## To evaluate logarithms and solve logarithmic equations.

RECALL: In section 1.4 we learned what the inverse of a function is.

- What is the inverse of the equation $y=2 x-5$ ?
- What properties do their sets of ordered pairs have?
- What properties do their respective graphs have?


## LOGARITHMIC FUNCTION:

- A logarithmic function is the INVERSE of an exponential function.
- Recall that an exponential function is written as $y=c^{x}$ where where $c>0$ and $c \neq 1$., therefore the inverse function (called the logarithmic function) would be written as: $\square$ where $c>0$ and $c \neq 1$. X must also be positive ( $\mathrm{x}>0$ )
- The problem becomes an algebraic issue as there is no way to solve for y in this new equation (no way to get the y by itself). Therefore we have a completely different way to write this equation that means the same thing mathematically BUT ALLOWS US TO SOLVE FOR Y!


This new equation is read as follows: y equals the log of x to the base c .

NOTE: What this new equation is asking can be described by looking back at it's original left version - "what exponent (y) do I give to my base (c) to get my answer (x)?"

Example \#1: Rewrite each of the following in logarithmic form:

| Exponential Form | Logarithmic Form |
| :--- | :--- |
| $2^{3}=8$ |  |
| $2^{4}=16$ |  |
| $2^{0}=1$ |  |
| $2^{-1}=\frac{1}{2}$ |  |
| $3^{4}=81$ |  |
| $\left(\frac{3}{4}\right)^{2}=\frac{9}{16}$ |  |
| $\left(\frac{1}{2}\right)^{-5}=32$ |  |

Example \#2: Evaluate the following logarithms:
a) $\log _{2} 8$
b) $\log _{3} 81$
C) $\log _{9} 9$
d) $\log _{3} \frac{1}{9}$
e) $\log _{5}(-25)$
f) $\log _{10} 1000$
g) $\log _{6} 1$
h) $\log _{2} \sqrt{8}$
i) $\log _{3} 9 \sqrt{3}$
k) $\log _{\frac{1}{2}}(64)$
I) $\log _{\sqrt{5}} 125^{\frac{3}{2}}$

## SOME RULES THAT MIGHT HELP YOU WITH SOME OF THE ABOVE EXAMPLES:

- $\log _{c} 1=0$ since in exponential form $c^{0}=1$.
- $\log _{c} c=1$ since in exponential form $c^{1}=c$
- $\log _{c} c^{x}=x$ since in exponential form $c^{x}=c^{x}$
- $c^{\log _{c} x}=x, x>0$, since in logarithmic form $\log _{c} x=\log _{c} x$


## Example \#3:

Evaluate $\log 100$.

## NOTE:

- The Base seems to be missing in this question.
- Mathematicians often numbers in the most basic situations - for example x really should be $x^{1}$, the $\sqrt{x}$ really should be $\sqrt[2]{x}$
- When a base appears to be missing from a logarithm, the base that has been excluded is always base 10. This question really should look like $\log _{10} 100$
- Logarithms that are to the base 10 can be solved by the same method we used in example 1, however they are known as THE COMMON LOG and are also built into your calculator. You should see a LOG button on your calculator. To evaluate a number to the Common Log/Base 10, you can just press log 100 and get the answer.

Example \#4: Solve the following logarithmic equations
a) $\log _{x} 64=2$
b) $\log _{3}(x+4)=2$
c) $\log _{125} x=\frac{2}{3}$

### 8.1 Day 1 ASSICNMENT

8.1 FA: P380 \#2, 3, 4, 8a, 9a, 12
8.1 MLA: P380 \#6, 7, 13, 14a, 20, 17
8.1 ULA P380 \#5, 18, 19,

### 8.1 Day 2 Graphs of Logarithmic Functions

## To graph logarithmic functions

## Example \#1:

a) Use a table of values to graph $y=2^{x}$

$$
y=2^{x}
$$

| $\mathbf{x}$ | $\mathbf{f ( x )}$ |
| :--- | :--- |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |



Identify the following for $\mathrm{y}=2^{\mathrm{x}}$ :

