

To Identify and Sketch the graph of a Rational Function with Denominator Degree One Using Transformations.

RATIONAL FUNCTION: A RATIONAL FUNCTION is a function $f(x)$ of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$. The base rational function is written as $f(x) = \frac{1}{x}$

INVESTIGATION:

Use technology to investigate the transformations between $f(x) = \frac{1}{x}$ and \rightarrow transform using $f(x) = af(b(x-h)) + k$

Graph the following using technology:

a) $f(x) = \frac{1}{x}$

- Why does the definition of a rational function specify that $q(x) \neq 0$?

b) $f(x) = \frac{3}{x}$ or this could be written as $f(x) = 3\left(\frac{1}{x}\right)$.

- What transformative variable does the 3 represent in this question? _____ What type of transformation is it? _____ What is the mapping? $(x, y) \rightarrow (\quad , \quad)$

c) $y = \frac{3}{x-4}$ or this could be written as $y = 3\left(\frac{1}{x-4}\right)$.

- What transformative variable does the 4 represent in this question? _____ What type of transformation is it? _____ What is the entire mapping? $(x, y) \rightarrow (\quad , \quad)$

d) $y = \left(\frac{3}{x-4}\right) + 5$.

- What transformative variable does the 5 represent in this question? _____ What type of transformation is it? _____ What is the entire mapping? $(x, y) \rightarrow (\quad , \quad)$

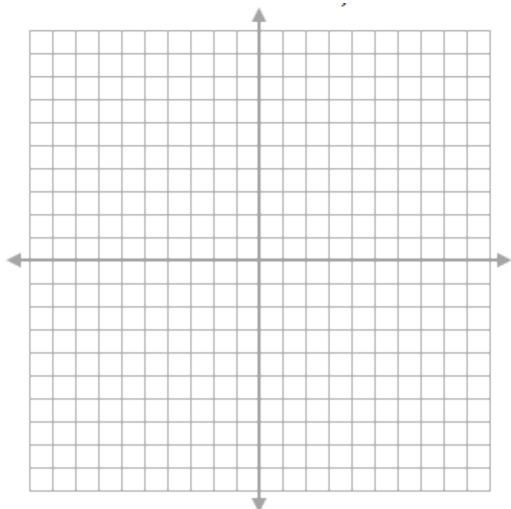
- Given a base RATIONAL FUNCTION of $f(x) = \frac{1}{x}$, we can transform it in the same way we transformed other base functions such as $y = x^2$, $y = \sqrt{x}$, $y = |x|$

- The transformation for RATIONAL FUNCTIONS can be written specifically as a new function

$$g(x) = \frac{a}{b(x-h)} + k$$

Example #1: Sketch $f(x) = \frac{1}{x}$ using a table of values.

x	y
-4	
-2	
-1	
$-\frac{1}{2}$	
0	
$\frac{1}{2}$	
1	
2	
4	



What happens to the graph as the value of x gets very close (approaches) zero from the left?

What about when it approaches zero from the right?

What happens to the graph as the value of y approaches zero from above?

What about when it approaches zero from below?

An **asymptote** is a line that the graph of a relation approaches as a limit or boundary.

- The graph of a rational function **never** crosses a **vertical asymptote** but it **may or may not cross a horizontal asymptote**.
- Asymptotes are drawn using dashed lines
- At what values do vertical asymptotes occur?
- Equation of the vertical asymptote of $f(x) = \frac{1}{x}$:
- Equation of the horizontal asymptote of $f(x) = \frac{1}{x}$:
- The **end behaviour** is what happens when x approaches a large positive value or a large negative value ($\pm\infty$)

BASE FUNCTION CHARACTERISTICS: $f(x) = \frac{1}{x}$

non-permissible value(s):

behaviour near non-permissible value(s):

end behaviour:

domain:

range:

equation of vertical asymptote:

equation of horizontal asymptote:

x intercept:

