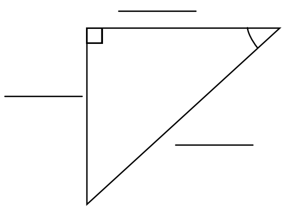
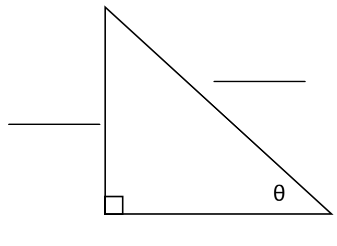
# PreAP FDN 20 CH 3 PREVIEW: REVIEW OF RIGHT TRIANGLES

Concepts: REVIEW

- Discovery Activity
- Setting your Calculator!

**LABELLING RIGHT TRIANGLES:**






**FORMULA'S FOR RIGHT TRIANGLES:**

$$\text{Sine of } \theta = \frac{\text{opposite side}}{\text{hypotenuse}}; \qquad \text{Sin}A = \frac{\text{opp}}{\text{hyp}}$$

$$\text{Cosine of } \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}; \qquad \text{Cos }A = \frac{\text{adj}}{\text{hyp}}$$

$$\text{Tangent of } \theta = \frac{\text{opposite side}}{\text{adjacent side}}; \qquad \text{Tan }A = \frac{\text{opp}}{\text{adj}}$$

$$a^2 + b^2 = c^2$$

**EX #1:** Calculate the following (these questions will always be calculated to 4 decimal places).

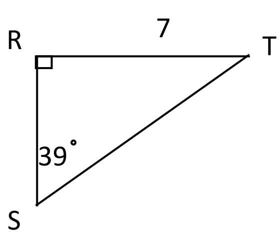
a) sin30° b) cos24.6° c) tan58°

**EX #2:** Find the value of $\theta$ to the nearest tenth.

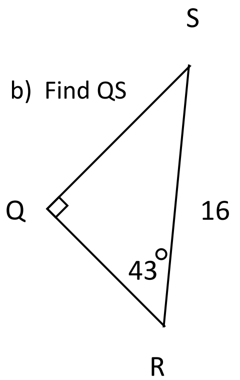
a) $\sin\theta = 0.4826$ b) $\cos\theta = 0.7486$ c) $\tan\theta = 2.4868$

**EX #3:** Find the value of the unknown to the nearest tenth.

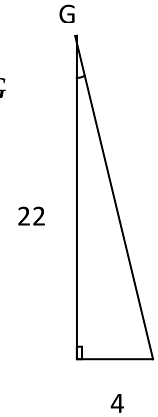
a) Find RS



b) Find QS



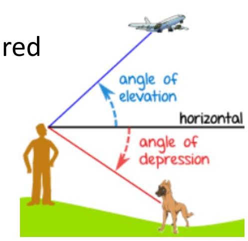

c) Find $\measuredangle G$




**EX #4:** Solve $\Delta PQR$, given $\angle Q = 90°$, $p$ = 15 cm and $r$ = 12 cm. Round to the nearest tenth of a unit.

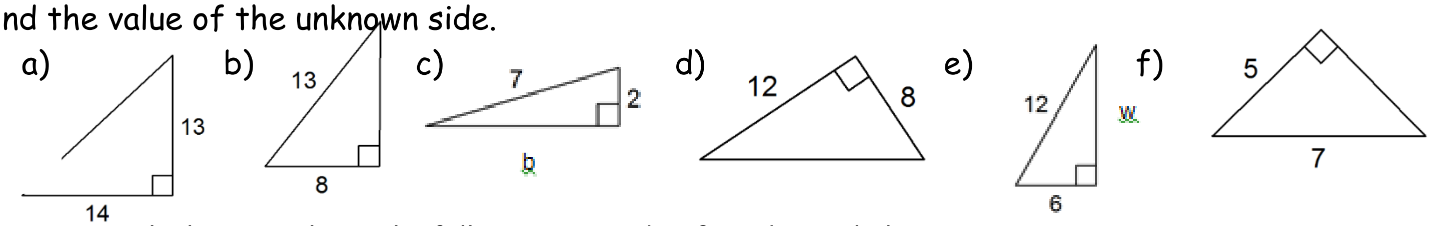
**Angle of Elevation (Also known as Angle of Inclination):** The angle measured between a horizontal line and a line angling upwards.

