5.1 Representing Relations

In this section, we will review how to represent Relations in the following ways: as a set of ordered pairs, an arrow diagram, a table and a graph.

Online Video Lesson: **https://goo.gl/9bZs3E**

ACTIVITY #1: Turn to page 256 and complete the activity "Make Connections"

SET: A collection/list of distinct objects.
EX: A List of Teachers: {Ms. Carignan, Mr. McMillan, Mr. Adams, Ms. Moroz, Ms. Sebastian}
EX: A List of Subjects: {English, Math, Science, Wellness}

ELEMENT: What you individually call each item in the SET.
EX: Mr. McMillan is one element in the Random List of Teachers

NOTE: *The list of elements in the set are usually enclosed in braces (curly brackets). The order in which the elements are listed does not matter.*

RELATION: Something that associates the elements of one set to the elements of another set

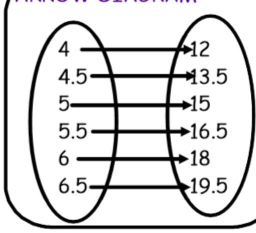
RELATIONS CAN BE EXPRESSED IN THE FOLLOWING WAYS:

WORDS
Three times the length of your ear, e, is equal to the length of your face, f (from chin to hairline)

EQUATION
f = 3e

ORDERED PAIRS
(4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)

ARROW DIAGRAM



GRAPH

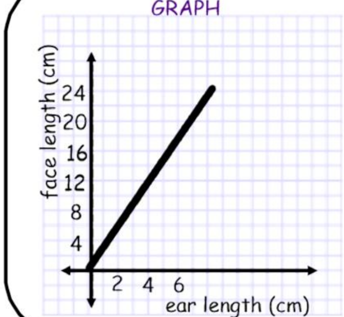


TABLE OF VALUES

Ear Length (cm)	Face Length (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

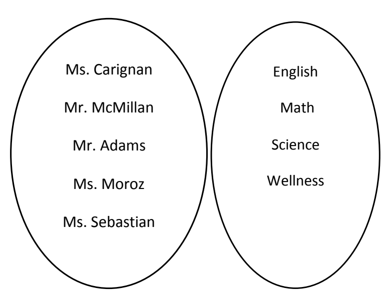
Example #1: Using the examples given in the definitions on the previous page, represent the relationship between the set of Teachers to the Set of Subjects in the following ways:

a) In a Table

Teacher Name	Subject

b) As a list of Ordered Pairs

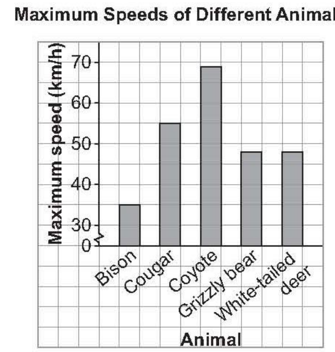
c) As an arrow diagram



When the elements of one or both sets are numbers, the relation can be represented as a bar graph.

Example #2: Using the information represented by the following bar graph, re-represent the relation as:

a) A table



b) A list of ordered pairs

c) An arrow diagram

CALCULATORS NOT ALLOWED!!! Label this assignment properly!

5.1 *FA (Foundational Assign)(C23)
P 262 #4, 6, 7, 10, 12 & P 167 #21

5.1 MLA (Mid Level Assignment) (C14, 15)
P 262 #13

NC 5.1 Assignment for Concepts #23

5.2A & 5.5A Properties of Functions

NC 5.2A & 5.5A Concept: #23, 24, 25

In this section, we will learn about DOMAIN, RANGE and FUNCTIONS. These are terms we use when we are dealing with questions about Relations between two Sets.

Online Video Lesson: **https://goo.gl/y9MWZR**

There is a special class of relations, called functions, where two quantities depend on each other in a particular way.

- The amount of tension on a guitar string determines the musical note played.
- The channel displayed on your television screen depends on the number you enter into the remote.

Describe other examples from your daily life where two quantities depend on each other in a particular way.

INVESTIGATION

Study the following relations. They are categorized as functions and non-functions.

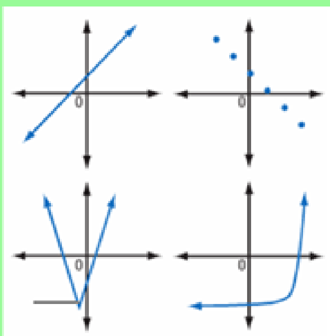
These 8 relations ARE functions

x	y
5	10
6	15
7	20

x	y
11	3
21	3
31	3

$\{(-2, -5), (0, 4), (2, 13), (4, 22)\}$

$\{(10, 10), (12, 10), (14, 12), (16, 12)\}$



What is similar about the functions?

What is similar about the non-functions?

How can you tell whether or not a relation is a function?

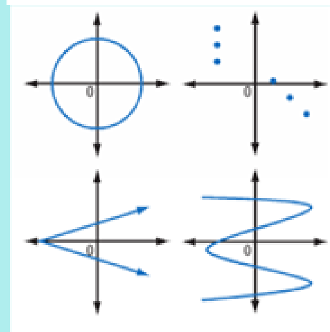
These 8 relations ARE NOT functions

x	y
6	10
6	15
7	20

x	y
3	11
3	21
3	31

$\{(10, 10), (12, 10), (12, 14), (12, 16)\}$

$\{(7, 5), (7, 8), (9, 11), (11, 14)\}$

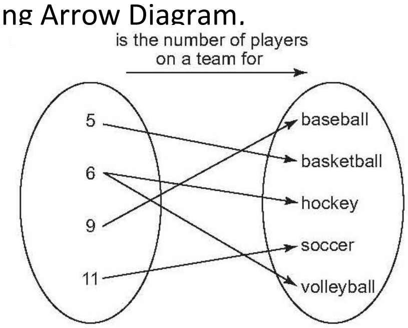


DOMAIN: The list of all elements in the first set of a relation.

RANGE: The list of all elements in the second set of a relation.

FUNCTION: A specific type of relation that occurs when each element in the DOMAIN is associated with exactly one element in the range. This means that you don't see any repeated elements in the first column of a table, in the first numbers of a set of ordered pairs, or more than one arrow coming from any element in the first oval of an arrow diagram.

