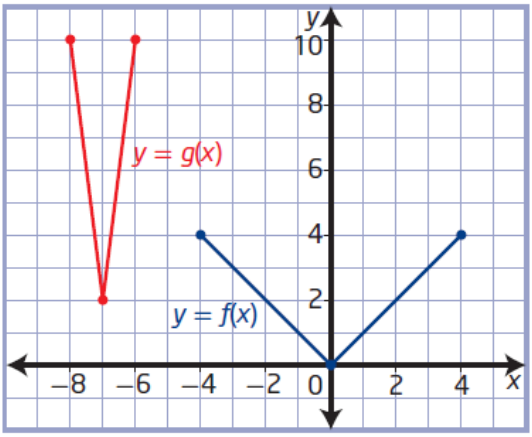
→ Identify the values of *h, k, a, b*

→ Describe the transformation in words

→ Write the mapping notation

→ Create a table of values for f(x) and g(x) using the mapping notation

Example #4: The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



- Decide if there is any reflection. If so, it is vertical, horizontal or both?

- Compare distances between key points to determine the stretches. Compare the domains and ranges of each function to help determine *a* and *b*.

- Identify the value of h and k by examining the translations. Pick a point that is NOT affected by a reflection or stretch (you can use an appropriate x or y intercept)

- Write your answer in the an equation of the form y = a(f(b(x – h)) + k

- What do you think the base function is? Using the base function for the original graph, what is our new equation?

1.3 ASSIGNMENT (Must use loose graph paper):

1.3 FA: P39 #1, 2, 3, 4, 5a, 6, 7acd, 8a, 9ce, 10abc
1.3 MLA: P39 #5b, 7bef, 8b, 9abdf, 11, 12, 13, 14
1.3 ULA: P39 #15, 16, 17 , 18, C2

1.4 Inverse of a Relation

To graph and find the equation of the inverse of a given relation.

- The inverse of a relation is found by interchanging the x coordinates and the y coordinates of the ordered pairs of the relation. In mapping notation this looks like (x, y) $\longrightarrow$ (y, x)
- To find the inverse equation, interchange the x and y variable in the original equation and solve for the new "y". The original relation is y = f(x) while the new function is called x = f(y). In order to have the equation solved for y we actually rewrite this new relation as $f^{-1}(x) = y$ (inverse of f of x equals y).
  - **Note:** This -1 looks like an exponent but it is NOT AN EXPONENT. This is because f represents a function, not a variable. (Remember that when we have an exponent of -1, it means to reciprocate as in $x^{-1} = \frac{1}{x^1}$. The -1 in $f^{-1}(x)$ is NOT AN EXPONENT AND THEREFORE THERE IS NO RECIPROCAL PROPERTY THAT APPLIES).
- The inverse of a function is itself not necessarily a function. Remember we can test to see if a relation is a function by checking either to see that each element in the domain (x) has a unique element in the range (y) (no repeats in x) or to see if a graph passes the vertical line test (VLT)

Example #1:

- Given the graph of the following relation, sketch the graph of the inverse relation in a different colour.

- State the domain and range of the relation and of its inverse. What do you notice?

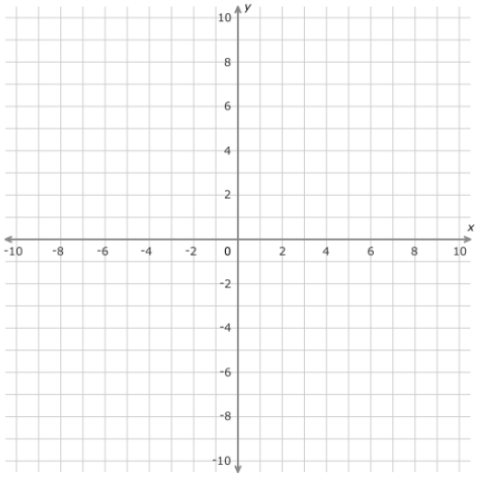
- Determine whether the relation and its inverse are functions. Is there any way you could predict whether or not the inverse graph will be a function just by looking at the original function?

- Is there any other relationship that exists between the two graphs? Are there any invariant points?

Example #2:

Consider the relation $f(x) = (x + 2)^2$.

a) Graph the relation. How can we use the idea of transformations on base functions to graph the given function? Is this relation a function?



b) Find the equation of the inverse and graph it. What is the easiest way to graph the inverse relation? Is the inverse a function?

c) How could you restrict the domain so that the inverse IS a function?

Example #3:

Algebraically determine the equation of the inverse of each function.

a) $f(x) = 3x + 6$

b) $f(x) = x^2 - 4$

1.4 ASSIGNMENT (Must use loose graph paper):

1.4 FA: P51 #1b, 2b,3, 4, 5ace, 6, 9ac

1.4 ULA: P52 #5f, 9d, 12a, 13a, 15ab

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

1. **Section 1.1**

- https://goo.gl/5Tgrni
- https://goo.gl/hYn44P
- https://goo.gl/b5cU6j
- https://goo.gl/vh3JUC
- https://goo.gl/FVQs9F
- https://goo.gl/xpkfEs
  Note! In the last two videos they are using *c* and *d* instead of *h* and *k*.

2. **Section 1.2**

- https://goo.gl/S6HzYt
- https://goo.gl/5jXBtD
- https://goo.gl/KnB81T
- https://goo.gl/3Kgb9C
- https://goo.gl/3azaGj
- https://goo.gl/3azaGj

3. **Section 1.3**

- https://goo.gl/dkgbQf
- https://goo.gl/1cWL7E
- https://goo.gl/FH9oED

4. **Section 1.4**

- https://goo.gl/iCqbC2
- https://goo.gl/2udijP
- https://goo.gl/4UN4HY
- https://goo.gl/eerQ1b