**Angle of Depression:** The angle measured between a horizontal line and a line angling downwards (NOTE: this angle must always start from a horizontal line)




**Ex #5:** Ms. C is standing in the courtyard which is 32 feet away from the school and directly across from her classroom. She looks up to the second floor and sees the windows of her classroom which are 16 feet high (and is annoyed when she sees her students hanging out of the windows instead of doing their work). What is the angle of inclination that Ms. C is looking up at? Leave your answer to the nearest hundredth. Please disregard Ms. C's height (she's pretty short anyway)

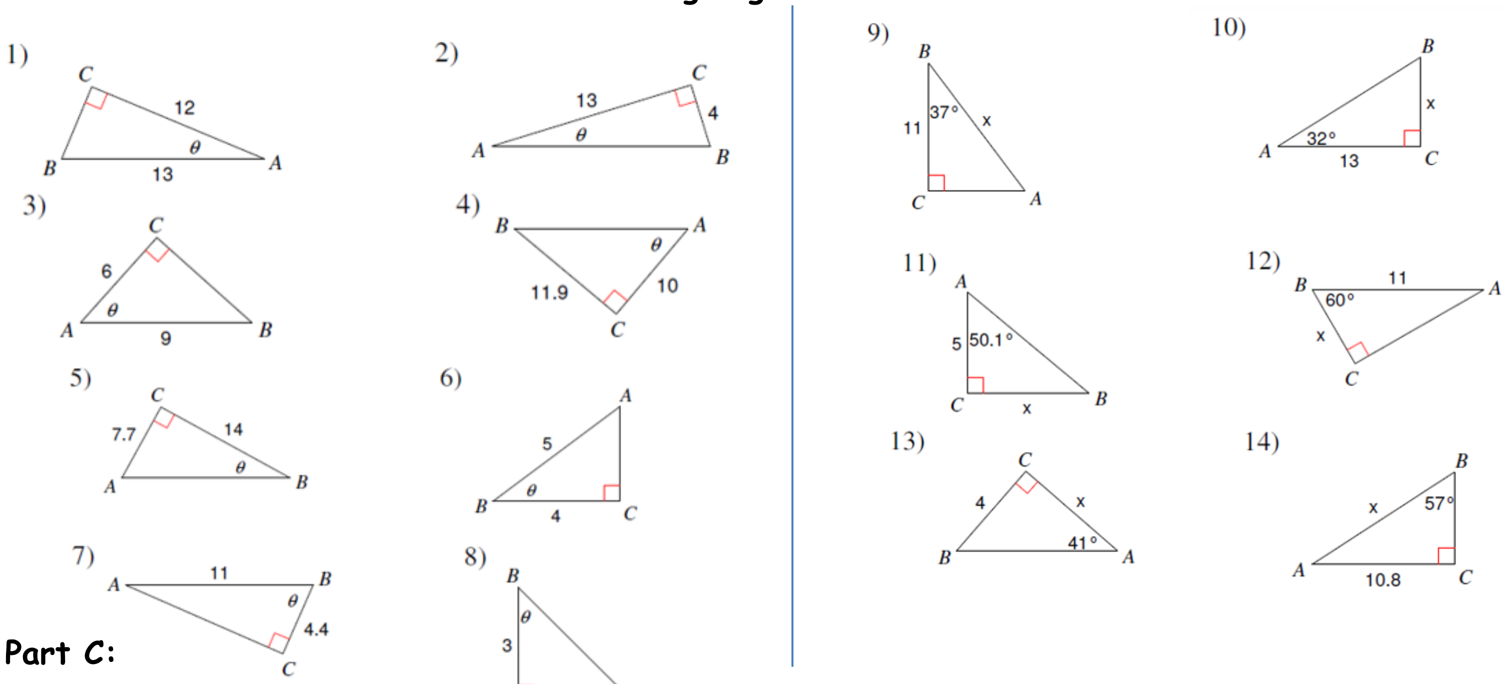
1. Find the value of the unknown side.