Example #1: Answer the following questions using the following Arrow Diagram.



a) State the Domain

b) State the Range

c) Is the given relation is a function? Why or why not?

Example #2: The following list of ordered pairs describes the relationship between certain months and number of students in Ms. C's Math 10 class that have their birthdays in that month.

{(January, 4), (February, 7), (March, 3), (April, 4), (May, 6)}

a) State the Domain

b) State the Range

c) Is the given relation a function? Why or Why not?

Example #3: Rewrite the ordered pairs in Example 2 by exchanging the Domain with the Range.

a) What is the new Domain?

b) What is the new Range?

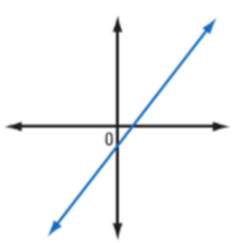
c) Is the new Relation a Function? Why or why not?

Vertical Line Test

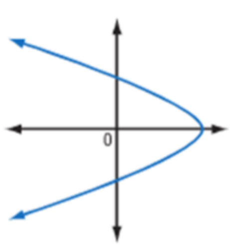
* a test to see if a graph represents a function
* if any vertical line intersects the graph at more than one point, the relation is not a function.

Example #4: For each pair of relations, decide which relation is a function and which relation is not a function. Explain your choice.

a) A



B



Did You Know?

The term *function* was developed and used by German mathematician Gottfried Wilhelm Leibniz (1646–1716). The notation we use today was created by Swiss mathematician Leonhard Euler (1707–1783). Euler contributed to many branches of mathematics.

Example #3: The following table shows the fines for speeding in a 60 km/h zone:

a) What is the Domain?

b) What is the Range?

c) Is the relation a function? Why or why not?

Speed, *s* (km/h)	Fine, *f* ($)
75	89
80	124
85	150
90	177
100	264
110	351

CALCULATORS NOT ALLOWED!!! Label this assignment properly!

5.2A & 5.5A *FA (Foundational Assign)
P 270 #4, 5, 10 11, 12, 13
Extra questions in this handout (do on looseleaf) (Omit 1F for now)

C 5.2A & 5.5A Assignment for Concepts #23, 24, 25

1. Determine whether each relation is a function or is not a function. Give a reason for your answer.

a) $(-1, 2), (0, 1), (1, 2), (2, 5)$

b) $(3, 12), (4, 12), (5, 14), (6, 14)$

c) $(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$

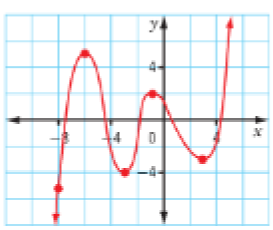
d)

x	*y*
0	0
1	−1
1	1
4	−2
4	2

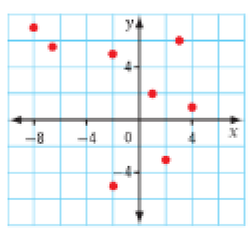
e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

f)



g)



2. a) For above question 1d) identify the domain and the range.
   b) For above question 1g) identify the domain and the range.

5.2B & 5.5B Function Notation

NC 5.2B & 5.5B Concept: #24, 25, 26

In this section we will learn how to write a function both as Equation in Two Variables and in Function Notation. We will also learn how to find the Value of a Function at different points.

Online Video Lesson: https://goo.gl/4jDY16

EQUATION IN TWO VARIABLES ⟷ FUNCTION NOTATION

We can use any other letter such as g, h or k to name a function

- Any function that can be written as an equation in two variables can be function notation. For example, to write the equation $d = 4t + 5$ in function notation, we may write $d(t) = 4t + 5$. t represents an element of the domain and d(t) represents an element of the range.

$$d = 4t + 5 \longleftrightarrow d(t) = 4t + 5$$

- When we write an equation that is not related to a context, we use x as the independent variable and y as the dependent variable. Then an equation in two variables, such as $y = 3x - 2$ may be written as $f(x) = 3x - 2$.

$$y = 3x - 2 \longleftrightarrow f(x) = 3x - 2$$

- Conversely, we may write an equation in function notation as an equation in two variables. For example, the equation $C(n) = 300 + 25n$, we write $C = 300 + 25n$. And, for the equation $g(x) = -2x + 5$ we write $y = -2x + 5$
-

$$C(n) = 300 + 25n \longleftrightarrow C = 300 + 25n$$

$$g(x) = -2x + 5 \longleftrightarrow y = -2x + 5$$

Example #1:

a) Write the function $f(x) = -5x + 11$ as an equation in 2 variables.

b) Write the equation $C = 2.54i$ in function notation.

Example #2:

The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function. Write the equation in function form.

b) Determine the value of V(600). What does this number represent?

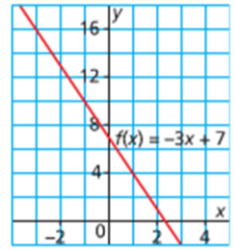
c) Determine the value of d when $V(d) = 26$. What does this number represent?

Example #3:

Here is a graph of the function f(x) = -3x + 7

a) Determine the range value when the domain value is -2

b) Determine the domain value when the range value is 4.



Independent Variable - The variable for which values are selected (typically the x values)

Dependent variable - The variable whose values depend on those of the independent variable (the y values)

Discreet Data – Data that when graphed, is not connected because it doesn't make sense. Typically discreet data is data that does not involve fractions or decimals.

Continuous Data - Data that when graphed, has connected points. Continuous data includes all decimal or fractional points between pieces of data.

Example #4: The relationship between the number of slices of pizza ordered and the total cost is given by the equation C = 2s + 1

a) Fill out the table that will show the relationship between the number of slices and the cost for 1 – 5 slices.

Slices (S)	c = 2s + 1	Cost (C)

b) What is the independent variable?

c) What is the dependent variable?

d) Find the value of the equation c = 2s + 1 when s = 40

e) Write the given equation in function notation.

f) Use the function notation equation to re-find the first two rows of the column.

g) Use the function notation equation to find s(40). Which of the above questions does this replicate?

h) Find the value of s when c(s) = 67.

