Ch 5 INTEGRAL CALCULUS 30L PRACTICE QUESTIONS (5.1 – 5.6)

The following questions are additional practice questions that will help you prepare for the 30L Chapter 5 test.The questions that allow for graphing calculator will be indicated within the instructions and/or by the image of a calculator beside the question. Please note that many questions given within the calculator section do not actually require the use of the calculator to answer the question most efficiently – it is quite possible that the calculator will be of no use at all. Questions that do not include an image of a calculator generally mean that NO calculator is to be used. .

1. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined
(B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist
(D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

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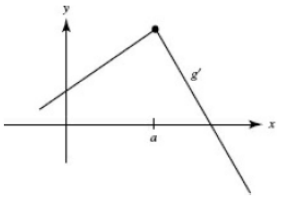
2. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?



I. g is continuous.
II. g is differentiable.
III. g is increasing.

(A) I only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

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3. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

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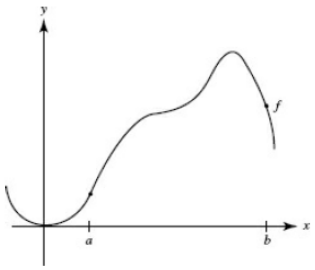
4. At how many points on the interval $[a, b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none
(B) 1
(C) 2
(D) 3
(E) 4

For Questions 5 and 6 $f'(x) = x \sin x - \cos x$ for $0 < x < 4$.

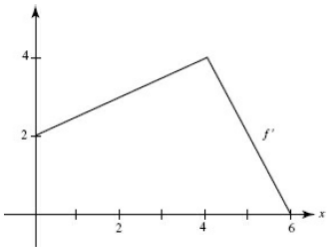
5. f has a local maximum when x is approximately

(A) 0.9
(B) 1.2
(C) 2.3
(D) 3.4
(E) 3.7

6. The graph of f has a point of inflection when x is approximately

(A) 0.9
(B) 1.2
(C) 2.3
(D) 3.4
(E) 3.7

7. The graph of f' is shown below. If we know that $f(2) = 10$, then the local linearization of f at $x = 2$ is $f(x) \approx$



(A) $\frac{x}{2} + 2$
(B) $\frac{x}{2} + 9$
(C) $3x - 3$
(D) $3x + 4$
(E) $10x - 17$

8. Suppose $f'(x) = x^2(x - 1)$. Then $f''(x) = x(3x - 2)$. Over which interval(s) is the graph of f both increasing and concave up?

I. $x < 0$
II. $0 < x < \frac{2}{3}$
III. $\frac{2}{3} < x < 1$
IV. $x > 1$

(A) I only
(B) II only
(C) II and IV
(D) I and III
(E) IV only

9. Which of the following statements is true about the graph of $f(x)$ in Question 8.

(A) The graph has no relative extrema.
(B) The graph has one relative extremum and one inflection point.
(C) The graph has two relative extrema and one inflection point.
(D) The graph has two relative extrema and two inflection points.
(E) None of the preceding statements is true.

10. At what point in the interval [1, 1.5] is the rate of change of $f(x) = \sin x$ equal to its average rate of change on the interval?

(A) 0.995
(B) 1.058
(C) 1.239
(D) 1.253
(E) 1.399

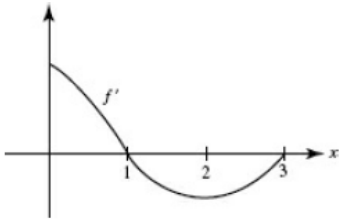
11. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