2. Using your calculator, evaluate the following. Round to **four** decimal places.
   a) sin 9° = ________ b) cos 15° = _______ c) tan 75° = _______

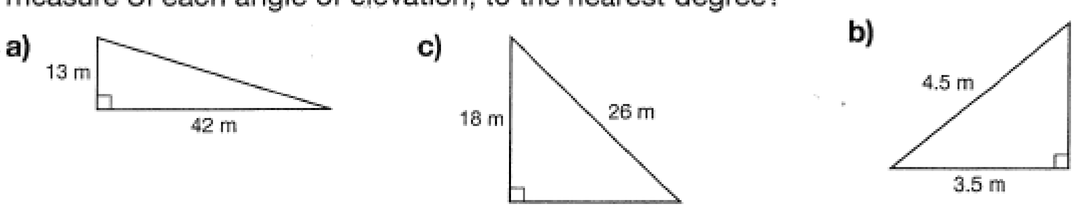
3. Using your calculator, find the measure of the angle. Round to the nearest degree (**no decimal**).
   a) sin A = 0.6428 b) tan B = 5.6713 c) cos C = 0.9336

**PART B: Find the measure of each missing angle or side to the nearest tenth**

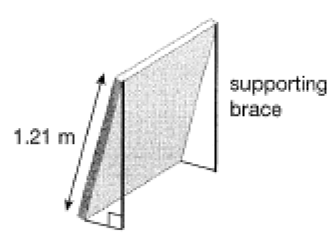



**Part C:**

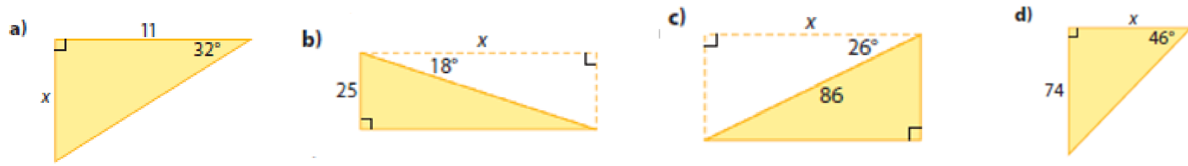
**1.** Label the angle of elevation, $x°$, on each diagram. What is the measure of each angle of elevation, to the nearest degree?




2. Bailey installs solar panels in Saskatoon. She adjusts the angle to match the season. In winter, the angle of elevation for panels in Saskatoon is 75.8°. The panels are 1.21 m tall. The roof is flat. How tall should the supporting brace be?

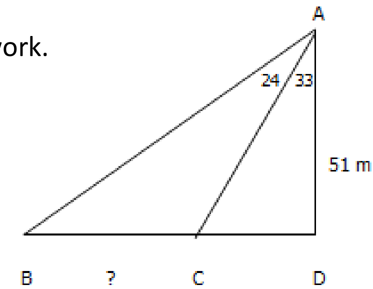


3. Highlight or circle the ANGLE OF DEPRESSION. Use this information to find the value of x.




4. Mike plants the rope of his kite in the ground so he can tie his shoes. The rope measures 10 m and creates an angle of $70^\circ$ with the ground. At what height from the ground is the kite?

5. Find the length of BC. Show ALL your work.



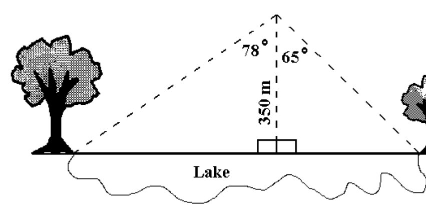

6. At 1pm, a building creates a 20 m shadow. The sun's rays form an $80^\circ$ angle with the ground. Draw this scenario and determine is the height of the building?

   b) Around 3pm, the sun has descended and now forms a shadow that is twice as long. What angle do the sun's rays and the ground now form?

7. A lighthouse is at the top of a cliff. The top of the lighthouse is 33 m above sea level. The angle of depression formed with a small fishing boat is 24°. At what distance from the base of the cliff is the boat located? Show ALL your work.

8. A tree makes a shadow 7.4m long on level ground when the angle of elevation of the sun is 29°. Determine the **height** of the **tree**.

**9.** To find the length of a lake, a surveyor measures off a distance of 350 m straight back from the shore. At this point, he observes that straight lines to trees at either end of the lake make angles of 78° and 65° with the perpendicular path back from the lake. Find the **length of the lake**.

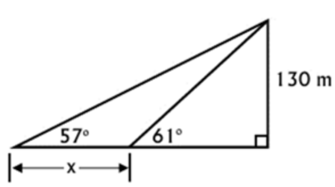



10. A flagpole is located on top of a building. From a point 10m from the foot of the building, the angle of elevation of the top of the flagpole is 30° and the angle of elevation of the bottom of the flagpole is 25°. Determine the height of the flagpole.