Example #5:

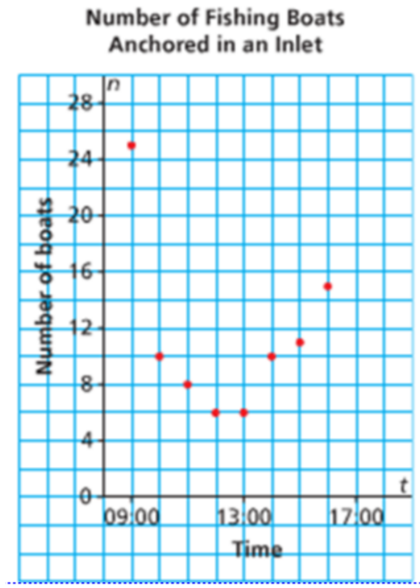
This graph shows the number of fishing boats, n, anchored in an inlet in the Queen Charlotte Islands as a function of time, t.

a) Identify the dependent variable and the independent variable.

Justify the choices.

b) Why are the points on the graph not connected? Explain.

c) Determine the domain and range of the graph.

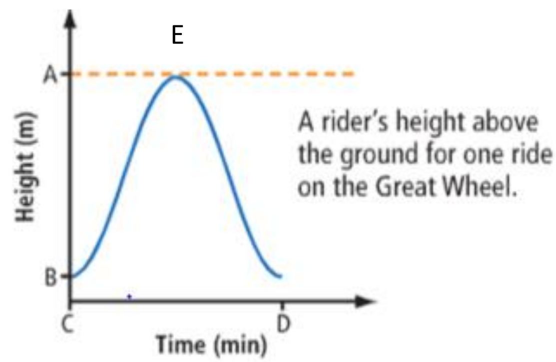


Example #6:

The Great Wheel is being built in Beijing in the People's Republic of China. When finished, it will be the largest Ferris wheel in the world. The wheel will have a diameter of 193 m and will reach a maximum height of 208 m. The graph shows a rider's height relative to the ground for a 20-min ride through one rotation.

a) What are the values of points A, B, C, D, E and what do they represent?

b) Describe the domain and the range of the graph?

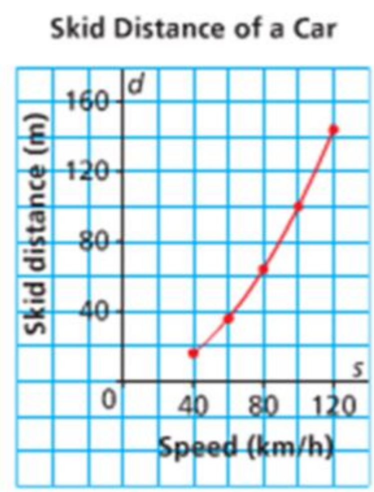


Example #7:

When police officers investigate a car crash, they estimate the speed the car was travelling by measuring the skid distance.

a) Why are the points on the graph connected?

b) Estimate and describe the domain and range of the graph. Are there any **restrictions** on the domain and range? Explain.

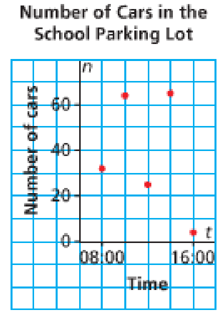


CALCULATORS NOT ALLOWED!!! Label this assignment properly!

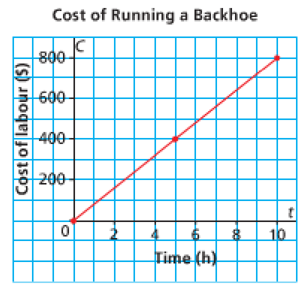
5.2B & 5.5B *FA (Foundational Assign) (C 24, 25)
EXTRA HANDOUT On next page

C 5.2B & 5.5B
Assignment for
Concepts #24, 25

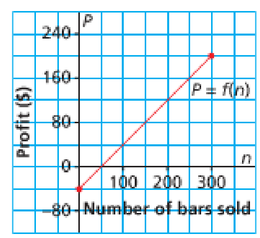
1. This graph shows the number of cars, n, in the school parking lot as a function of time, t.
   a) Identify the independent and dependent variables. Justify your choices.
   b) Why are the points on the graph not connected?
   c) Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.



2. St. Adolphe, Manitoba, is located in the flood plain of the Red River. To help prevent flooding, backhoes were used to build dikes around houses and farms in the town. This graph shows the labour costs for running a backhoe.
   a) Identify the independent and dependent variables. Justify your choices.
   b) Why are the points on the graph connected?
   c) Write the domain and range. Are there any restrictions on the domain and range? Explain.



3. Northlands School Outdoor Club had a fundraiser to help purchase snowshoes. The club had 300 power bars to sell. This graph shows the profit made from selling power bars.
   a) Describe the domain and range for the function. Are there any restrictions on the domain and range? Explain.



4. This graph shows the recommended maximum heart rate of a person, R beats per minute, as a function of her or his age, a years, for a stress test.
   b) Write the domain and range. Are there any restrictions on the domain and range? Explain.

5.5C DOMAIN AND RANGE CONT.

NC 5.5C Concept: #25, 26

WORDS can be used to describe the values that are allowed. For example, the domain is the set of all real numbers between 0 and 10, inclusive. The range is the set of all real numbers greater than 20.

A LIST is a useful way to give the domain and range for discrete data when there are not many numbers in the set. For the relation (0, 0), (1, 5), (3, 7), (5, 7) the domain is {0, 1, 3, 5} and the range is {0, 5, 7}

NUMBER LINES give a picture of the values that are allowed. For example, this number line represents all numbers between 0 and 10, inclusive.

The solid circles at 0 and 10 indicate that these points are included.

This number line represents all numbers greater than 20.

The open circle at 20 indicates that the value 20 is not included. The arrow pointing to the right indicates that there is no upper limit.

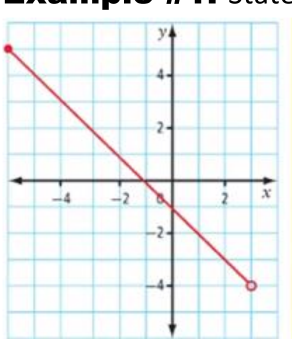
This number line represents the discrete list of numbers -2, 0, 4, 8, and 10

SET NOTATION is a formal mathematical way to give the values of the domain and range.

