

## Calculus 30 Final Exam Review Answer Key

- A. 1. a) -1      b) DNE      c) 2      d) 1      e) DNE

B.

- |                   |          |        |         |                   |                    |         |
|-------------------|----------|--------|---------|-------------------|--------------------|---------|
| 1. $\frac{1}{2}$  | 2. 32    | 3. -1  | 4. 8    | 5. 12             | 6. 4               | 7. DNE  |
| 8. 0              | 9. a) -1 | b) DNE | 10. DNE | 11. $\infty$      | 12. $\infty$       | 13. 0   |
| 14. $\frac{2}{3}$ | 15. 0    | 16. 0  | 17. -5  | 18. $\frac{1}{4}$ | 19. $-\frac{1}{2}$ | 20. DNE |

- C. 1.  $4x - 3$       2.  $\frac{1}{\sqrt{2x-1}}$

D.

$$\begin{aligned}
 1. \quad & \frac{\partial y}{\partial x} = 4x - 1 \\
 2. \quad & \frac{\partial y}{\partial x} = 24x^{23} \\
 3. \quad & \frac{\partial y}{\partial x} = 8x + 4 \\
 4. \quad & \frac{\partial y}{\partial x} = x^3 + x^2 + x + 1 \\
 5. \quad & \frac{\partial y}{\partial x} = -6x^{-4} - 3x^{-\frac{3}{2}} \\
 6. \quad & \frac{\partial y}{\partial x} = 4(x+1)(x^2 + 2x - 1) \\
 7. \quad & \frac{\partial y}{\partial x} = -2(2x-1)^{-2}(x+2)^{-2} - 2(2x-1)^{-1}(x+2)^{-3} \\
 & = -2(2x-1)^{-2}(x+2)^{-3}(3x+1) \\
 8. \quad & \frac{\partial y}{\partial x} = (x^2 + 2x)^{\frac{1}{2}} + (x+3)(x+1)(x^2 + 2x)^{-\frac{1}{2}} \\
 & = (x^2 + 2x)^{-\frac{1}{2}}(2x^2 + 6x + 3) \\
 9. \quad & \frac{\partial y}{\partial x} = \frac{(6t-2)(5-3t^2) - 6t(3t^2-2t)}{(5-3t^2)^2} \\
 & = \frac{-10 + 30t - 6t^2}{(5-3t^2)^2} \\
 10. \quad & \frac{\partial y}{\partial x} = \frac{(2x-1)(1-x^2)^{\frac{1}{2}} - \frac{1}{2}(-2x)(1-x^2)^{-\frac{1}{2}}(x^2-x)}{1-x^2} \\
 & = -1(1-x^2)^{-\frac{3}{2}}(x^3 - 2x + 1) \\
 11. \quad & \frac{\partial y}{\partial x} = -\frac{3}{2}x^2(27-x^3)^{-1/2} \\
 12. \quad & \frac{\partial y}{\partial x} = -\frac{1}{3}(9x^2+2)(3x^3+2x+1)^{-4/3}
 \end{aligned}$$