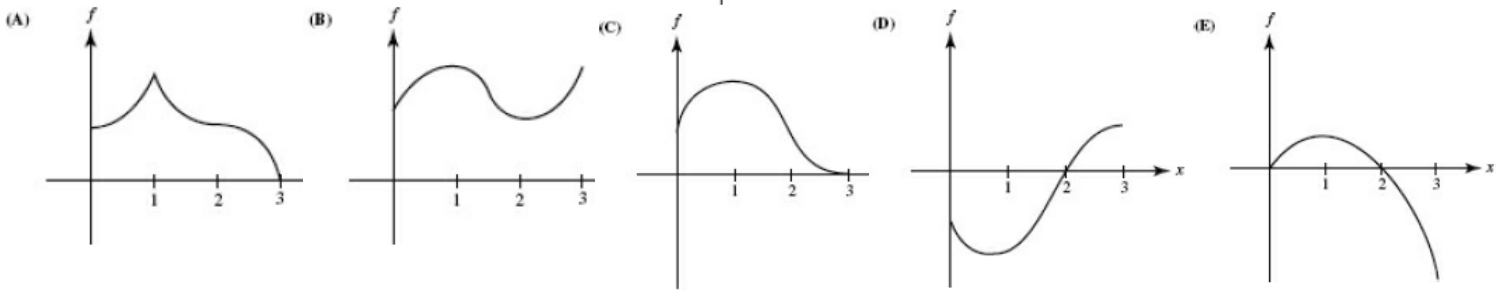
(A) $\lim_{x \to a} f(x)$ exists.
(B) $\lim_{x \to a} f(x) = f(a)$.
(C) $f'(a)$ exists.
(D) $f(a)$ is defined.
(E) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

12. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

A. The change in temperature during the first day
B. The change in the temperature during the 24th hour.
C. The average rate at which the temperature changed during the 24th hour
D. The average rate at which the temperature is changing during the first day
E. The rate at which the temperature is changing at the end of the 24th hour.

13. Given f' as graphed, which could be the graph of f?





14. On the interval $0 < x < 10$, how many relative minimums does the graph of $g(x)$ have if $g'(x) = \frac{\sin x}{x+2}$?

A. 0
B. 1
C. 2
D. 3
E. 4

15. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) −1.58 (B) −1.63 (C) −1.67 (D) −1.89 (E) −2.33

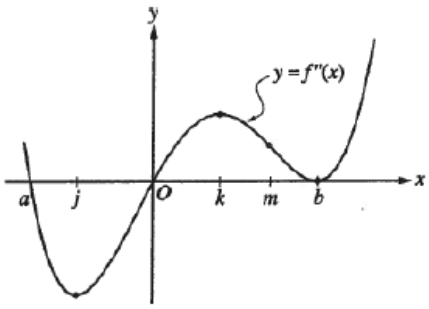
x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

16. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

A. $-2<x<2$ only
B. $-2 \le x \le 2$ only
C. $-1 \le x \le 1$ only
D. $x \ge -2$
E. $x \le -2$ or $x \ge 2$
F. $x \ge 2$ only

17. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of $f''(x)$ is shown to the right. For what values of x does the graph of $f'(x)$ have a relative maximum?

A. j and k only
B. a and b only
C. a only
D. 0 only
E. a and 0 only



18. A table of function values for a twice differentiable function, $f(x)$, is pictured to the right. Which of the following statements is/are true if $f(x)$ has only one zero on the $-3 \le x \le 3$? Use only the given points to answer!

I. $f'(x) < 0$ on the interval $-3 < x < 3$.
II. $f(x)$ has a zero between $x = 1$ and $x = 3$.
III. $f''(x) > 0$ on the interval $-3 < x < 3$.

x	$f(x)$
−3	10
−1	8
1	2
3	−13

A. I only B. I and II only C. III only

D. II and III only E. I, II and III

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19. The function f has a first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1+x+x^3}$. What is the x–coordinate of the point of inflection of the graph of f? (CALCULATOR PROBLEM)

A. 1.008
B. 0.473
C. 0
D. −0.278
E. The graph has no points of inflection.

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20. If $h(x)$ is a twice differentiable function such that $h(x) < 0$ for all values of x, then at what value(s) does the graph of $g(x)$ have a relative maximum if $g'(x) = \left(9 - x^2\right) \cdot h(x)$?

A. $x = 3$ and $x = -3$ B. $x = 3$ only
C. $x = 9$ only D. $x = -3$ only
E. $g(x)$ does not have a relative maximum

21. For $t \geq 0$, the velocity of a particle moving along the x – axis is given by $v(t) = e^{\tan t} + t^2 - 5$. Which of the following statements is/are true?

I. The particle first changes directions at $t = 1$.

II. On the interval $0 < t < 1$, the mean value theorem guarantees a time t at which the instantaneous acceleration is equal to the average acceleration of the particle.