Set Notation	What It Means
The domain: $\{x \mid x \leq 10, x \in R\}$	{} is the type of brackets used for a set. $\in$ means "is an element of". $\mid$ means "such that". The statement is read as follows: x is an element of the real numbers such that x is less than or equal to 10.
The range: $\{y \mid y > 20, y \in R\}$	The statement is read as follows: y is an element of the real numbers such that y is greater than 20.

INTERVAL NOTATION used different brackets to indicate an interval. This style of bracket,], is used if the end number is included. This style of bracket,), is used if the end number is not included. The infinity symbol, ∞, is used if there is no end point. A domain of all numbers between 0 and 10, inclusive, would be given as [0, 10]. A range of all numbers greater than 20 would be given as $(20, \infty)$

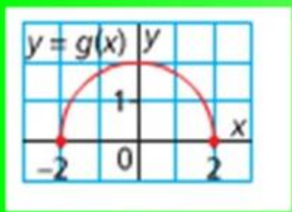
Example #1: State the domain and range of the following using two different notations.



Is a list of numbers in {} a suitable way to list the domain and range for this function? Why or why not?

Example #2:

Choose any way to describe the domain and range of the relation. Is it a function?



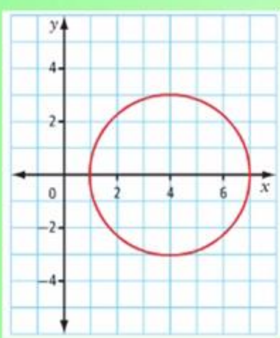
When a relation is continuous (a solid line) it makes more sense to use set notation or interval notation to describe the domain and range.

When a relation is not continuous (just points) it makes more sense to use a list to describe the domain and range.

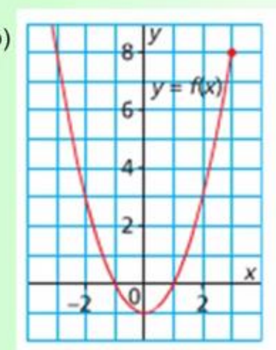
Example #3:

Use set notation and interval notation to describe the domain and range of each relation.

a)



b)



a) i) Is this relation a function?

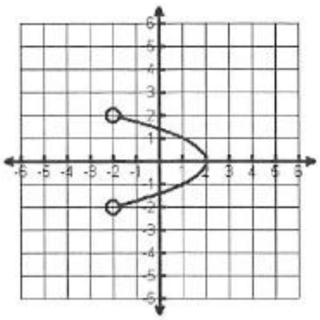
b) i) Is this relation a function?

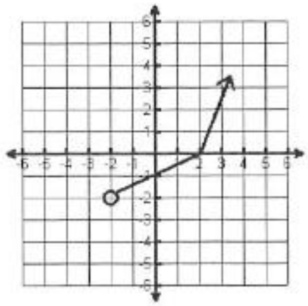
ii) What would the domain and range be if the graph had an open circle at the right end of the graph?

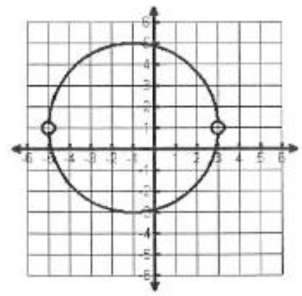
iii) What would the domain and range be if the graph had an open circle at the both ends of the graph?

Example #4:

State whether the following are functions and give the domain and range in both interval and set notation. Are they functions?







CALCULATORS NOT ALLOWED!!! Label this assignment properly

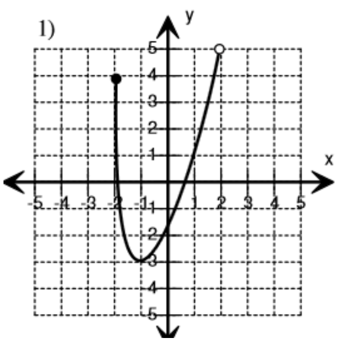
5.5C *FA (Foundational Assign)(C25 & 26)
Assignment below (on looseleaf)

NC 5.5C Assignment
for Concepts #25 & 26

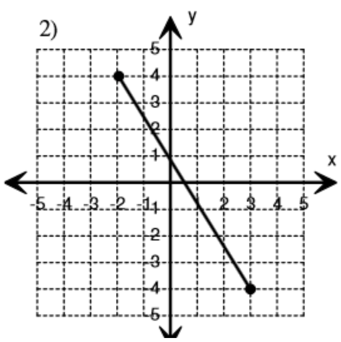
Part A:

Find the Domain and Range for each graph. ANSWER USING INTERVAL NOTATION: Is the graph a function?

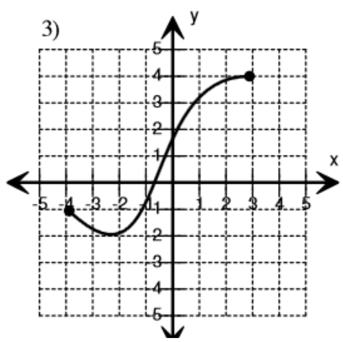
1)



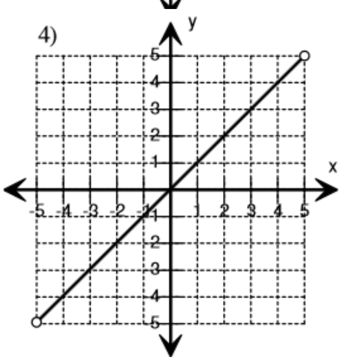
2)



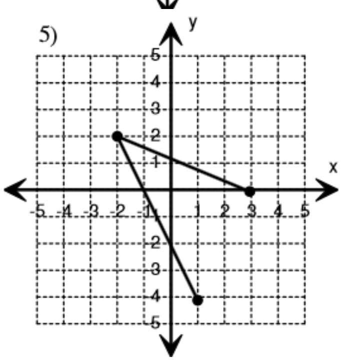
3)



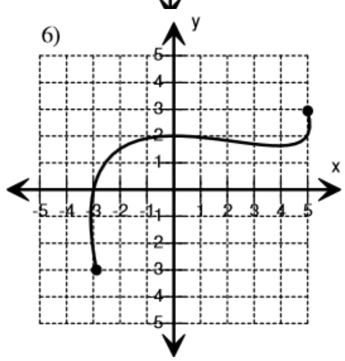
4)