11. Find the largest angle of a triangle with sides 9,10 and 11 (do after law of cosines).

12. A bridge is 150m above the water. From the ends of the bridge the angles of depression of a buoy moored in the water directly below the bridge are 32° and 47°. How long is the bridge?

13. Determine the value of x in the given diagram.

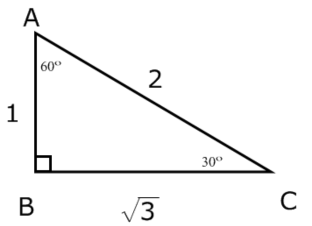


Concepts: # 22, 23

- An **oblique** triangle is a triangle that does not have a right angle and an **acute** triangle is a triangle where none of the angles is larger than 90°
- In a right triangle we can use the formula's for sine, cosine, tangent and the Pythagorean theorem. None of these formula's will work in an oblique triangle. What can use to find unknown sides and angles in an oblique acute triangle?

## Investigating the Sine Law/The Law of Sines

**PART 1: Right Triangles**

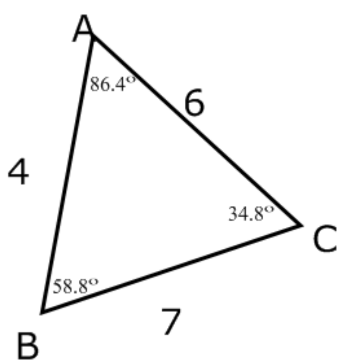



Using the given diagram, find each of the following ratios:

$\frac{a}{\sin A}$ $\frac{b}{\sin B}$ $\frac{c}{\sin C}$

What do you notice?

**Part 2: Now try with an acute triangle**




Using the given diagram, find each of the following ratios:

$\frac{a}{\sin A}$ $\frac{b}{\sin B}$ $\frac{c}{\sin C}$

What do you notice?

http://www.mathopenref.com/lawofsines.html - App to show

**The Sine Law:**
Given $\Delta ABC$, the Sine Law (Not the same as the Sine Ratio) says that:

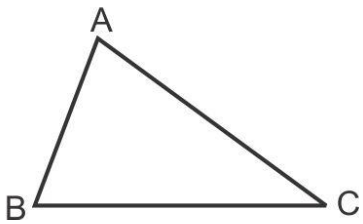
**or**

*Note:* You use only two of the three ratios to find a missing measure!

The ***Sine Law*** is used to solve triangles if:

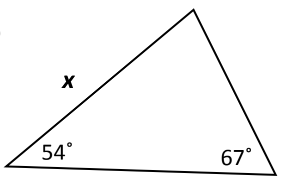
- you know the length of a side of a triangle and the measure of any two angles, you can find the measure of the other two sides.
- you know the length of two sides and the measure of a non-included angle, you can find the measure of the measure of the other non-included angle *provided the triangle can exist** (*Later you will learn about the ambiguous case*

**PROOF OF THE SINE LAW:**

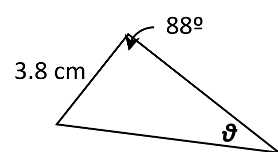



**Ex #1:** Determine the indicated side length or angle to the nearest tenth of a unit. **(CONCEPT 22)**

a)




b)



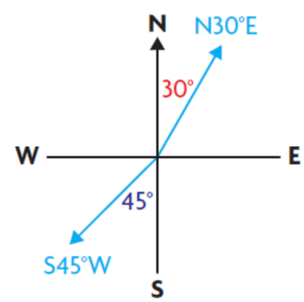

**Ex #2:** Solve $\Delta XYZ$, given $\angle X = 85º$, $x = 15$ cm and $y = 12$ cm. **(CONCEPT 22)**

**Ex #3:** A radio tower is supported by two wires on opposite sides. On the ground, the ends of the wire are 280 m apart. One wire makes a 60° angle with the ground. The other makes a 66° angle with the ground. How long is the shorter wire? **(Concept #24)**

**Ex #4:** Cape Knox is located 215.0 km due south of Cape Ommaney, British Columbia. A hovercraft leaves Cape Ommaney on a heading of S22ºW. A tug boat leaves Cape Knox and travels on a heading of N72ºW. How far from Cape Knox will their paths cross? **( Concept #24)**

Communication | ***Tip***

Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.