- Domain
- Range
- X intercept
- $Y$ intercept
- Whether the graph represents an INCREASING or a DECREASING function
- The equation of the horizontal asymptote


## REMEMBER THAT THE LOGARITHMIC FUNCTION IS THE INVERSE OF THE EXPONENTIAL FUNCTION

b) Let's graph $y=\log _{2} x$ on the same axis

| $\mathbf{x}$ | $\mathbf{f ( x )}$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Identify the following for $y=\log _{2} x$ :

- Domain
- Range
- X intercept
- Y intercept
- Whether the graph represents an INCREASING or a DECREASING function
- The equation of any asymptote:


### 8.1 Day 2 ASSTCNMENT

8.1 Day 2 FA: P380 \#1, 8b, 9b, 10
8.1 Day 2 MLA: P280 \#14b, 15, 16, 17

### 8.2 Transformations of Logarithmic Functions

## To use transformations to graph logarithmic functions

Given the base function $y=\log _{c} x$, multiple transformations can be applied to create the general transformation

$$
\text { equation of } y=a \log _{c}(b(x-h))+k
$$

Example \#1: Use your previous knowledge about transformations to predict the transformations of the graph of $y=\log _{3}(x+9)+2$
a) Write the mapping notation and use tables to sketch the graph.

b) Identify the following for $y=\log _{3}(x+9)+2$ :

- Domain
- Range
- X intercept
- Y intercept
- Whether the graph represents an INCREASING or a DECREASING function
- The equation of any asymptote:

QUESTION: What characteristics of log function indicate it will be a decreasing function?

Example \#3: Describe the transformations that would occur in the following function: $y=-\log _{2}(2 x+6)$

Example \#4: The lighter (red) graph can be generated by stretching the darker (blue) graph of $y=\log _{4} x$. Write the equation that describes the lighter (red) graph.
a)

b)

c) Only a horizontal translation has been applied to the graph of $y=\log _{4} x$ so that the graph of the transformed image passes through the point ( 6,2 ). Determine the equation of the transformed image.

Example \#5: Write the equations that correspond to the following transformations of $y=\log _{5} x$
a) Reflected in the $x$ axis and translated 1 unit down and 4 units left
b) vertically stretched by a factor of 3 , stretched horizontally by a factor of 2 and translated 2 units right

## Example \#6:

There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, $F$, of flower species that a butterfly feeds on and the number of butterflies observed, $B$, can be modelled by the function $F=-2.641+8.958 \log B$.
a) How many flower species would you expect to find if you observed 100 butterflies?

b) Predict the number of butterfly observations in a region with 25 flowers.

### 8.2 ASSICNMENT

### 8.2 FA: P389 \#1, 2, 3, 4, 6, 7

8.2 MLA: P289 \#8, 9, 10, 11, 13, 14
8.2 ULA: P289 \#15, 16, 17, C1, C2

### 8.3 DAY 1 Laws of Logarithms

## To use transformations to determine equivalent expressions for given logarithmic statements

## INVESTIGATION:

1. a) Show that $\log (1000 \times 100) \neq(\log 1000)(\log 100)$.
b) Use a calculator to find the approximate value of each expression, to four decimal places.
i) $\log 6+\log 5$
ii) $\log 21$
iii) $\log 11+\log 9$
iv) $\log 99$
v) $\log 7+\log 3$
vi) $\log 30$
c) Based on the results in part b), suggest a possible law for $\log M+\log N$, where $M$ and $N$ are positive real numbers.

PRODUCT LAW OF LOGS: $\log _{c} M N=$
d) Use your conjecture from part c) to express $\log 1000+\log 100$ as a single logarithm.
2. a) Show that $\log \frac{1000}{10} \neq \frac{\log 1000}{\log 10}$
b) Use a calculator to find the approximate value of each expression, to four decimal places.
i) $\log 12$
ii) $\log 35-\log 5$
iii) $\log 36$
iv) $\log 72-\log 2$
v) $\log 48-\log 4$
vi) $\log 7$
c) Based on the results in part b), suggest a possible law for $\log M-\log N$, where $M$ and $N$ are positive real numbers.
QUOTIENT LAW OF LOGS: $\quad \log _{c} \frac{M}{N}=$
d) Use your conjecture from part c) to express $\log 1000-\log 100$ as a single logarithm.
3. a) Show that $\log 1000^{2} \neq(\log 1000)^{2}$.
b) Use a calculator to find the approximate value of each expression, to four decimal places.
i) $3 \log 5$
ii) $\log 49$
iii) $\log 125$
iv) $\log 16$
v) $4 \log 2$
vi) $2 \log 7$
c) Based on the results in part b), suggest a possible law for $P \log M$, where $M$ is a positive real number and $P$ is any real number.