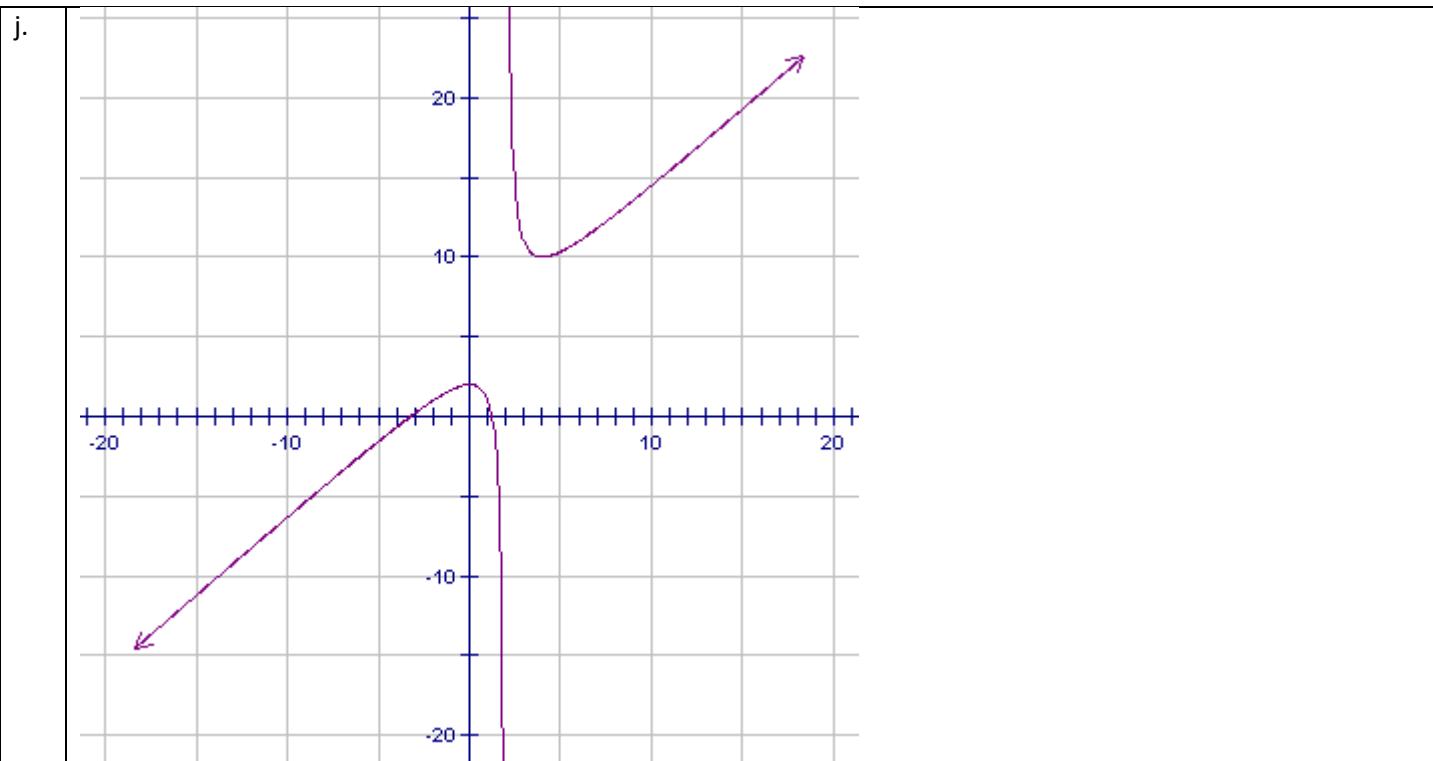
$$\begin{aligned}
 13. \quad & \frac{\partial y}{\partial x} = 2x(2x^2-7)^{1/3} + \frac{1}{3}x^2(4x)(2x^2-7)^{-2/3} \\
 & = \frac{2}{3}x(2x^2-7)^{-2/3}(8x^2-21) \\
 14. \quad & \frac{\partial y}{\partial x} = -2(x-6)^{-2} - 3x^{-2} \\
 15. \quad & \frac{\partial y}{\partial x} = 3(x^2+5)^{-2} - 4x(3x+2)(x^2+5)^{-3} \\
 & = -(x^2+5)^{-3}(9x^2+8x-15) \\
 16. \quad & 2y \frac{\partial y}{\partial x} = 2x \frac{\partial y}{\partial x} + 2y \\
 & \frac{\partial y}{\partial x} = \frac{y}{y-x} \\
 17. \quad & 2xy^2 - y^3 = x^2 \\
 & 2y^2 + 4xy \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 2x \\
 & \frac{dy}{dx} = \frac{2x-2y^2}{4xy-3y^2} \\
 18. \quad & 2x = yx + y^2 \\
 & 2 = \frac{dy}{dx}x + y + 2y \frac{dy}{dx} \\
 & \frac{dy}{dx} = \frac{2-y}{x+2y}
 \end{aligned}$$

- 19 a.  $m = 28$   $y = 28x - 35$   
 b.  $m = 224$   $y = 224x - 476$   
 c.  $m = 13/242$   $y = \frac{13}{242}x - \frac{135}{121}$