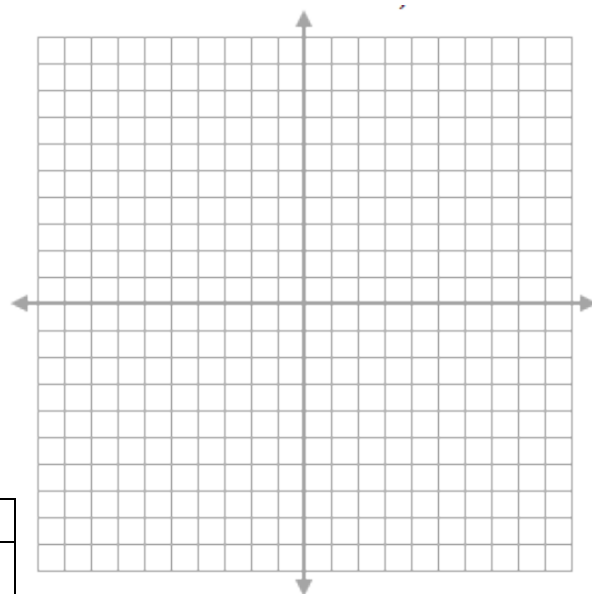
y intercept:

Example #2: Given the transformation $2f(x + 1) - 3$

a) Write the new equation of this transformation of $f(x) = \frac{1}{x}$

b) Sketch and describe the characteristics of the transformed function.

x	y
-4	
-2	
-1	
-1/2	
0	
1/2	
1	
2	
4	



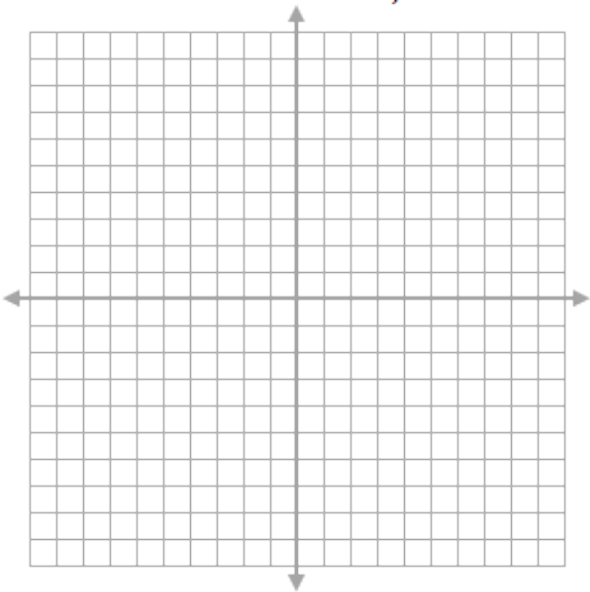
CHARACTERISTIC	f(x) =
MAPPING:	
Non-Permissible Values (NPV)	
Behavior near NPV	
End Behavior	
Domain	
Range	
Vertical Asymptote	
Horizontal Asymptote	
X Intercept	
Y Intercept	

Example #3:

Graph the following function using mappings and identify its characteristics. $y = \frac{2x + 2}{x - 4}$

NOTE: The equation is not given in the standard form. Rewrite it in the form $y = af(b(x - h)) + k$ or $g(x) = \frac{a}{b(x - h)} + k$

x	y
-4	
-2	
-1	
-1/2	
0	
1/2	
1	
2	
4	



CHARACTERISTIC	f(x) =
MAPPING:	
Non-Permissible Values (NPV)	
Behavior near NPV	
End Behavior	
Domain	
Range	
Vertical Asymptote	
Horizontal Asymptote	
X Intercept	
Y Intercept	

Describe how to find the vertical asymptote:

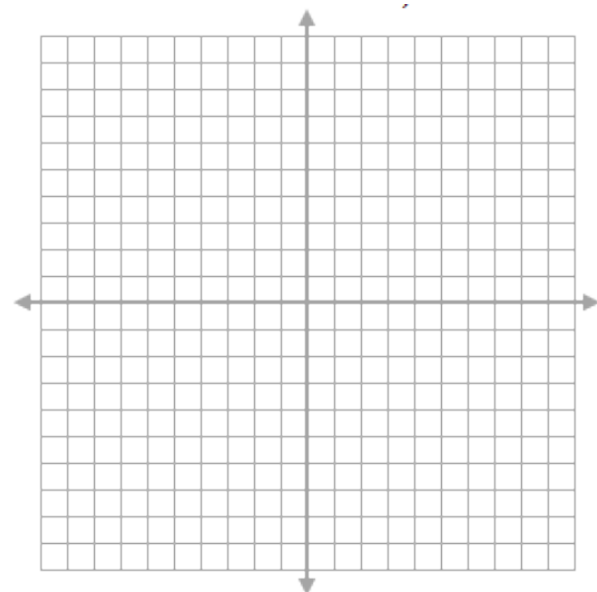
How did we find the horizontal asymptote in this question:

Is there an easier way to find the equation of the horizontal asymptote if we don't need to rewrite the equation into standard form?