## POWER LAW OF LOGS: $\quad \log _{c} M^{P}=$

d) Use your conjecture from part c) to express 2 log 1000 as a logarithm without a coefficient.


Example \#1: The laws of common logarithms are also true for any logarithm with a base that is a positive real number other than 1 . Without using technology, evaluate each of the following.
a) $\log _{6} 18+\log _{6} 2$
b) $\log _{2} 40-\log _{2} 5$
c) $4 \log _{9} 3$

Example \#2: Rewrite as a single log.
a) $5 \log _{3} 2$
b) $\log _{8} 35-\log _{8} 5$
c) $\log _{9} 6+\log _{9} 7$
d) $\log _{7} 24+\log _{7} 4-\log _{7} 3$
e) $-\log _{7} 24-\log _{7} 4-\log _{7} 3$
f) $6 \log _{5} 2-2 \log _{5} 4$

Example \#3: Rewrite as a single log and evaluate.
a) $4 \log _{4} 2-2 \log _{4} 8$
b) $\log _{6} 8+\log _{6} 9-\log _{6} 2$
c) $3 \log _{\frac{2}{3}} 2-3 \log _{\frac{2}{3}} 3$
d) $\log _{7} 7 \sqrt{7}$
e) $2 \log _{2} 12-\left(\log _{2} 6+\frac{1}{3} \log _{2} 27\right)$

## Turn to page 392 in your textbook!

- How do slide rules work: https://goo.gl/YtBU2m
- Slide Rule Scene in Apollo 13: https://vimeo.com/34664087


### 8.3 Day 1 ASSICNMENT

Use your properties of logarithms to rewrite each logarithmic statement as the logarithm of a single number or expression. Assume that all logarithms are defined.

1. $\log _{8} 3+\log _{8} 11$
2. $\log _{2} 3+\log _{2} 5+\log _{2} 6$
3. $\log _{a} x+\log _{a} y+\log _{a} z$
4. $\log _{b}(x-1)+\log _{b}(x+3)$
5. $\log _{4} 20-\log _{4} 10$
6. $\log _{5} 160-\log _{5} 16$
7. $\quad \log _{v}(h+1)-\log _{v}(h-1)$
8. $3 \log _{6} 4$
9. $2 \log _{7} 11$
10. $\log x+\log y-\log z$
11. $\log x-\log y-\log z$
12. $-\log x-\log y-\log z$
13. $2 \log _{11} x+3 \log _{11} y+4 \log _{11} z$
14. $\frac{1}{2} \log _{3} x+\frac{2}{3} \log _{3} y$
15. $\log _{k}\left(x^{2}-y^{2}\right)-\log _{k}(x+y)$
16. $-2 \log a-3 \log b+4 \log c$
17. $\log a+\log (a+1)-\log \left(a^{3}-a\right)$

Use your properties of logarithms to rewrite each logarithmic statement as the logarithm of a single number. Then evaluate that logarithm without the use of a calculator.
28. $\log _{2} 80-\log _{2} 5$
29. $\log _{5} 50+\log _{5} 5-\log _{5} 2$
30. $\log _{10} 2+\log _{10} 50$
31. $\log _{4} 32+\log _{4} 2$
32. $\log _{3} 9^{10}$
33. $\log _{5} 125^{-13}$
34. $\quad \log _{3}(3 \sqrt[3]{3})$
35. $\log _{12} 8+\log _{12} 9+\log _{12} 2$
36. $\log _{15} 450-\log _{15} 2$
37. $3 \log _{7} 4-2 \log _{7} 8$
38. $\log _{4} 48+\log _{4} 8+\log _{4}\left(\frac{2}{3}\right)$
39. $\log _{2} 24+\log _{2} 4-\log _{2} 3$
40. $\log _{3} 36+\log _{3} 18-\log _{3} 24$
41. $\log _{2} 20-\log _{2} 5+\log _{2} 8$
42. $\log _{5} \sqrt{10}+\log _{5} \sqrt{\frac{25}{2}}$