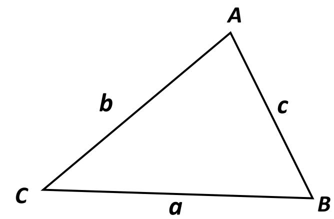

## CH 3.2 ASSIGNMENT : Concept #22, 23

**FA P124 Concept 22: #2, 3aef, 6ac, 7, 8 Concept #24: #4, 5, 9, 10, 12, 13**
**MLA P124 #8, 15,**
**ULA P124 # 17, 18, 19**

Concepts: # 22, 23

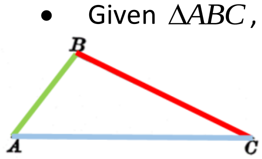
- To use the **Law of Sines** we needed to know a "complete set of information" (an angle measure and the measure of the side opposite that angle)
- If you don't know a "complete set of information" we can use the **Law of Cosines**

**Ex #1:** Use the Pythagorean Theorem to derive the Law of Cosines




**The Cosine Law:**

- Given $\Delta ABC$, the Cosine Law (Not the same as the Cosine Ratio) says that:

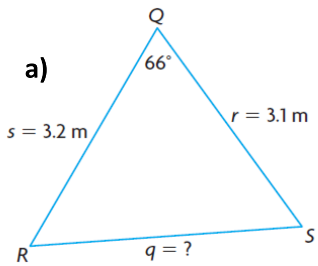



- We use the above formulas to find a side when given 2 sides and the included angle (SAS)
- We can also use the above formulas to find an angle when given all 3 sides ***(SSS).*** If we want to find an angle, we often manipulate the formula so it looks like:

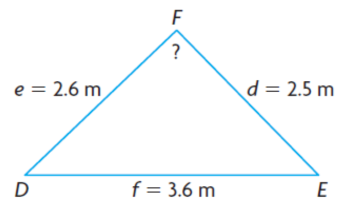
**DO YOUR WORK TO MANIPULATE THE FORMULA HERE:**

**Ex #2:** Determine the indicated side length or angle to the nearest tenth of a unit

**a)**




**b)**




**Ex #3:** Solve $\triangle ABC$ given $a$ = 6 cm, $b$ = 7 cm and $\angle C$ = 103˚. Round all answers to the nearest tenth of a unit.

**Ex #4:** During a hockey game, a player on the blue line shoots a puck toward the 1.83-m-wide net from a point that is 20.3 m from one goal post and 21.3 m from the other goal post. Within what angle must he shoot to hit the net? Answer to the nearest tenth of a degree.

## CH 3.3 ASSIGNMENT : Concept #22, 23

**FA P137 #2, 3, 4b,5b, 7ad, 8, 9**
**MLA P138 #10-17**

### Summary when to use Sine Law vs Cosine Law

**Law of Sines**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

a) Use when given 2 angles and a side **( ASA or AAS)**
b) Use when given2 sides and an angle opposite a given side. **(SSA)**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Law of Cosines**

a) Use to find a side when given 2 sides and the included angle **(SAS)**

$a^2 = b^2 + c^2 - 2bc\text{Cos}A$
$b^2 = a^2 + c^2 - 2ac\text{Cos}B$
$c^2 = a^2 + b^2 - 2ab\text{Cos}C$

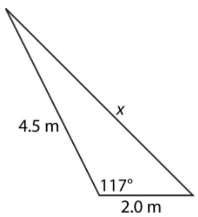
b) Use to find an angle when given 3 sides ***(SSS)***

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

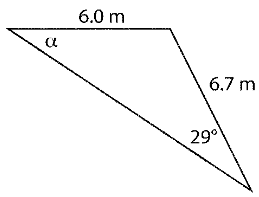
$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

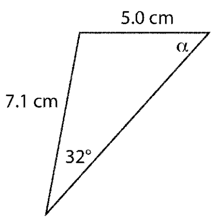
1. What law or property would you use to determine the indicated side length?