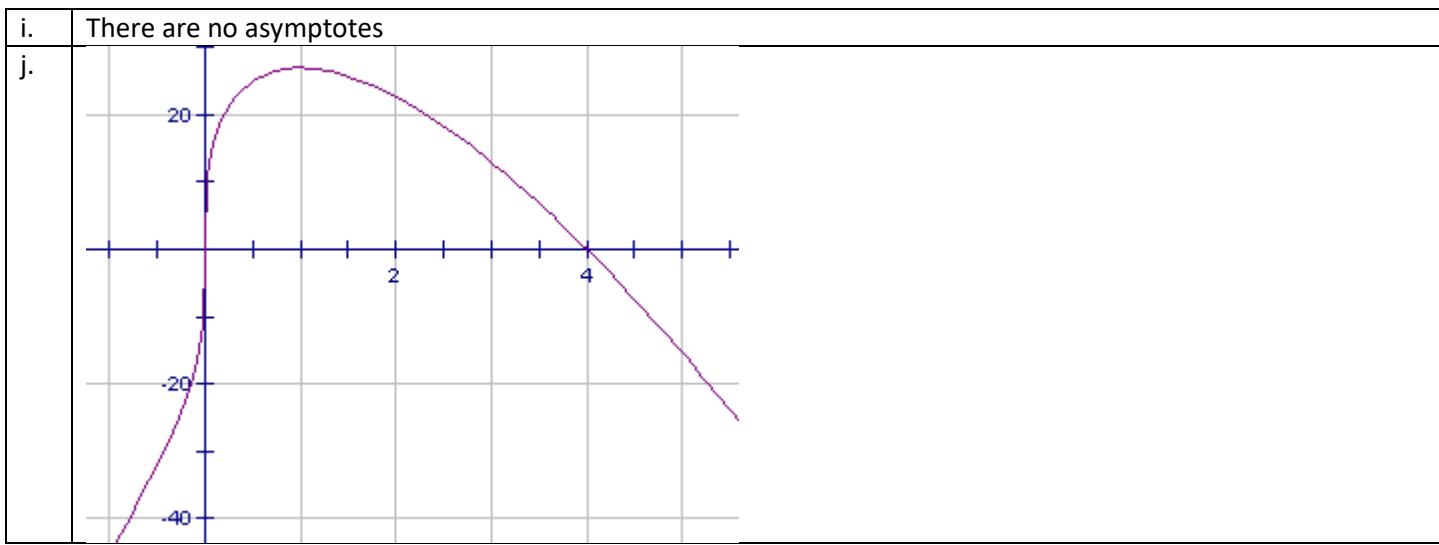
20. (Part 1)

A.	$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$  $\begin{array}{ccccccc} \leftarrow & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & \rightarrow \\ \text{---} & 0 & + & + & + & + & + & + & + & + & + & + & (x+4) \\ \text{---} & 0 & + & + & + & + & + & + & + & + & + & + & (x-2) \\ \leftarrow & + & + & + & + & 0 & - & - & - & - & - & - & 0 & + & + & + & + & + & + & + & + & + & + & 3(x+4)(x-2) \end{array}$
b.	Increasing on $(-\infty, -4) \cup (2, \infty)$ because $f'(x) > 0$ , Decreasing on $(-4, 2)$ because $f'(x) < 0$
c.	The critical numbers are $x = -4, 2$
d.	There is a relative maximum at $(-4, 80)$ because $f'(x)$ changes from $+ 0 -$ at $x = -4$ and a relative minimum at $(2, -28)$ because $f'(x)$ changes from $- 0 +$ at $x = 2$
e.	$f''(x) = 6x + 6 = 6(x+1)$  $\begin{array}{ccccccc} \leftarrow & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & \rightarrow \\ \text{---} & 0 & + & + & + & + & + & + & + & + & + & + & 6 \\ \text{---} & 0 & + & + & + & + & + & + & + & + & + & + & (x+1) \\ \leftarrow & + & + & + & + & + & 0 & - & - & - & - & - & 6(x+1) \end{array}$
f.	Concave down on $(-\infty, -1)$ because $f''(x) < 0$ Concave up on $(-1, \infty)$ because $f''(x) > 0$
g.	$f(-1) = (-1)^3 + 3(-1)^2 - 24(-1) = -1 + 3 + 24 = 26$ Inflection point at $(-1, 26)$ because $f''(x)$ changes signs at $x = -1$
h.	$0 = x^3 + 3x^2 - 24x$ $0 = x(x^2 + 3x - 24)$ $0 = x$ or $0 = x^2 + 3x - 24$ {apply the quadratic formula} $x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-24)}}{2(1)}$ $x = \frac{-3 \pm \sqrt{103}}{2}$ $x$ intercepts at $0, \frac{-3 + \sqrt{103}}{2}, \frac{-3 - \sqrt{103}}{2}$ $y = (0)^3 + 3(0)^2 - 24(0) = 0$ $y$ intercept at $0$
i.	There are no asymptotes
j.	