5)



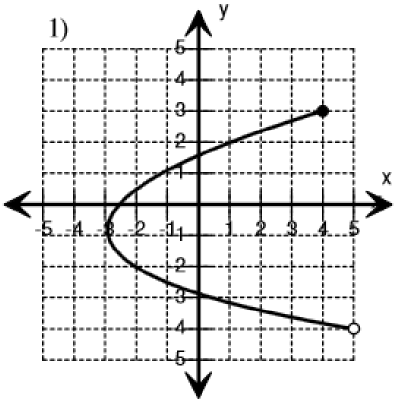
6)



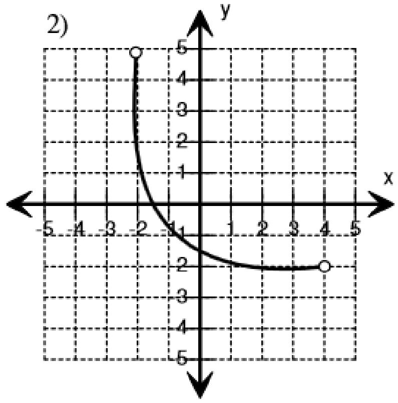
Part B:

Find the Domain and Range for each graph. Give your answer in SET NOTATION FORM.

1)



2)



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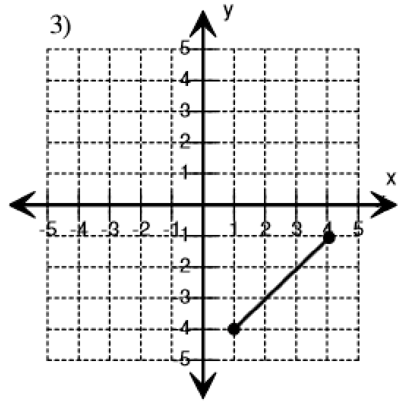
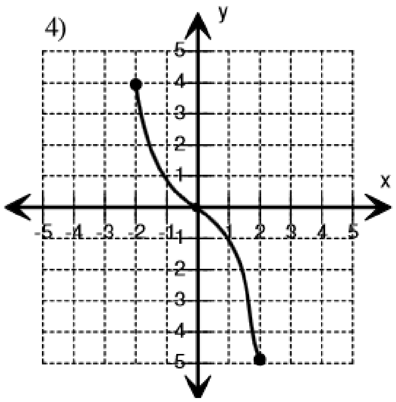
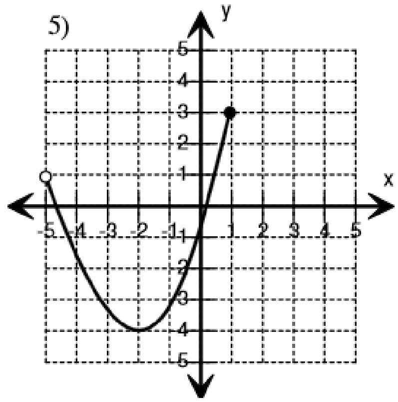


4)



5)



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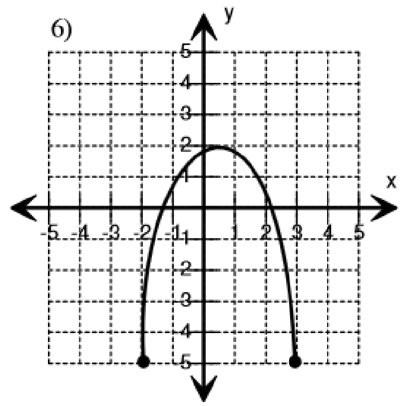

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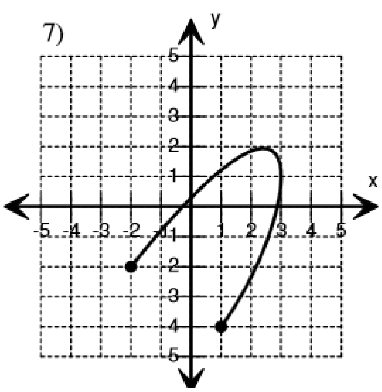

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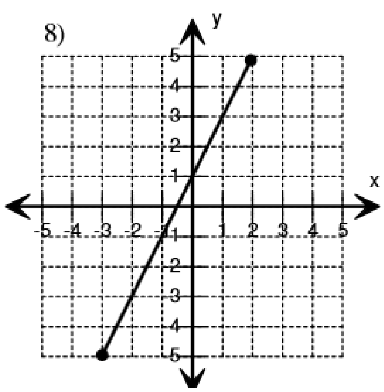

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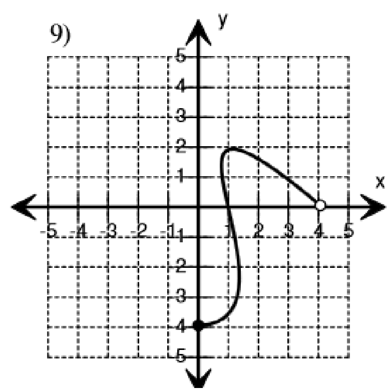


5.3 Interpreting and Sketching Graphs

NC 5.3 Concept: #23, 24, 25, 26

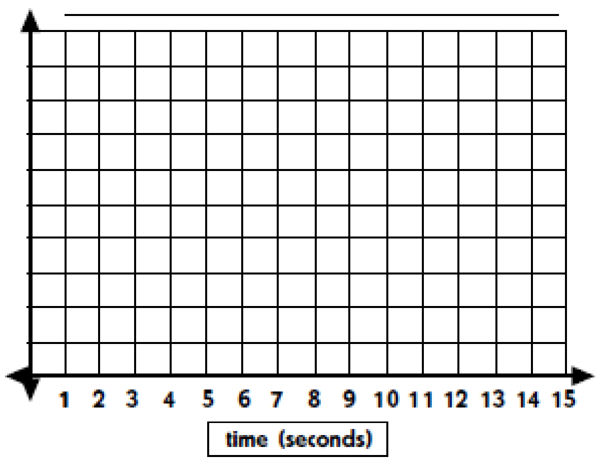
In this section we will learn how to describe what is happening in a given graph as well as sketch a possible graph for a given situation.

http://www.graphingstories.com/

GRAPHING STORIES

______________________ (name)

1.