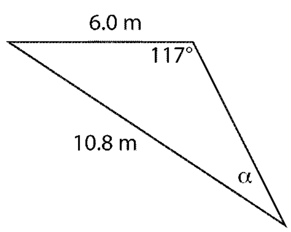

2. What law or property would you use to determine the indicated angle measure?



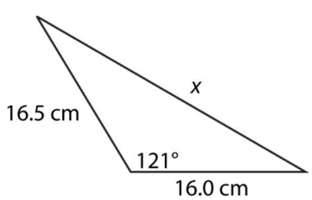

3. Determine the indicated angle measure to the nearest degree.



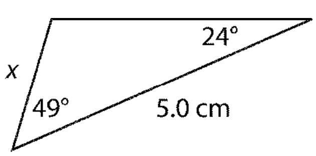

4. Determine the indicated angle measure to the nearest degree.




5. Determine the indicated side length to the nearest tenth of a centimetre.




6. Determine the indicated side length to the nearest tenth of a centimetre.



## Exploring the Primary Trigonometric Ratios of Obtuse Angles

1. Using your calculator, complete the following table:

| $\theta$ | sin$\theta$ | cos$\theta$ | tan$\theta$ | $(180^0-\theta)$ | Sin$(180^0-\theta)$ | Cos$(180^0-\theta)$ | Tan$(180^0-\theta)$ |
|---|---|---|---|---|---|---|---|
| $100^0$ | | | | | | | |
| $110^0$ | | | | | | | |
| $120^0$ | | | | | | | |
| $130^0$ | | | | | | | |
| $140^0$ | | | | | | | |
| $150^0$ | | | | | | | |
| $160^0$ | | | | | | | |
| $170^0$ | | | | | | | |
| $180^0$ | | | | | | | |

Answer each of the following using one of these responses: **THE SAME** or

**THE SAME BUT OPPOSITE IN SIGN**

a) The sine of an angle ($\theta$) and its supplement (180 - $\theta$) are ________________

b) The tangent of an angle ($\theta$) and its supplement (180 - $\theta$) are ____________________

c) The cosine of an angle ($\theta$) and its supplement (180 - $\theta$) are ______________________

**FORMULAS BASED ON THE ABOVE INFO:**

$\sin\theta =$ _______________

$\cos\theta =$ _______________

$\tan\theta =$ _______________

WHY DOES THIS MATTER?

**Ex #1:** Determine the measure of 2 angles between 0-180 that have the following sine ratios. Round to the nearest degree.

a) 0.34

b) 4/7

**Ex #2:** Which of the following are valid? Why?

a) sin50 = sin130

b) cos20 = cos 160

c) tan 40 = -tan140

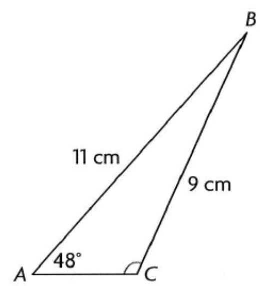
**Ex #3:** Calculate to 4 decimals. Predict another angle that will have an equal or opposite trig ratio.

a) sin63

b) cos 47

c) tan 18

**Ex #4:** Determine the measure of angle C to the nearest degree. Note: This triangle is obtuse




**Ex #5:** Find x


## CH 4.1/4.2 ASSIGNMENT : Concept #22, 23

**FA P163 #1, 2, 3** **P172 #5, 11,13, 14, 15** **P 175 #8**

Concepts: # 24

**http://teachhighschoolmath.blogspot.ca/2013/04/triangle-challenge-ambiguous-case.html**

- Complete the pipe cleaner activity on the handout

## SUMMARY OF THE AMBIGUOUS CASE:

- When you are given an angle, its adjacent side and it's opposite side **(SSA or ASS)**, sometimes there is more than one possible triangle that can be drawn. Due to this possibility, we say that when given SSA or ASS, there exists an ambiguity regarding the correct drawing. We call this "the ambiguous case".

**CASE 1:** The side opposite to our given angle is an altitude (is perpendicular)

**CASE 2:** The side opposite to our given angle is longer than the side that is adjacent to our given angle

**CASE 3:** The side opposite to our given angle is shorter than the adjacent side to the given angle but longer than it would be in CASE 1 (if it were an altitude)

**CASE 4:** The side opposite our given angle is shorter than the CASE 1 scenario – shorter than it would be if were an altitude.