Example #3:

a) Sketch a graph of the function $y = \frac{4x-5}{2x-6}$ by finding

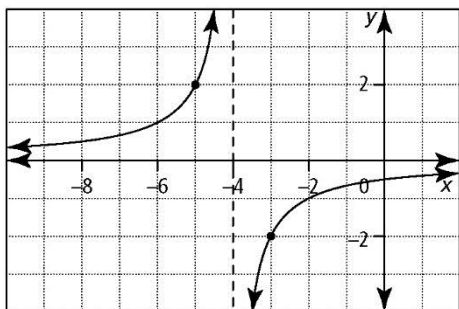
- the x intercepts
- the y intercepts (and another test point)
- the vertical and horizontal asymptotes.



b) Can you write the equation in standard form?

Example #4:

Write the equation for the function in the form $y = \frac{a}{b(x-h)} + k$.



9.1A ASSIGNMENT

9.1A FA: P442 #1, 3, 4ab(no technology), 5

9.1A MLA: P442 #9, 10, 11

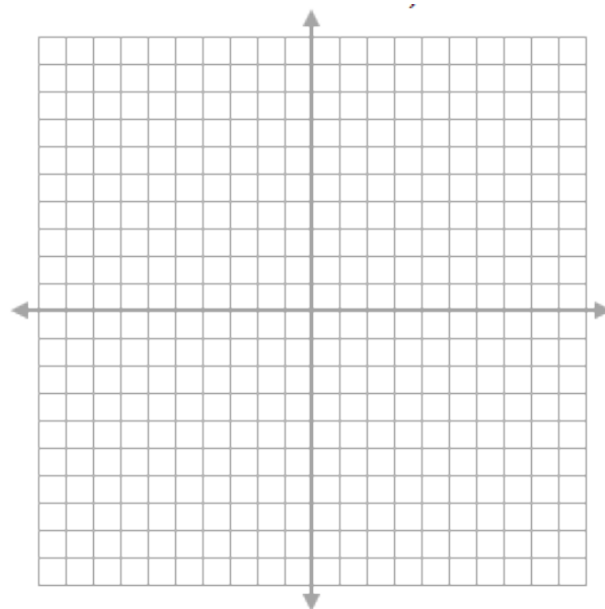
9.1A ULA: 20, 21

To Identify and Sketch the graph of a Rational Function Using Transformations.

Example #1: Sketch $f(x) = \frac{1}{x^2}$ as a base function using a table of values and identify its characteristics.

x	f(x)
-2	
-1	
-.5	
0	
.5	
1	
2	

CHARACTERISTIC	f(x) =
Non-Permissible Values	
Behavior near NPV	
End Behavior	
Domain	
Range	
Vertical Asymptote	
Horizontal Asymptote	
X Intercept	
Y Intercept	



BASE FUNCTION CHARACTERISTICS: $f(x) = \frac{1}{x^2}$

non-permissible value(s):

behaviour near non-permissible value(s):

end behaviour:

domain:

range:

equation of vertical asymptote:

equation of horizontal asymptote:

x intercept:

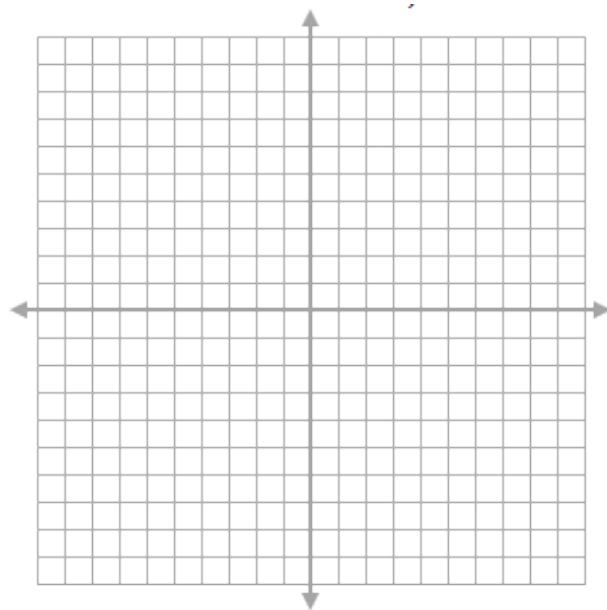
y intercept:

TRANSFORMATION EQUATION:

Example #2:

a) Sketch the graph of the function $y = \frac{4}{x^2 + 6x + 9} - 2$ by transforming the graph of $y = \frac{1}{x^2}$ and using mapping

x	f(x)
-2	
-1	
-.5	
0	
.5	
1	
2	



CHARACTERISTIC	f(x) =
MAPPING:	
Non-Permissible Values (NPV)	
Behavior near NPV	
End Behavior	
Domain	
Range	
Vertical Asymptote	
Horizontal Asymptote	
X Intercept	
Y Intercept	

b) Manipulate this equation so that it is in the form of a rational function of a single fraction $f(x) = \frac{p(x)}{q(x)}$

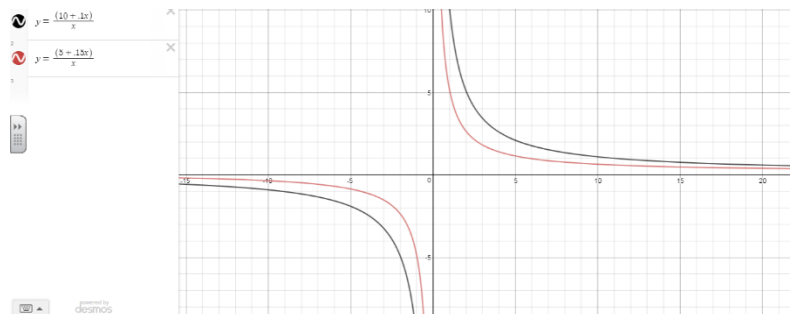
Example #3: The rational function $y = \frac{a}{x-5} + k$ passes through the points (3, 5) and (7, 2). Determine the values of a and k.

Example #3: A mobile phone service provider offers several different prepaid plans. One of the plans has a \$10 monthly fee and a rate of 10¢ per text message sent or minute of talk time. Another plan has a monthly fee of \$5 and a rate of 15¢ per text message sent or minute of talk time. Talk time is billed per whole minute.

a) Represent the average cost per text or minute of each plan with a rational function.

b) Graph the functions using technology.

c) What do the graphs show about the average cost per text or minute for these two plans as the number of texts and minutes changes?



d) Which plan is the better choice?

9.1B ASSIGNMENT

9.1B FA: P442 #2, 6, 7a

9.1B ULA: P442 #7bd, 8, 13, 14, 16, 17

9.1B ULA: P442 #19, 22, C2

To Identify Vertical and Horizontal Asymptotes & Points of Discontinuity (Holes)**INVESTIGATION:**

Using technology, sketch the following graphs and visually determine the location of the horizontal asymptote:

a) $y = \frac{3}{x-2}$

b) $y = \frac{3x+1}{x-5}$

c) $y = \frac{3x^2+1}{x^2-5}$

d) $y = \frac{3x^3+1}{x^2-5}$

d) $y = \frac{3x+1}{x^2-5}$

e) $y = \frac{3}{x-2} - 4$

CHARACTERISTICS OF RATIONAL FUNCTIONS

HORIZONTAL ASYMPTOTES FOR RATIONAL FUNCTIONS: A RATIONAL FUNCTION in the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$ has horizontal asymptotes of the following:

CASE 1:

If the degree of the numerator is less than the degree of the denominator, the graph has a horizontal asymptote at $y = 0$

Example: $y = \frac{-2}{x+3}$

CASE 2:

If the degree of the numerator is the same as the degree of the denominator, the graph has a horizontal asymptote at