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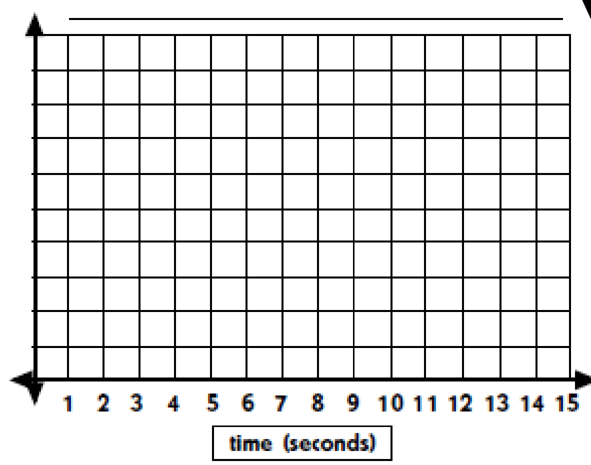

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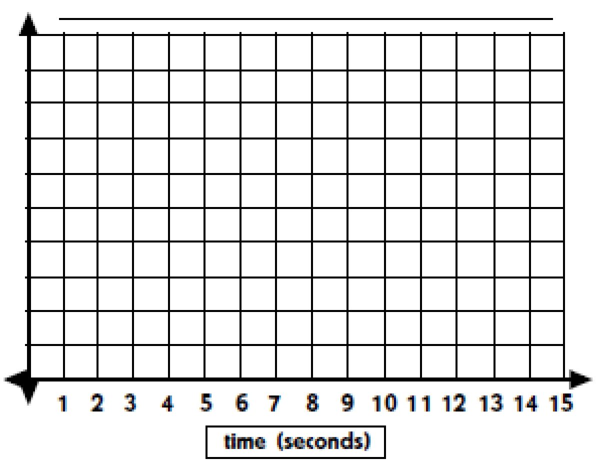

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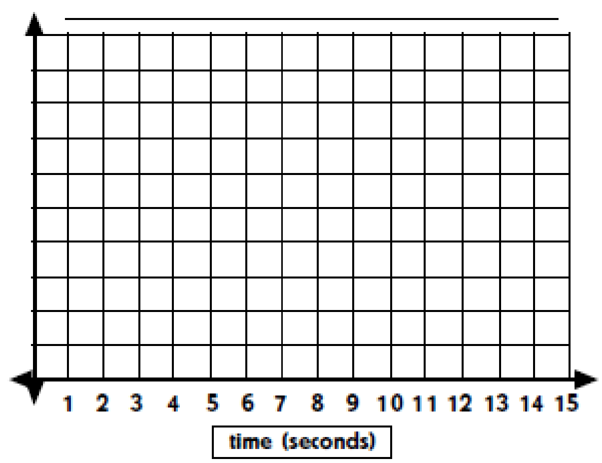

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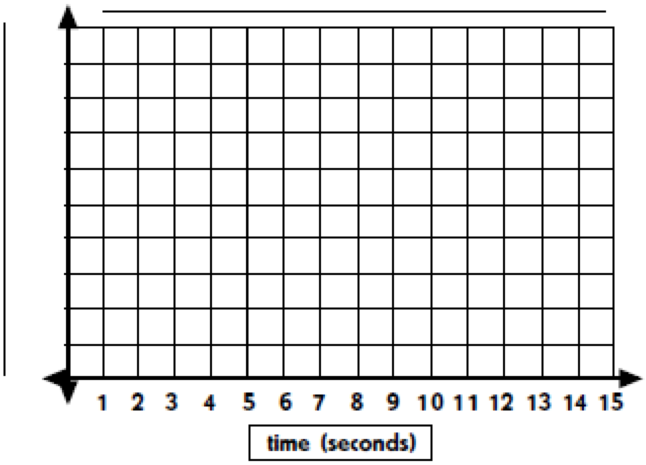

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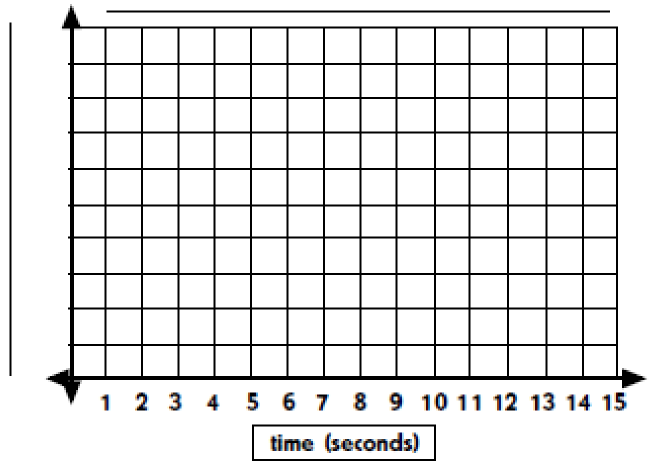

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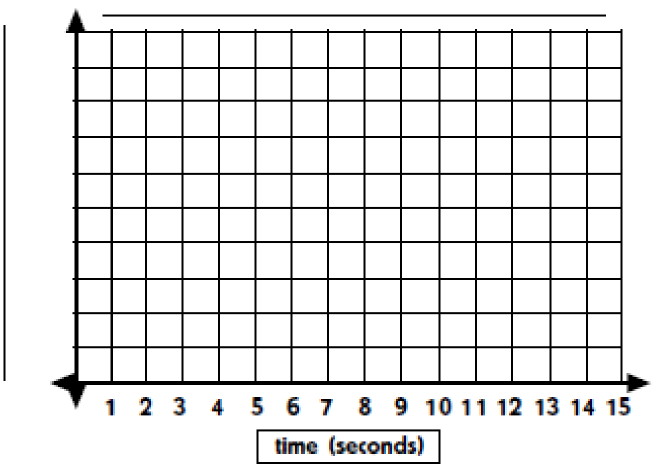