**Ex #1:** Determine the number of triangles that are possible

a) $\measuredangle A = 40°, a = 3, b = 4$

b) $\measuredangle D = 120°, d = 2, e = 3$

c) $\measuredangle A = 30°, a = 4, b = 12$

d) $\measuredangle C = 99°, a = 30, c = 27$

e) $\measuredangle A = 30°, a = 25, b = 50$

f) $\measuredangle B = 27°, b = 25, c = 30$

**Ex #2:** Solve $\triangle BAD$ given that $\measuredangle B = 37.7°, b = 30, c = 30, d = 42$

**SUMMARY AND STEPS:**

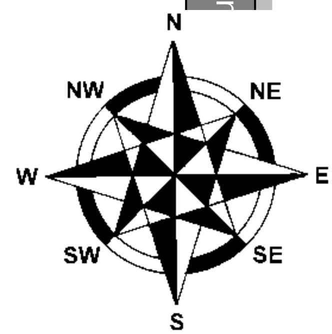
## CH 4.3 : Concept #24

**FA  P183 #1- 5**

## Problems involving Directions

***Using Directions:***

- The compass is a circle divided into points and position and each of which is given a name.
- Directions can also be given using a ***positive acute angle*** formed by the line of travel with the ***north-south line***.
- **Note:** Directions always use a vertical line as the initial side!




***Examples:***

N50ºE N50ºW S50ºE

**Required Steps:**

1. Draw and label a diagram.
2. Describe unknown value(s).
3. Set up (an) equation(s) to represent the given information.
4. Solve the equation(s).
5. Answer in a complete sentence. Include units in your answer.

## EXTRA WORD PROBLEMS : Concept #23

1. A roof truss is 9.8 m wide. If the angles formed by the roof beams are 15° and 18°, find the length of the longest roof beam.

2. A hot-air balloon is flying above BC Place Stadium. Marie is standing due north of the stadium and can see the balloon at an angle of elevation of 64º. Roy is due south of the stadium and can see the balloon at an angle of elevation of 49º. The horizontal distance between Maria and Roy is 500 m. Determine the distance, to the nearest tenth, that the hot-air balloon is from Marie.

3. A telephone pole of length 8 m is located on the side of a hill that is inclined at an angle of 21° with the horizontal. The pole is supported by a wire of length 12 m that is attached to a stake in the ground that is directly uphill from the pole so that the pole remains vertical. What is the angle between the wire and the hill? Give your answer to two decimal places.

4. The Delta Regina Hotel is the tallest building in Saskatchewan. From the window of her room on the top floor at a height of 70 m, Suzie observes a car moving toward the hotel. If the angle of depression changes from 18º to 35º during the time that Suzie observes the car, determine the distance that the car has travelled. Answer to the nearest tenth.

5. Max decided to ski to a friend's cabin. He skied 6.8 km in the direction N35°E. He rested, then skied S30°E and arrived at the cabin. The cabin is 10.2 km from his home, as the crow flies. Determine, to the nearest tenth, the distance he travelled on the second leg of his trip.

6. Cape Knox is located 215.0 km due south of Cape Ommaney, British Columbia. A hovercraft leaves Cape Ommaney on a heading of S22ºW. A tug boat leaves Cape Knox and travels on a heading of N72ºW. How far from Cape Knox will their paths cross?

7. Two ships leave the same harbour at the same time, one at a speed of 12 km/h on a course of N20ºE and the other at a speed of 15 km/h on a course of S74ºW. How far will the ships be after 4 hours? Give your answer to two decimal places.

## SOLUTIONS

1. a) 19.1 b) 10.2 c) 6.7 d) 14.4 e) 10.4 f) 4.9
2. a) 0.1564 b) 0.9659 c) 3.7321
3. a) 40 b) 80 c) 21

**PART B:** 1) 23 2) 17.1 3) 48.2 4) 50 5) 28.8 6) 36.9 7) 66 8) 45 8a) 15 8b) 61.5 9) 13.8 10) 8.1 11) 6 12) 5.5 13) 4.6 14) 12.9

**PART C:**

1. a) 17 c) 44 b) 39
2. 1.17
3. a) 6.9 b) 76.9 c) 77.3 d) 71.5
4. 9.4
5. 45.4
6. a) 113.4 b) 71
7. 74.1
8. 4.1
9. 2397.2
10. 1.11
11. 48
12. 380
13. 12.3
14. 45.4

1. 90.996° 2. 5.560m

1. 409.9m 2. 38.49° 3. 115.5m

1. 11.0 km 2. 80.7 3. 96.4