III. At $t = 2$, the speed of the particle is decreasing.

A. II and III only
B. I and III only
C. III only
D. I, II and III

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22. A particle moves along a line so that at time t, where $0 \leq t \leq \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

A. 2.55
B. 0.74
C. 1.32
D. –5.19

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23. The position of a particle moving along the x – axis is given by the function $p(t) = (t-1)\cos(2t)$. At what value of t does the particle change directions the second time on the interval $0 < t < 3$?

A. 0.543
B. 1.386
C. 0.892
D. 1.839

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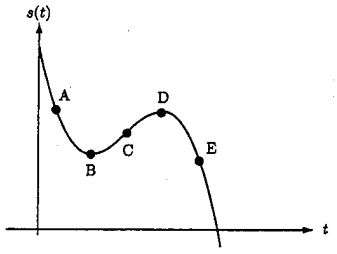
24. The graph below shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time, t. Which of the following points marked is closest to the point where the acceleration first becomes negative?



A. A
B. B
C. C
D. D

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25. A particle moves along the x – axis so that at any time $t \geq 0$ its velocity is given by the function $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

A. 1.500
B. 29.453
C. 20.453
D. 74.860

THE REMAINDER OF THE ASSIGNMENT REQUIRES WORK AND JUSTIFICATION TO BE SHOWN

26. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c on the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following values could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

27. For $t\geq 0$, the temperature of a cup of coffee in degrees Fahrenheit t minutes after it is poured is modeled by the function $F(t)=68+93(0.91)^t$. Find the value of $F'(4)$. Using correct units of measure, explain what this value means in the context of the problem. **[CALC]**

28. A particle moves along the x-axis so that any time $t>0$, its velocity is given by $v(t)=2t\ln t-t$.

a. Write an expression for the acceleration of the particle.

b. What are the values of t for which the particle is moving to the right? Justify your answer.

c. Is the particle speeding up or slowing down at $t = 1$? Show the analysis that leads to your conclusion.

d. Find the absolute minimum velocity of the particle. Show the analysis that leads to your conclusion.

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29. The function $f'(x) = \cos(\ln x)$ is the first derivative of a twice differentiable function, $f(x)$.

a. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a relative maximum. Justify your answer.

b. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a relative minimum. Justify your answer.

c. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a point of inflection. Justify your answer.

30. For the functions in exercises 1 and 2, determine if the Mean Value Theorem holds true for $0 < c < 5$? Give a reason for your answer. If it does hold true, find the guaranteed value(s) of c. **[CALC]**

a) $f(x) = -2 + \frac{1}{2}|x-3|$

b) $g(x) = -2x + \sin^2 x$

31. The price of a share of stock in dollars over a week is given by the function $P(t) = \sqrt{2t+1} + 2\cos t + 20$ where t is measured in days and $0 \le t \le 5$.

a. Find the average rate of change of the price of the stock over [0, 5]. Use correct units.

b. Apply the Mean-Value Theorem to P on [0, 5] and explain the result in the context of the problem situation.

c. On what value of t over the 5-day period is the price of the stock increasing the fastest?

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32. Administrators at a hospital believe that the number of beds in use is given by the function

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50,$$

where t is measured in days. **[CALC]**

a. Find the value of $B'(7)$. Using correct units of measure, explain what this value means in the context of the problem.

b. For $12 \le t \le 20$, what is the maximum number of beds in use?

33. Determine whether $g(x) = \sin 2x + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, \pi]$ If so, find all numbers c in (a, b) such that $f(b) - f(a) = f'(c)(b-a)$. **NO SCIENTIFIC OR GRAPHING CALCULATOR.**

34. For questions $5 - 8$, use the table given below which represents values of a differentiable function g on the interval $0 \le x \le 6$. Be sure to completely justify your reasoning when asked, citing appropriate theorems, when necessary.

x	0	2	3	4	6
$g(x)$	-3	1	5	2	1

a) Estimate the value of $g'(2.5)$.

b) If one exists, on what interval is there guaranteed to be a value of c such that $g(c) = -1$? Justify your reasoning.

c) If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 0$? Justify your reasoning.

d) . If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 4$? Justify your reasoning.