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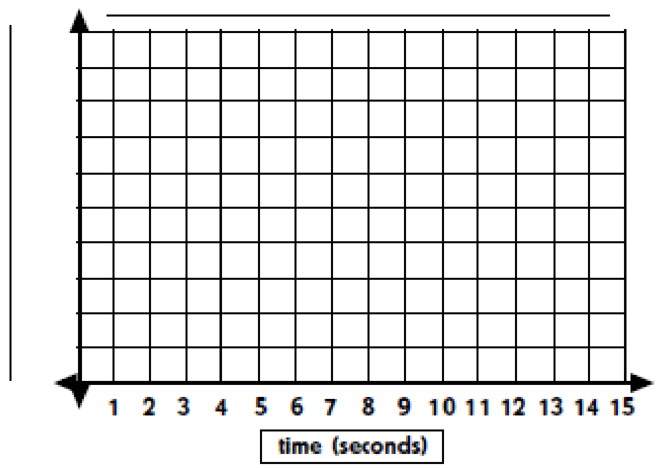
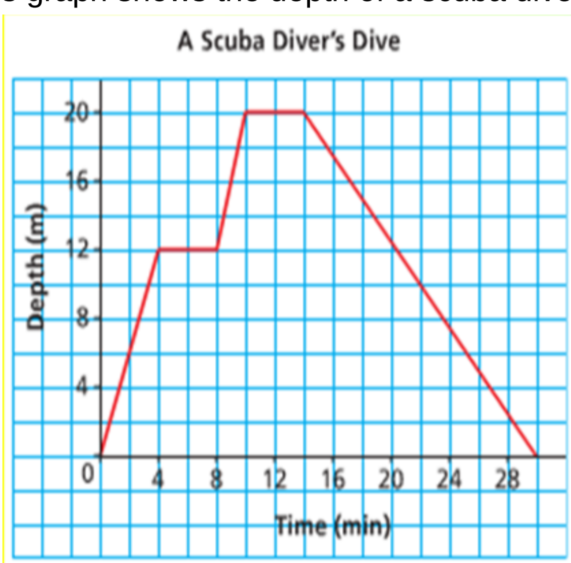


Online Video Lesson: **https://goo.gl/qE12U8**

Example #1: In math, a graph provides much information.

This graph shows the depth of a scuba diver as a function of time.



How many minutes did the dive last?

At what times did the diver stop her descent?

What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

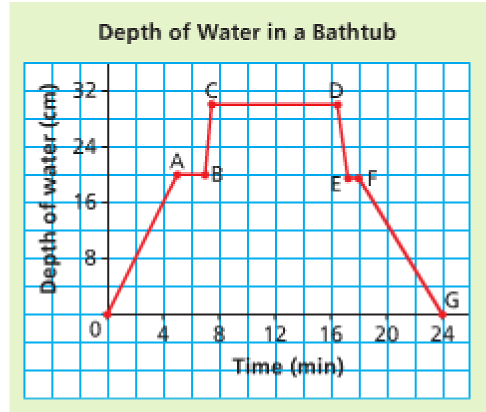
What is the domain?

What is the range?

INVESTIGATION

This graph shows the depth of water in a bathtub as a function of time.

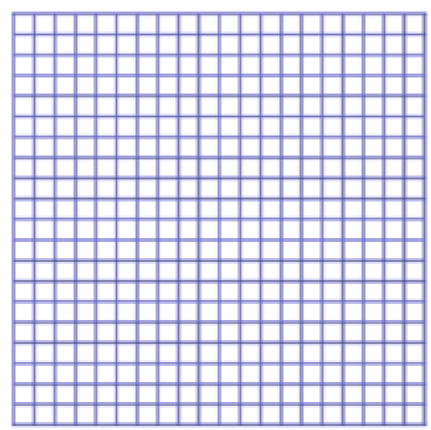
- What does each segment of the graph represent?



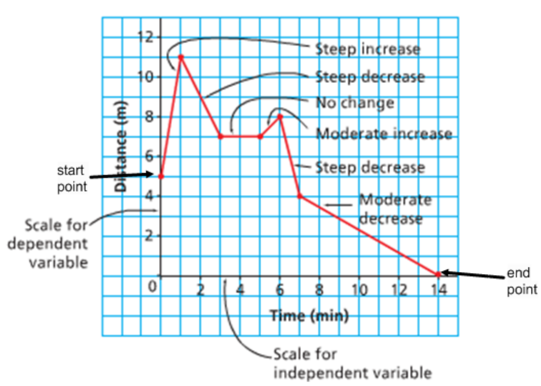
- Compare your description with that of your partner. Are both your stories the same?
- Should they be? Explain.
- What is the domain and range?

Sketch a graph to represent this situation:

- You put the plug in the bath and turn on the taps.
- Then you leave the bathroom and return to discover that the bath has overflowed.
- Then you turn off the taps and pull out the plug to let out some water. You put the plug back in

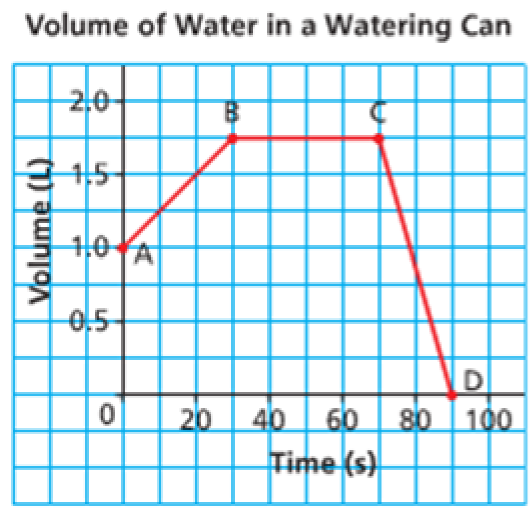


MAKING THE CONNECTION...

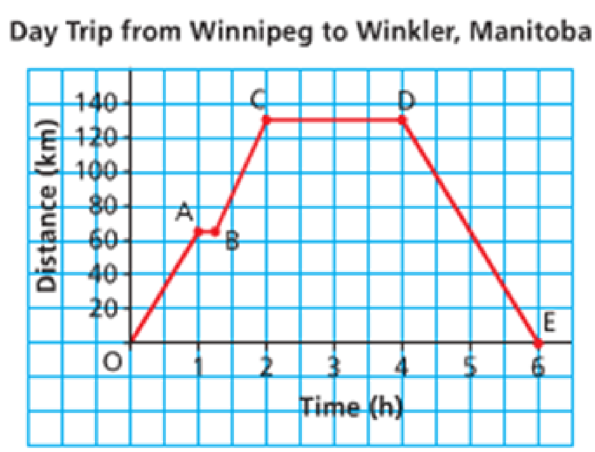


Example #2:
The graph shows how the volume of water in a watering can changes over time.