## SOLUTIONS TO 8.3 DAY 1

1. $\log _{8} 33$
2. $\log _{2} 90$
3. $\log _{a}(x y z)$
4. $\log _{b}\left(x^{2}+2 x-3\right)$
5. $\log _{4} 2$
6. $\log _{5} 10$
7. $\log _{v}\left(\frac{h+1}{h-1}\right)$
8. $\log _{6} 64$
9. $\log _{7} 121$
10. $\log _{3} \sqrt[3]{25}$
11. $\log _{6} \frac{1}{4}$
12. $\log _{8} 243$
13. $\log q^{p}$
14. $\log _{2} w^{a+8}$
15. $\log 4$
16. $\log 2$
17. $\log 2$
18. $\log _{3} 108$
19. $\log _{7} \frac{1}{4}$
20. $\log \left(\frac{x y}{z}\right)$
21. $\log \left(\frac{x}{y z}\right)$
22. $\log \left(\frac{1}{x y z}\right)$
23. $\log _{11}\left(x^{2} y^{3} z^{4}\right)$
24. $\log _{3}\left(x^{\frac{1}{2}} y^{\frac{2}{3}}\right)$
25. $\log _{k}(x-y)$
26. $\log \left(\frac{c^{4}}{a^{2} b^{3}}\right)$
27. $\log \left(\frac{1}{a-1}\right)$
28.4
28. 3
30.2
29. 3
30. 20
31. -39
32. $\frac{4}{3}$
33. 2
34. 2
37.0
35. 4
36. 5
40.3
37. 5
38. $\frac{3}{2}$
39. $\log _{5} 4-\log _{5} 7$
40. $\log _{3} 8+\log _{3} a$
41. $\log _{2} 9+\log _{2} x+\log _{2} y$
42. 

$\log _{6} 5+\log _{6} h-\log _{6} 3-\log _{6} t$
47. $3 \log _{7} t$
48.
$\log _{8} 4+6 \log _{8} s+7 \log _{8} h$
49.
$\log _{w} 12+3 \log _{w} a-2 \log _{w} b$
50.
$\log _{3} 6+\frac{1}{2} \log _{3} 3-\log _{3} 11$
51. $\log _{a}(x-4)+\log _{a}(x+3)$
52. $-3 \log _{1} a-2 \log _{t} b$
53. $2 \log _{4}(x+5)$
54.
$\log _{s}(m-6)+\log _{s}(m+3)-$ $\log _{s}(m+7)-\log _{5}(m-2)$
55. 1.38685
56. 2.66096
57. 0.52068
58. -2.44566
59. -28.43316
60. -0.73912
61. 1.31396
62. -1.23599
63. (a)
$\log (12+4)=1.204119983$
$\log 12+\log 4=1.681241237$
63. (b)
$\log \left(\frac{12}{4}\right)=0.4771212547$
$\frac{\log 12}{\log 4}=1.79248125$
63. (c)
$\log \left(12^{2}\right)=2.1585362492$
$(\log 12)^{2}=1.164632162$
64. There are several possibilities. One suggestion is to write $\log 11$ as $\log \left(\frac{22}{2}\right)$. Then you can use your log properties to obtain
$\log \left(\frac{22}{2}\right)=\log 22-\log 2$.
65. to be discussed 66. (a)
$\log _{a} b=\frac{\log _{b} b}{\log _{b} a}=\frac{1}{\log _{b} a}$
66. (b)
$\left(\log _{c} b\right)\left(\log _{a} c\right)$

$$
\begin{aligned}
& =\left(\frac{\log _{a} b}{\log _{a} c}\right)\left(\log _{a} c\right) \\
& =\log _{a} c \\
& 67 . \\
& \log _{b} b^{x}=x \log _{b} b=x(1)=x \\
& 68.0 \\
& 69 \\
& \text { (a) } r+s \\
& \text { (b) } 4 s \\
& \text { (c) } s-r \\
& \text { (d) } \\
& \log 30=\log (3 \cdot 10) \\
& =\log 3+\log 10=s+1
\end{aligned}
$$