$y = \frac{a}{b}$ where a is the leading coefficient of $p(x)$ and b is the leading coefficient of $q(x)$

Example: $y = \frac{2x-3}{3x+9}$ HORIZONTAL ASYMPTOTE AT: _____

CASE 3:

If the degree of the numerator is greater than the degree of the denominator, the graph will have an oblique (slant) asymptote found by completing synthetic division:

Example: $y = \frac{x^2}{x+1}$ OBLIQUE ASYMPTOTE AT: _____

NOTE: If the function is written in the form $g(x) = \frac{a}{b(x-h)} + k$, the horizontal asymptote is found at $y = k$

Example #1: Use the equation of each function to predict whether its graph has a horizontal or oblique asymptote. Find their equations.

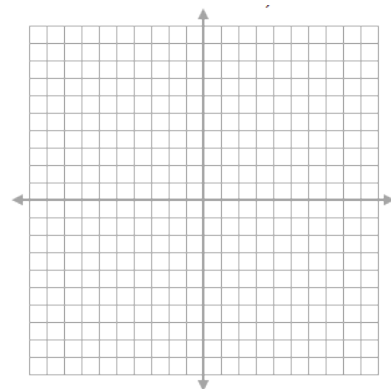
a) $y = \frac{x}{9 - x^2}$

b) $y = \frac{-x^2 + 4}{x - 3}$

INVESTIGATION:

Consider the function $y = \frac{x^2 - 3x}{x - 3}$.

- Would you expect it to be a linear graph, a quadratic graph or a graph with asymptotes (rational graph)?
- Can you predict any of the characteristics of the graph?
- Use technology to graph the function. Any surprises?
- Go to your table function – is there anything interesting that happens at $x = 3$?
- Sketch the graph showing all important details. Use algebra to determine the resulting equation of the original function.



VERTICAL ASYMPTOTES AND POINTS OF DISCONTINUITY (HOLES) FOR RATIONAL

FUNCTIONS: A RATIONAL FUNCTION in the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$ and where $p(x)$ and $q(x)$ are BOTH IN FACTORED FORM

- A. **Vertical Asymptote(s)** will exist if there are no common factors between $p(x)$ and $q(x)$. The equation(s) of the asymptote(s) are identified by the non-permissible values of the function.

EXAMPLE: $y = \frac{x^2 + 2x}{x - 3}$

- B. **Points of Discontinuity (Holes in the Graph)** will exist if there is at least one common factor between $p(x)$ and $q(x)$. To find these points you solve the common factors by setting them equal to zero.

C.

EXAMPLE: $y = \frac{x^2 - x - 6}{x - 3}$

- D. **BOTH VERTICAL ASYMPTOTE(S) AND POINT(S) OF DISCONTINUITY:** If you remove common factors and still have a rational expression (if you still have an x in the denominator), you have both point(s) of discontinuity and vertical asymptote(s).

EXAMPLE: $y = \frac{x^2 - 2x - 8}{x^2 + x - 2}$

Example #2: Use the equation of each function to predict whether its graph has vertical asymptote(s), point(s) of discontinuity or both.

a) $y = \frac{x}{9 - x^2}$

b) $y = \frac{-x^2 + 4}{x^2 - 5x + 6}$

c) $y = \frac{x^3 - 1}{x - 1}$

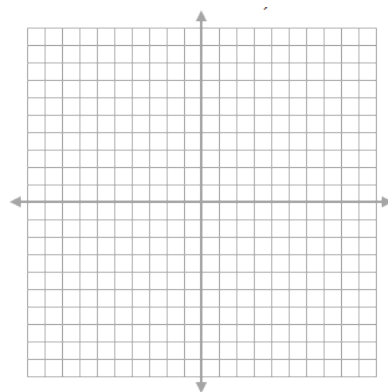
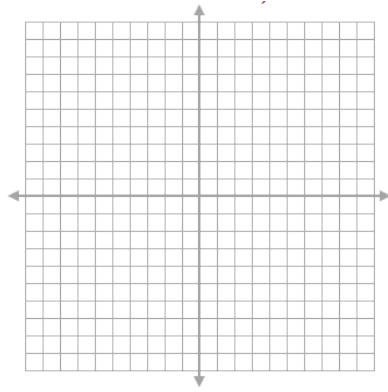
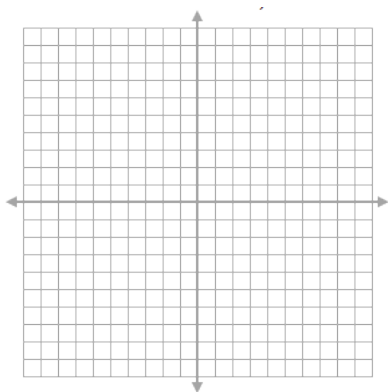
Example #3: For each function below:

- i) Determine any non-permissible values of x and whether each indicates a hole or vertical asymptote.
- ii) Determine the equations of any horizontal or oblique asymptotes.
- iii) Use the above information as well as the x and y intercepts to sketch the graph.

a) $y = \frac{x^2 + x - 2}{x + 1}$

b) $y = \frac{x^2 - 3x + 2}{1 - x^2}$

c) $y = \frac{x^2}{x^2 + 2x - 8}$



Example #4: Compare the behaviours of $f(x) = \frac{x^2 - 2x}{4 - 2x}$ and $g(x) = \frac{x^2 + 2x}{4 - 2x}$

	$f(x) = \frac{x^2 - 2x}{4 - 2x}$	$g(x) = \frac{x^2 + 2x}{4 - 2x}$
Non Permissible Value(s) (NPV)		
What feature exists at the NPV		
What is the behaviour at the NPV		

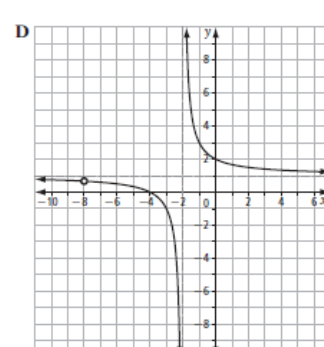
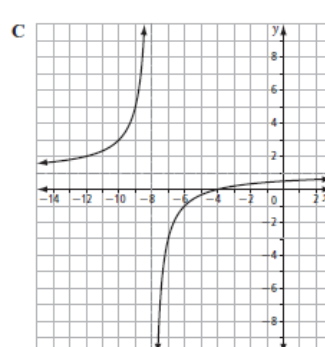
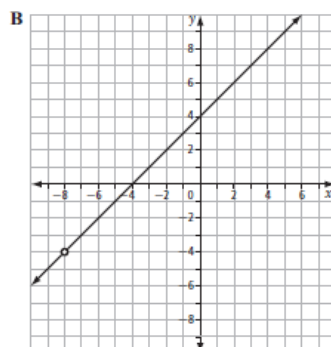
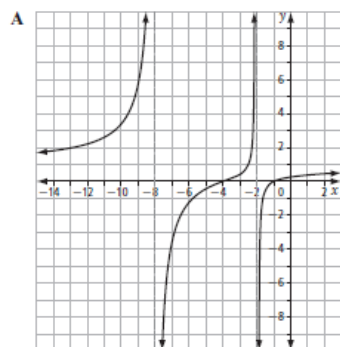
Example #5: Match the following. Justify your answers.

a) $f(x) = \frac{x + 4}{x + 8}$

b) $f(x) = \frac{x^2 + 12x + 32}{x + 8}$

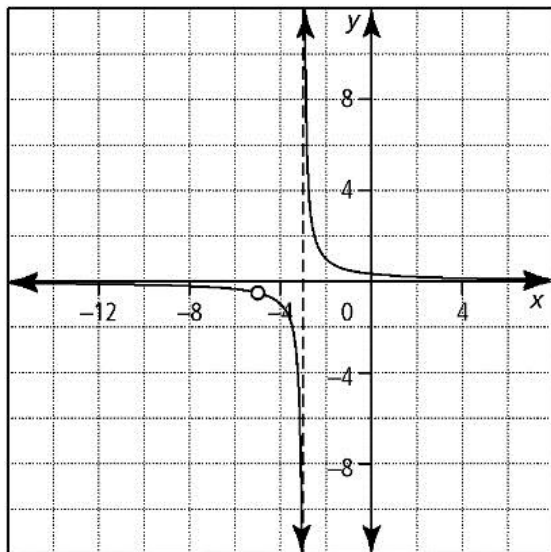
c) $f(x) = \frac{x^2 + 12x + 32}{x^2 + 10x + 16}$