Ba.	$f'(x) = \frac{(2x+2)(x-2)-1(x^2+2x-4)}{(x-2)^2} = \frac{2x^2-4x+2x-4-x^2-2x+4}{(x-2)^2} =$ $f'(x) = \frac{x^2-4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$ <p>The sign chart for <math>f'(x)</math> shows the following intervals:    - For <math>x &lt; -2</math>, the expression is positive (blue '+' signs).    - At <math>x = -2</math>, there is a jump from positive to negative (red '-' sign).    - For <math>-2 &lt; x &lt; 0</math>, the expression is negative (red '-' sign).    - At <math>x = 0</math>, there is a jump from negative to positive (blue '+' sign).    - For <math>0 &lt; x &lt; 4</math>, the expression is negative (red '-' sign).    - At <math>x = 4</math>, there is a jump from negative to positive (blue '+' sign).    - For <math>x &gt; 4</math>, the expression is positive (blue '+' sign).</p>
b.	Increasing on $(-\infty, 0) \cup (4, \infty)$ because $f'(x) > 0$ , Decreasing on $(0, 4)$ because $f'(x) < 0$
c.	The critical numbers are $x = 0, 4$
d.	There is a relative minimum at $(4, 10)$ because $f'(x)$ changes from $-0+$ at $x = 4$ and a relative maximum at $(0, 2)$ because $f'(x)$ changes from $+0-$ at $x = 0$
e.	$f''(x) = \frac{(2x-4)(x-2)^2 - 2(x-2)(x^2-4x)}{(x-2)^4} = \frac{(x-2)[(2x-4)(x-2) - 2(x^2-4x)]}{(x-2)^4}$ $f''(x) = \frac{(2x-4)(x-2) - 2(x^2-4x)}{(x-2)^3} = \frac{2x^2-4x-4x+8-2x^2+8x}{(x-2)^3} =$ $f''(x) = \frac{8}{(x-2)^3}$ <p>The sign chart for <math>f''(x)</math> shows the following intervals:    - For <math>x &lt; 2</math>, the expression is negative (red '-' sign).    - At <math>x = 2</math>, there is a jump from negative to positive (blue '+' sign).    - For <math>x &gt; 2</math>, the expression is positive (blue '+' sign).</p>
f.	Concave down on $(-\infty, 2)$ because $f''(x) < 0$ Concave up on $(2, \infty)$ because $f''(x) > 0$
g.	No inflection point
h.	$0 = \frac{x^2+2x-4}{x-2}$ $0 = x^2 + 2x - 4 \quad \{ \text{apply the quadratic formula} \}$ $x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$ <p style="color: red;">x intercepts at <math>-1 + \sqrt{5}, -1 - \sqrt{5}</math></p> $y = \frac{(0)^2 + 2(0) - 4}{(0) - 2} = 2$ <p style="color: red;">y intercept at 2</p>
i.	$x = 2$ is a vertical asymptote



Ca.	$f'(x) = 12x^{-2/3} - 12x^{1/3} = 12x^{-2/3}(1-x) = \frac{1-x}{12x^{2/3}}$ $\frac{1-x}{12x^{2/3}}$
b.	Increasing on $(-\infty, 0) \cup (0, 1)$ because $f'(x) > 0$ Decreasing on $(1, \infty)$ because $f'(x) < 0$
c.	The critical numbers are $x = 1, 0$
d.	There is a relative maximum at $(1, 0)$ $f'(x)$ changes from +0- at $x = 1$
e.	$f'(x) = \frac{1-x}{12x^{2/3}}$ $f''(x) = \frac{-1(12x^{2/3}) - 8x^{-1/3}(1-x)}{144x^{4/3}} = \frac{4x^{-1/3}[-3x - 2(1-x)]}{144x^{4/3}} = \frac{4x^{-1/3}(-x-2)}{144x^{4/3}} = \frac{-(x+2)}{36x^{5/3}}$ $\frac{-(x+2)}{36x^{5/3}}$
f.	Concave down on $(-\infty, 2) \cup (0, \infty)$ because $f''(x) < 0$ Concave up on $(-2, 0)$ because $f''(x) > 0$
g.	$f(-2) = 36(-2)^{1/3} - 9(-2)^{4/3} = -54\sqrt[3]{2}$ because $f''(x)$ changes sign at $x = -2$ Inflection point at $(-2, -54\sqrt[3]{2}) \approx (-2, -90.7)$
h.	$0 = 36x^{1/3} - 9x^{4/3}$ $0 = x^{1/3}(36 - 9x)$ $0 = x$ or $4 = x$ x intercepts at 0, 4 $y = 36(0)^{1/3} - 9(0)^{4/3} = 0$ y intercept at 0



20. (Part 2)

a) Local Max at  $x = -2$  is  $40/3$  or  $13\frac{1}{3}$  because  $f''(x) < 0$  when  $x = -2$  and

Local Min at  $x = 5$  is  $-263/6$  or  $-43.8\bar{3}$  because  $f''(x) > 0$  when  $x = 5$ .

b) Local min at  $x = 1$  is 0 because  $f''(x) > 0$  when  $x = 1$ , Local min at  $x = 2$  is 0 because  $f''(x) > 0$  when  $x = 2$ ,

Local max at  $x = 1.5$  is  $1/16$  because  $f''(x) < 0$  when  $x = 1.5$

c) No local extrema because  $f'(0) = 0$  (no min or max).