(e) $2 r+2 s$
(f) $-5 r$
(g) $1-r$

### 8.3 DAY 2 Laws of Logarithms

To use transformations to determine equivalent expressions for given logarithmic statements

Example \#1: Expand each log as far as possible:
a) $\log _{5}\left(\frac{4}{7}\right)$
b) $\log _{23}(7 \cdot 9 \cdot 25)$
c) $\log _{5}\left(\frac{3^{8} x 3^{11}}{7^{4}}\right)$
d) $\log _{7} \sqrt[3]{x}$
e) $\log _{5} \frac{x y}{z}$
f) $\log _{6} \frac{1}{x^{2}}$
g) $\log _{4} \frac{x^{3} y}{4 z}$
h) $\log \frac{x^{3}}{y \sqrt{z}}$

Example \#2: Write each expression as a single logarithm in simplest form. State the restrictions on the variable.
a) $\log _{7} x^{2}+\log _{7} x-\frac{5 \log _{7} x}{2}$
b) $\log _{5}(2 x-2)-\log _{5}\left(x^{2}+2 x-3\right)$

Example \#3: Write each expression as a single logarithm in simplest form.
a) $\log _{6} 2 x^{7}+\log _{6} 3 x^{2}+\log _{6} 9$
b) $\log 4 x \sqrt{y}-\log x^{2} \sqrt{y}$
c) $\log _{7} x^{4}+\frac{1}{3}\left(\log _{7} x^{2}-\log _{7} \sqrt{5 x}\right)$

### 8.3 Day 2 ASSIGNMENT

8.3 Day 2 FA: P400 \#1, 3, 7, 8
8.3 Day 2 MLA: P400 \#6, 10, 11, 12
8.3 Day 2 MLA: P400 \#9, 13, 16, 18, 19, 20, C2

## To solve logarithmic and exponential equations.

Note: A logarithmic equation is an equation containing the logarithm of a variable.

## SOLVING LOGARITHMIC EQUATION: START BY STATING RESTRICTIONS

Remember these basic statements of equality:

- If $\log _{c} L=\log _{c} R$, then $L=R$
!! Given that $c, L, R>0$ and $\mathrm{c} \neq 1$
- If $L=R$, then $\log _{c} L=\log _{c} R$
- $\log _{c} L=R$ can be written as $L=c^{R}$

1. Method 1 - Solve Graphically

- Graph each side of the original equation as its own function. The intersection point(s) represent the solutions

2. Method $\mathbf{2}$ - If both sides have the same base, solve algebraically

- If you have $\log _{c}($ Polynomial 1$)=\log _{c}($ Polynomial 2$)$, let Polynomial $1=$ Polynomial 2 and solve

3. Method $\mathbf{3}$ - Convert to exponential form and solve the resulting exponential equation
4. Method 4 - Use the laws of logs to change both sides of the equation to single logarithms that each have same base and use method 2 to solve

Example \#1: Solve $\log _{6}(2 x-1)=\log _{6} 11$ graphically.

- Graph $y=\log _{6}(2 x-1)$ and $y=\log _{6} 11$ on the same set of axes and find the $x$-coordinate of the point of intersection. (See graph below)

Our focus is to solve algebraically!