- What is the starting volume?
- Which point is this?
- Describe segment AB and what that means.
- Describe segment BC and what that means.
- Describe segment CD and what that means.
- What does point D represent?
- What is the domain and range?



Example #3:
Describe the journey for each segment of the graph.



The distance between Winnipeg and Winkler is 130 km.

What was the total driving time? Explain.

What are the dependent and the independent variables?

Example #4:

Josaphee leaves her home and walks to the store. After buying a drink, she slowly jogs to her friend's house. Josaphee visits with her friend for a while and then runs directly home. Using the distances shown, draw a distance-time graph that shows Josaphee's distance from her house. Explain each section of your graph.

CALCULATORS NOT ALLOWED!!! Label this assignment properly!

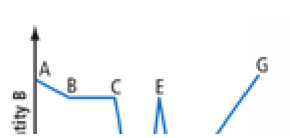
5.3 *FA (Foundational Assign)(C23, 24, 25, 26)
P 281 #1, 3, 5, 7, 10, 11, 13, 16

5.3 MLA (Mid Level Assignment) (C14, 15)
P 282 #14, 17, 18 & Extra Questions (Below)

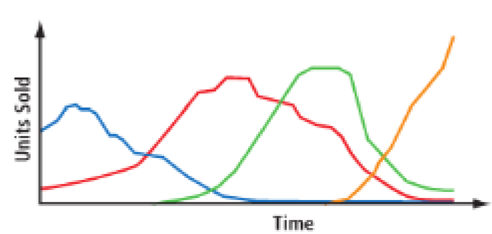
NC 5.3 Assignment for Concepts #23-26

Extra Questions: (on looseleaf)

1. The graph shows how quantity B is changing relative to quantity A. Describe each section of the graph as representing a constant increase, a constant decrease, an increase that is not constant, a decrease that is not constant, or no change. Explain your answers.



2. Formats for distributing recorded music have changed through the years. Study the multi-line graph. Predict which line represents each format: vinyl albums, cassette tapes, compact discs, and digital downloads. Explain your choices.



SOLUTIONS TO EXTRA QUESTIONS FOR 5.2A & 5.5A

1 a) Function (No repeats in domain)
b) Function (No repeats in domain)
c) Function (No repeats in domain)
d) Not function (1 & 4 repeat in Domain)
e) Function (No repeats in domain)
f) Function (passes VLT)
g) Function (fails VLT at x = -2)

2. a) D = {0, 1, 4} R = {-2, -1, 0, 1, 2}
b) D = {-8, -7, -2, 1, 2, 3, 4} R = {-5, -3, 1, 2, 5, 6, 7}

SOLUTIONS TO EXTRA QUESTIONS FOR 5.2B & 5.5B

1. a) Independent variable is time t, dependent variable is number of cars n
b) Cars can only be counted in whole numbers and don't allow for decimals
c) D:{8, 10, 12, 14, 16} R:{5, 25, 32, 64, 65} Approx values

2. a) Independent variable is time t, dependent variable is cost of labour, c
b) Both time and cost allow for decimal answers
c) D:[0, 10] R:[0, 800] Restrictions are $t \geq 0$, $c \geq 0$

3. D:[-40, 180] R:[0, 300] Restrictions are $x \geq 0$, no restrictions on profit

4. D:[10, 90] R:[110, 175] Restrictions are $a \geq 0$, $r \geq 25$ (this is an approximate lowest possible heart rate)

SOLUTIONS TO EXTRA QUESTIONS FOR 5.5C

Part A:
1. D:[-2, 2) R:[-3, 5) Not a function (fails VLT at x = -2)
2. D:[-2, 3] R:[-4, 4] Function, passes VLT
3. D:[-4, 3] R:[-2, 4] Function, passes VLT
4. D:(-5, 5) R:(-5, 5) Function, passes VLT
5. D:[-2, 3] R:[-4, 2] Not function Fails VLT
6. D:[-3, 5] R:[-3, 3] Not function, fails VLT at x - -3
7. D:[-2, 3] R:[-4, 2] Not function fails VLT
8. D:[-3, 2] R:[-5, 5] Function, passes VLT
9. D:[0, 4) R:[-4, 2) Not function, fails VLT

Part B: BE CAREFUL CHECKING THE INEQUALITY SIGNS!
1. $D:\{x|-3 \leq x < 5, x \in R\}$ $R:\{y|-4 < y \leq 3, y \in R\}$
2. $D:\{x|-2 < x < 4, x \in R\}$ $R:\{y|-2 < y < 5, y \in R\}$
3. $D:\{x|1 \leq x \leq, 4x \in R\}$ $R:\{y|-4 \leq y \leq -1, y \in R\}$
4. $D:\{x|-2 \leq x \leq 2, x \in R\}$ $R:\{y|-5 \leq y \leq 4, y \in R\}$
5. $D:\{x|-5 < x \leq 1, x \in R\}$ $R:\{y|-4 \leq y \leq 3, y \in R\}$
6. $D:\{x| \leq x \leq 3, x \in R\}$ $R:\{y|-5 \leq y \leq 2, y \in R\}$
7. $D:\{x| \leq x \leq 3, x \in R\}$ $R:\{y|-4 \leq y \leq 2, y \in R\}$
8. $D:\{x|-3 \leq x \leq 2, x \in R\}$ $R:\{y|-5 \leq y \leq 5, y \in R\}$
9. $D:\{x|0 \leq x < 4, x \in R\}$ $R:\{y|-4 \leq y \leq 2, y \in R\}$

SOLUTIONS TO EXTRA QUESTIONS FOR 5.5C

1. AB – Constant change
   BC – no change
   CD – decrease that is not constant
   DE- increase that is not constant
   EF – constant decrease
   FG – constant increase

2. Graph 1 – vinyl albums
   Graph 2 – Cassette tape
   Graph 3 – CD's
   Graph 4 – Digital
   Discuss why