21.

$$(x)(x) = x^2$$

$$Y = x - x^2$$

$$Y' = 1 - 2x$$

$$0 = 1 - 2x$$

$$x = 1/2$$

Please show the sign analysis here as well!

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

The number that exceeds its square by the greatest amount is  $\frac{1}{2}$  because  $Y'$  changes from  $+0-$  at  $x = 1/2$ . The amount that it exceeds its square by is  $\frac{1}{4}$ .

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

22.

$$60 = l + 2w$$

$$l = -2w + 60$$

$$A = lw$$

$$A = (-2w + 60)(w)$$

$$A = -2w^2 + 60w$$

$$A' = -4w + 60$$

$$A' = -4(w - 15)$$

$$0 = w - 15$$

$$w = 15$$

$$l = -2(15) + 60$$

$$l = 30$$

$$A = (30)(15)$$

$$A = 450m^2$$

Please show the sign analysis here as well!

The maximum dimensions are 30 m by 15 m (they are a maximum because A' changes from +0- at w = 15 m). The maximum area is 450 m<sup>2</sup>

23.

$$A = 2xy \quad x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3600 \quad y = \sqrt{3600 - x^2}$$

$$A = 2x(\sqrt{3600 - x^2})$$

$$A' = 2(3600 - x^2)^{\frac{1}{2}} + \left[ 2x\left(\frac{1}{2}\right)(3600 - x^2)^{-\frac{1}{2}}(-2x) \right]$$

$$A' = (3600 - x^2)^{-\frac{1}{2}} [2(3600 - x^2) - 2x^2]$$

$$A' = (3600 - x^2)^{-\frac{1}{2}} [7200 - 4x^2]$$

$$0 = (3600 - x^2)^{-\frac{1}{2}} [7200 - 4x^2]$$

$$0 = 7200 - 4x^2$$

$$x^2 = 1800$$

$$x = \sqrt{1800}$$

$$x = 30\sqrt{2}$$

$$y = \sqrt{3600 - (30\sqrt{2})^2}$$

$$y = \sqrt{3600 - 1800}$$

$$y = \sqrt{1800}$$

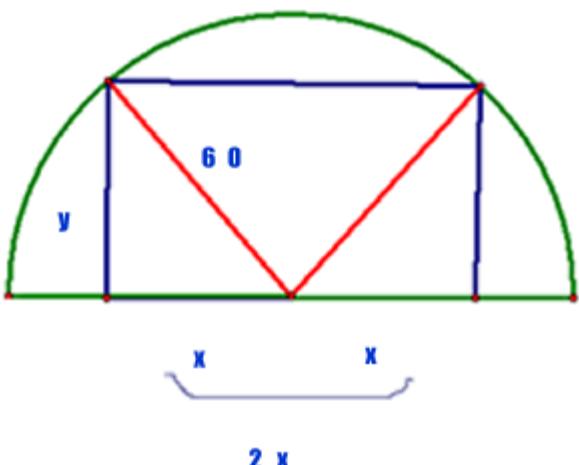
$$y = 30\sqrt{2}$$

max area is when  $x = 30\sqrt{2}$  and  $y = 30\sqrt{2}$

$$A = 2xy$$

$$A = 2(30\sqrt{2})^2$$

$$A = 3600m^2$$



Please show the sign analysis here as well!

The dimensions of the largest rectangle that can be inscribed in a semi-circle of radius 60 m are  $30\sqrt{2}$  m and  $30\sqrt{2}$  m because A' changes from +0- when  $x = 30\sqrt{2}$

The maximum area is 3600 m<sup>2</sup>

$$SA = 4xh + x^2$$

$$2700 = 4xh + x^2, \quad h = \frac{2700 - x^2}{4x}$$

$$V = x^2h, \quad V = x^2 \left( \frac{2700 - x^2}{4x} \right)$$

$$V = 625x - \frac{x^3}{4}$$

$$V' = 625 - \frac{3x^2}{4}$$

$$0 = 625 - .75x^2$$

$$-625 = -.75x^2$$

$$x^2 = 833.333, \quad x = 28.87 \text{ cm}$$

$$h = \frac{2700 - (28.87)^2}{4(28.87)}$$

$$h = 16.16 \text{ cm}$$

$$V = x^2h$$

$$V = 13466.70 \text{ cm}^3, \text{ rounding may change the answer slightly}$$

25.