Example \#2: Solve the following
a) $\log _{3}(x-4)=5$
b) $\log _{4}(x-3)+\log _{4}(x+3)=2$
c) $\log _{2}(2 x-1)-\log _{2}(x+7)=-3$
d) $\log (x+3)-\log 2=2 \log x$
e) $\log _{7} 4-\log _{7} x=\log _{7} 5-\log _{7}(x+3)$
d) $\log _{3}\left(x^{2}-8 x\right)^{5}=10$

## SOLVING EXPONENTIAL EQUATIONS WITH DIFFERENT BASES:

- Remember that you can do whatever you want to one side of an equation as long as you do it to another (besides dividing by zero!).
- In order to solve an exponential equation with different bases, follow these steps:

1. Take the log of both sides (remember that this really means $\log _{10}$ )
2. Use the laws of logs to isolate your variable
3. Solve for the variable

Example \#2: Solve the following:
a) $13^{x}=26$
b) $11^{-4 x}=104$
c) $4^{2 x}=3^{x-1}$
d) EXTRA CHALLENGE

$$
3\left(5^{2 x-6}\right)=\frac{2^{x+7}}{7^{x}}
$$

### 8.4 Day 1 ASSIGNMENT

Solve each of the following logarithmic equations. Be careful to reject those values of $x$ that would caust the logarithm to be undefined.
(1.) $\log x+\log 5=1$
2. $\log _{3} x-\log _{3} 4=2$
10. $\quad \log _{3}(2 x-5)-\log _{3}\left(x^{2}+4 x+4\right)=-2$
(11.) $\log _{4} 72-\log _{4} 9=x$
(3.) $\log _{5} x^{3}=2+\log _{5} x$
12. $\log _{5} 6=\log _{5} x-\log _{5} 7$
4. $\log 9 x+\log x=4$
(13). $\log _{7} 98+\log _{7} 3.5=x$
(5.) $\log _{6} x-\log _{6}(x-1)=\log _{6} 3$
14. $\log _{3} 63-\log _{3} 7 x=\log _{3} 2$
6. $\log _{8}(x+1)-\log _{8} x=\log _{8} 4$
(15.) $\log _{8} \sqrt{0.125}=x$
(7.) $\log _{7}(x+1)+\log _{7}(x-5)=1$
16. $\log _{5} 4+\log _{5}(2 x-3)=20$
8. $\log _{6}(x-3)+\log _{6}(x+2)=1$
(17.) $\log _{2} 2+\log _{2}(x+2)-\log _{2}(3 x-5)=3$
9. $\log _{2}(9 x+5)-\log _{2}\left(x^{2}-1\right)=2$
18. $\log _{5}(2 x+1)+\log _{5}(3 x-1)=2$

Solve each of the following exponential equations. Give your answers to 5 decimal places.
(19.) $3^{x}=18$
20. $8^{x}=25$
(21.) $4^{2 x}=90$
22. $5^{3 x}=63$
(23.) $29^{\frac{1}{2} x}=76$
24. $16^{\frac{3}{5} x}=73$
25. $4^{2 x}=3^{x-1}$
26. $6^{3 x}=2^{2 x-3}$
(27.) $\left(2^{2 x}\right)^{3}=4^{x+3}$
28. $\left(3^{x+1}\right)^{3}=2^{4-x}$
(29.) $(1.4)^{x}=(2.6)^{x+5}$
30. $(1.93)^{2 x}=3(4.1)^{x}$
(31.) $4\left(2^{x}\right)=3^{x+1}$
32. $(2.1)^{x+1}=5(3.6)^{2 x}$

## Digging Deeper

33. To try and simplify $\frac{\log a}{\log b}$, a careless student just "crossed out" the "log" getting $\frac{a}{b}$ which he then reduced to $\frac{2}{3}$. Amazingly, and coincidentally, that was the correct answer. What were the values of $a$ and $b$ ?

## SOLUTIONS:

1. $\{2\}$
2. $\{36\}$
3. $\{5\}$
4. $\left\{\frac{100}{3}\right\}$
5. $\left\{\frac{3}{2}\right\}$
6. $\left\{\frac{1}{3}\right\}$
7. $\{6\}$
8. $\{4\}$
9. $\{3\}$
10. $\{7\}$
11. $\left\{\frac{3}{2}\right\}$
12. $\{42\}$
13. $\{3\}$
14. $\left\{\frac{9}{2}\right\}$
15. $\left\{-\frac{1}{2}\right\}$
16. $\left\{\frac{5^{20}+12}{8}\right\}$
17. $\{2\}$
18. $\{2\}$
19. $\{2.63093\}$
20. $\{1.54795\}$
21. $\{1.62296\}$
22. $\{0.85809\}$
23. $\{2.57223\}$
24. $\{2.57909\}$
25. $\{-0.65629\}$
26. $\{-0.52130\}$
27. $\{1.50000\}$
28. $\{-0.13117\}$
29. $\{-7.71770\}$
30. $\{-11.45020\}$
31. $\{0.70951\}$
32. $\{-0.47667\}$
33. $a=\frac{9}{4} ; b=\frac{27}{8}$