d) $f(x) = \frac{x^2 + 5x + 4}{x^2 + 10x + 16}$

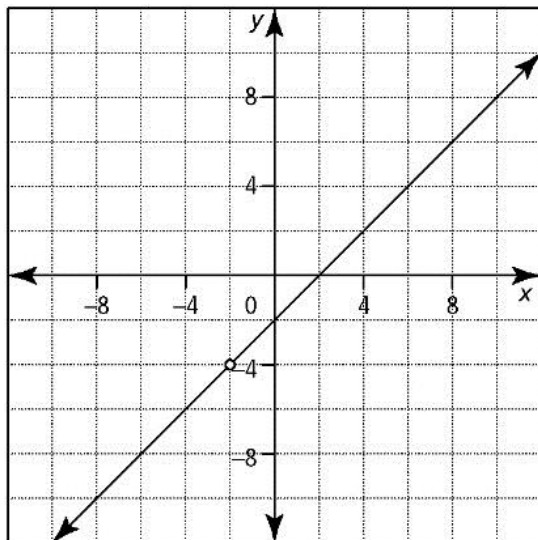


Example #6: Find the equations for the following graphs of rational functions. In order to find the equation be sure to first find the following: Point(s) of Discontinuity, Vertical Asymptote(s), Horizontal Asymptote, X intercept(s), Y intercept

a)



b)



9.2 ASSIGNMENT

9.2 FA: P451 #1, 4abc, 5, 6

9.2 MLA: P451 #4d, 7, 8, 9, 12, 14, 15, 16 (#23 USING TECHNOLOGY)

9.2 ULA: P451 #10, 11, 17, 19, 20, 21

To relate the roots of rational equations to the x intercepts of the graphs of rational functions, to identify extraneous roots and to approximate solutions to rational equations graphically.

Example #1: Find the roots to the following:

$$\frac{14}{x} - x + 5 = 0$$

STEPS TO SOLVING/FINDING THE ROOTS/FINDING THE X INTERCEPTS:

1. Factor any numerator or denominator that can be factored.
2. ***IDENTIFY ALL NON-PERMISSIBLE VALUES***
3. Algebraically “remove” all denominators by multiplying EACH term on both sides by the LOWEST COMMON DENOMINATOR.
4. Once all denominators have been “cancelled out”, add/subtract terms to get all terms to one side and zero on the other (Note: It is often easiest to move all terms to the side where x^2 will be positive)
5. Factor or use the quadratic equation to find the solution(s)/root(s)/x intercept(s)
6. Verify to be sure that none of your answers are extraneous solutions (make sure they aren’t non-permissible values in step 2)

Example #2: Solve the following equation to the nearest hundredth. Verify by using graphing software (2 different methods)

$$2 - \frac{3x}{2} = \frac{1 + 4x - x^2}{4x + 10}$$

Example #3: Solve $\frac{x+3}{2x-6} = 2x - \frac{x}{3-x}$

Example #3: Amber and Matteo are travelling separately from their home in Calgary to a wedding 400 km away. Amber leaves 1 h earlier than Matteo, but Matteo drives at an average speed 20 km/h faster than Amber. If they arrive at the wedding at the exact same time, what was the average speed at which each of them travelled?

a) Let x represent the time it takes Amber to travel to the wedding. Write an expression for the average speed that each person travels.

b) Write and solve an equation that represents the difference in their average speeds.

9.3 ASSIGNMENT

9.3 FA: P465 #3, 5ac(algebraically), 6abd(algebraically)

9.3 MLA: P465 #4(using technology), 7, 8, 10, 11, 13,

9.3 ULA: P465 #16*, 17, 15

LIST OF VIDEOS THAT MAY AID IN UNDERSTANDING

Section 9.1

- <https://goo.gl/Tdtbm4>
- <https://goo.gl/hMvr8K>

Section 9.2

- <https://goo.gl/cpYSdm>
- <https://goo.gl/kKdXwr>

Section 9.3

- <https://goo.gl/gqL8Kg>