$$f(x) = \left( \sqrt{x^2 + 40000} \right)(500) + (1000 - x)200$$

$$f(x) = 500(x^2 + 40000)^{\frac{1}{2}} + (200000 - 200x)$$

$$f'(x) = 250(x^2 + 40000)^{-\frac{1}{2}}(2x) - 200$$

$$f'(x) = \frac{500x}{\sqrt{x^2 + 40000}} - 200$$

$$0 = \frac{500x}{\sqrt{x^2 + 40000}} - 200$$

$$500x = 200\sqrt{x^2 + 40000}$$

$$2.5x = \sqrt{x^2 + 40000}$$

$$6.25x^2 = x^2 + 40000$$

$$5.25x^2 = 40000$$

$$x^2 = 7619.047$$

$$x = 87.29$$

$\therefore$  S should be 87.29m from V in order to minimize the total cost

Please show the sign analysis here as well!

The dimensions of the largest box with a square base and open top constructed using 2700  $\text{cm}^2$  of material are  $x = 28.87 \text{ cm}$  and  $h = 16.16 \text{ cm}$  because  $V'$  changes from  $+0-$  when  $x = 28.87 \text{ cm}$

The maximum volume is  $13466.70 \text{ cm}^3$

Please show the sign analysis here as well!

The minimal cost for the pipeline occurs when  $x = 87.29 \text{ m}$  because  $f'(x)$  changes from  $-0+$  when  $x = 87.29 \text{ m}$

26.(a)  $s(t) = 2t^2 + 5t$      $v(t) = 4t + 5$      $a(t) = 4$

(b)  $s(t) = (3t+1)^{1/2}$      $v(t) = \frac{3}{2}(3t+1)^{-1/2}$      $a(t) = -\frac{9}{4}(3t+1)^{-3/2}$

(c)  $s(t) = \frac{4t}{t^2+1}$      $v(t) = \frac{-4(t^2-1)}{(t^2+1)^2}$      $a(t) = \frac{8t(t^2-3)}{(t^2+1)^3}$

27. (a)  $v(t) = t^2 + 5t$      $a(t) = 2t + 5$

(b)  $v(4) = (4)^2 + 5(4) = 36 \text{ m/s}$

(c)  $a(3) = 2(3) + 5 = 11 \text{ m/s}^2$

(d)  $66 = t^2 + 5t$

$$0 = t^2 + 5t - 66$$

$$0 = (t+11)(t-6)$$

$$t \neq -11 \quad t = 6 \text{ s}$$

$$s(6) = \frac{1}{3}(6)^3 + \frac{5}{2}(6)^2 = 162 \text{ m}$$

(e)  $13 = 2t + 5$

$$8 = 2t$$

$$4s = t$$

$$v(4) = (4)^2 + 5(4) = 36 \text{ m/s}$$

28.) (a)  $f'(x) = \frac{1}{2x-5} \log_3 e(2) = \frac{2}{2x-5} \log_3 e$

(b)  $f'(x) = \frac{12}{x^3-3x} (\log e)(3x^2-3) = \frac{36(x^2-1)}{x^3-3x} (\log e)$

(c)  $f'(x) = \left( \frac{x-1}{x+1} \right) \log_{11} e \left( \frac{1(x-1)-1(x+1)}{(x-1)^2} \right) = \left( \frac{x-1}{x+1} \right) \log_{11} e \left( \frac{-2}{(x-1)^2} \right) = \left( \frac{-2}{(x+1)(x-1)} \right) \log_{11} e$

$$f(x) = x^{-3} \log_3 x$$

(d)  $f'(x) = -3x^{-4} \log_3 x + \frac{1}{x} \log_3 e(x^{-3}) = -3x^{-4} \log_3 x + \log_3 e(x^{-4}) =$

$$x^{-4} (-3 \log_3 x + \log_3 e) = x^{-4} \left( \log_3 \frac{e}{x^3} \right)$$

29.) Find the derivative of each of the following functions.