If $\frac{a}{b}=\frac{2}{3}$, then $3 a=2 b$.
Thus $a=\frac{2}{3} b$ (1). But
$\frac{\log a}{\log b}=\frac{2}{3}$. Thus
$3 \log a=2 \log b$ or
$\log a^{3}=\log b^{2}$. This
implies $a^{3}=b^{2}$ (2).
Substituting (1) into (2) and
solving gives our answer.
We must reject the solution
$b=0$, since the logarithm of 0 is undefined.

## To model and solve situations using exponential and logarithmic functions.

Example \#1: Palaeontologists can estimate the size of a dinosaur from incomplete skeletal remains. For a carnivorous dinosaur, the relationship between the length, $s$, in metres, of the skull and the body mass, $m$, in kilograms, can be expressed using the logarithmic equation $3.6022 \log s=\log m-3.4444$. Determine the body mass, to the nearest kilogram, of an Albertosaurus with a skull length of 0.78 m .

## Example \#2:

General Formula for Half Life Questions: Final Quantity = Initial Quantity (Factor of Change) ${ }^{\frac{\text { Time ( years })}{\text { Half Life (years) }}}$
When something living dies, the amount of radioactive carbon-14 (C-14) in its bones decreases. Archaeologists use this fact to determine the age of a fossil based on the amount of $\mathrm{C}-14$ remaining. The half-life of $\mathrm{C}-14$ is 5730 years.

When Ms. C taught at Balfour, she discovered (true story) that the skeleton used in the art department was not only an actual real skeleton, it was a skeleton that was dug up by a long ago science teacher and a group of students. Times were very different back in 1933 but Ms. C was shocked to learn that (according to newspaper clippings) the skeleton was actually taken from First Nation's land. She contacted the archeology department at the $U$ of $S$ and after a rather complicated process (which is another interesting story) the skeleton was picked up and taken to be carbon dated.

If the carbon remaining in the skeleton found at Balfour was $92 \%$ of what it originally was, how old was the skeleton that was found at Balfour?

Little bit of history - I went to Balfour myself and the art room was my homeroom throughout my years at high school. The skeleton (we called it Matilda) was hanging in its coffin box and was often taken out to be used in art displays. Here are the articles that I found in the archives room at Balfour - one was written by the Saskatoon Star and one by the Regina Leader. They were quite proud of the ingenuity of this science teacher and the society of the day celebrated his success rather than questioned the cultural appropriateness. Notice that the teacher had to request permission from the RCMP and the department of Indian Affairs to dig up this skeleton, but no permission had to be granted by any First Nation band themselves (the land in question is still held communally various First Nations but is not its own separate reserve community). The photograph of the skeleton was found in the 1933 Balfour yearbook. Unfortunately the skull went missing a week before the coroner picked up the skeleton from Balfour and the mystery of that has never been solved. The carbon dating would have been more accurate if they had been able to use the skull. The skeleton was laid to rest at a funeral ceremony in a cemetery north of Saskatoon.


PreCalculus 30/Ms. Carignan)

### 8.4 Day 2 ASSICNMENT

8.4 Day 2 FA: P412 \#6, 11, 13a, 15
8.4 Day 2 ULA: P412 \#13bc, 14

## LIST OF VIDEOS THAT MAY AIDE IN UNDERSTANDING

Section 8.1

- https://goo.gI/L2MZxY
- https://goo.gl/tdG7K4

Section 8.2

- https://goo.gl/1Ykvyr
- https://goo.gl/pSktXV


## Section 8.3

- https://goo.gl/R5SkBZ
- https://goo.gl/zfTLPB


## Section 8.4

- https://goo.gl/vEyKi7
- https://goo.gl/2bxy6W
- https://goo.gl/es7vhg