a.  $f'(x) = \frac{2x^2-5}{2x^3-5x+1}$

b.  $f'(x) = \frac{1}{\sqrt{1-6x}} \left( \frac{1}{2}(1-6x)^{-1/2}(-6) \right) = \frac{-3}{1-6x}$

c.  $f'(x) = \left( \frac{x+2}{x^2-2} \right) \left( \frac{2x(x+2)-1(x^2-2)}{(x+2)^2} \right) = \left( \frac{x+2}{x^2-2} \right) \left( \frac{x^2+4x+2}{(x+2)^2} \right) = \frac{x^2+4x+2}{(x^2-2)(x+2)}$

d.  $f'(x) = 3 \left[ \ln(x^3-1) \right]^2 \left( \frac{1}{x^3-1} (3x^2) \right) = \frac{9x^2 \left[ \ln(x^3-1) \right]^2}{x^3-1}$

e.  $f'(x) = \frac{1}{\ln(x^3+9x)} \left( \frac{1}{x^3+9x} (3x^2+9) \right) = \frac{3x^2+9}{(x^3+9x)\ln(x^3+9x)}$

30.) A salesperson at a car dealership began work in an unfamiliar city. The number of people whose names the salesperson could remember after working  $x$  weeks is given by the function  $f(x) = 12 \ln(x+1) + 1$ .

- (a)  $f(0) = 12 \ln((0)+1) + 1 = 1$
- (b)  $f(4) = 12 \ln((4)+1) + 1 \approx 20$
- (c)  $f'(x) = \frac{12}{x+1}$ .

(d)  $f'(4) = \frac{12}{4+1} = 2.4$  people/month - His ability to remember names is INCREASING at a rate of 2.4 people/month at  $x = 4$  months

$f'(9) = \frac{12}{9+1} = 1.2$  people/month His ability to remember names is INCREASING at a rate of 1.2 people/month at  $x = 9$  months

The rate at which the salesperson is meeting new people is reducing as the weeks go by. (e) The rate decreases as there are fewer new people that the salesman would meet and be able to recall.

31.) a.

$$f(x) = x^2 \ln x$$

$$f'(x) = 2x \ln x + \frac{1}{x}(x^2) = 2x \ln x + x$$

$$f''(x) = 2 \ln x + 2x\left(\frac{1}{x}\right) + 1 = 2 \ln x + 2 + 1 = 2 \ln x + 3$$

$$0 = 2 \ln x + 3$$

$$\ln x = -\frac{3}{2}$$

$$x = e^{-3/2}$$

Note that the function is undefined for  $(-\infty, 0)$ .

Using test points, you can determine the function is concave down for  $(0, e^{-3/2})$  and concave up for  $(e^{-3/2}, \infty)$   $(-\infty, \infty)$ , range  $[1, \infty)$

11.) a.  $f'(x) = 7^x (\ln 7)(3x^2)$

b.  $f'(x) = 5^{x^2+4x+3} (\ln 5)(2x+4)$

c.  $f'(x) = 11^x + x \cdot 11^x (\ln x) = 11^x (1 + x \ln x)$

d.  $f'(x) = \frac{1}{2}(x-1)^{-1/2} (4^{5x^4}) + (4^{5x^4})(\ln 4)(20x^3)(x-1)^{1/2} = \frac{1}{2}(x-1)^{-1/2} (4^{5x^4}) [1 + (\ln 4)(40x^3)(x-1)]$

32.) a. domain

34.) Find the derivative of each of the following functions.

a.  $f'(x) = 2e^{2x-5}$

b.  $f'(x) = 2(e^{3x+1})(e^{3x+1})(3) = 6(e^{3x+1})^2$

c.  $f'(x) = 2xe^x + x^2e^x = xe^x(2+x)$

d.  $f'(x) = \frac{e^{x^2-1} - e^{x^2-1}(2x)(x+1)}{(e^{x^2-1})^2} = \frac{e^{x^2-1}(1-2x^2+2x)}{(e^{x^2-1})^2} = \frac{-2x^2-2x+1}{e^{x^2-1}}$

e.  $f^{100}(x) = 2^{99}e^{2x}$ .

35a.	$T(0) = 80e^{-0.1(0)} + 20 = 80 + 20 = 100^\circ C$
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b.	$80 = 80e^{-0.1t} + 20$ $60 = 80e^{-0.1t}$ $0.75 = e^{-0.1t}$ $\ln(0.75) = -0.1t$ $\frac{\ln(0.75)}{-0.1} = t$ $2.87 \approx t$ <p>It would take a little less than 3 minutes to cool to 80°C</p>
c.	$T(11) = 80e^{-0.1(11)} + 20 = 80e^{-1.1} + 20 = 46.6^\circ C$
d.	$T'(t) = 80e^{-0.1t}(-0.1) = -8e^{-0.1t}$
e.	$T'(5) = -8e^{-0.1(5)} \approx -4.85$ After 5 minutes, the coffee is cooling at a rate of 4.85° / minute.
f.	$\lim_{t \rightarrow \infty} T(t) = 20$ The temperature will stay at 20 degrees after an extended time (room temp)

36.) a.  $f'(x) = 15\cos 5x$

b.  $f'(x) = 3^x (\ln 3)\cos(3^x)$

c.  $f'(x) = 4\sin^3 x \cos x$

d.  $f'(x) = \frac{1}{2}(\sin \sqrt{x})^{-1/2} (\cos \sqrt{x}) \left( \frac{1}{2}x^{-1/2} \right) = \frac{\cos \sqrt{x}}{4\sqrt{x}\sqrt{\sin \sqrt{x}}} = \frac{\cos \sqrt{x}}{4\sqrt{x}\sin \sqrt{x}}$

e.  $f'(x) = \frac{1}{\sin 3x}(3\cos 3x) = 3 \left( \frac{\cos 3x}{\sin 3x} \right) = 3 \cot 3x$

f.  $f'(x) = \frac{1}{2}(x+1)^{-1/2} \sin 5x^2 + 10x \cos 5x^2(x+1)^{1/2} = \frac{1}{2}(x+1)^{-1/2} [\sin 5x^2 + 20x \cos 5x^2(x+1)]$

g.  $f'(x) = (\cos[\sin(\sin 3x)]) (\cos(\sin 3x)) (\cos 3x)(3)$

37.) a.  $f'(x) = \frac{2}{3} \sin\left(\frac{2}{3}x\right)$

b.  $f'(x) = -\sin e^{11x} (e^{11x})(11) = -11e^{11x} \sin e^{11x}$

c.  $f'(x) = 6(\cos 10x)^{-1/2} (-\sin 10x)(10) = -\frac{60\sin 10x}{\sqrt{\cos 10x}}$

d.  $f'(x) = \cos x \cos 8x - 8 \sin 8x \sin x$

e.  $f'(x) = \frac{2x \cos(1-2x) - 2 \sin(1-2x)}{4x^2} = \frac{2(x \cos(1-2x) - \sin(1-2x))}{4x^2} = \frac{x \cos(1-2x) - \sin(1-2x)}{2x^2}$

$$2x - 2 \cos y \sin y \frac{dy}{dx} = 0$$

$$y \cos y = \sin x$$

$$\frac{dy}{dx} \cos y - y \sin y \frac{dy}{dx} = \cos x$$

b)  $\frac{dy}{dx} (\cos y - y \sin y) = \cos x$

$$\frac{dy}{dx} = \frac{\cos x}{\cos y - y \sin y}$$

38.) a)  $-2 \cos y \sin y \frac{dy}{dx} = -2x$

$$\frac{dy}{dx} = \frac{-2x}{-2 \cos y \sin y} = \frac{x}{\cos y \sin y}$$

E.

$$1. F(x) = \frac{1}{3}x^3 - \frac{1}{6}x^6 + 15$$

$$2. 3x^{\frac{1}{3}} + c$$

$$3. \frac{1}{6}x^3 + \frac{5}{4}x^2 - \frac{3}{2}x + c$$

$$4. \frac{15}{4}$$

$$5. \frac{1}{3}x^3 + x^2 + c$$

$$6. \frac{2}{3}(x^2 + 1)^{\frac{3}{2}} + c$$

$$7. \frac{4}{9}$$

$$8. \frac{2}{7}x^{\frac{7}{4}} - \frac{10}{3}x^{\frac{3}{4}} + c$$

$$9. \frac{1}{9}(x^2 - 5)^9 + c$$

$$10. \frac{-2}{3}(1 - x^3)^{\frac{1}{2}} + c$$

F.

$$1. \frac{68}{3}$$

$$2. \frac{32}{3}$$

$$3. \frac{1}{3}$$

$$4. \frac{4}{3}$$

G.

$$1. \text{(a) } 6 \text{ (b) } 0 \text{ (c) } \frac{9}{7} \text{ (d) } 0 \text{ (e) } \frac{4}{3} \text{ (f) } 0 \text{ (g) } 2 \text{ (h) } 3$$

H.

$$a) -4\cos\frac{1}{4}x + C$$

$$b) 2e^{\frac{x}{2}} + C$$

$$c) \frac{1}{2}\sin e^{2x} + C$$

$$d) \frac{1}{3}\sin^3 x + C$$

$$e) \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$$