Calculus 30 –Review (NO Graphing Calculator)
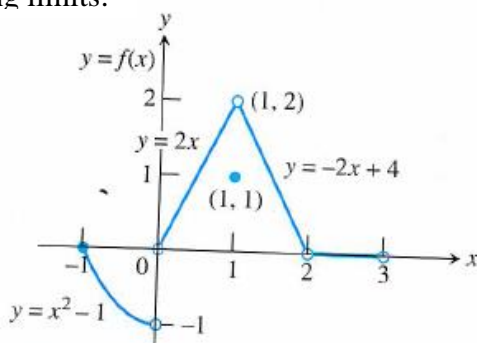
Find the following limits:

A. 1. a) $\lim\limits_{x\to 0^-}$

 b) $\lim\limits_{x\to 0}$

 c) $\lim\limits_{x\to 1}$

 d) $f(1)$

 e) $f(0)$

$y = f(x)$

$(1, 2)$

$y = 2x$

$y = -2x + 4$

$(1, 1)$

$y = x^2 - 1$

B. Find each limit

1. $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-4x+3}$

2. $\lim\limits_{x\to 5}\left(x^2+2x-3\right)$

3. $\lim\limits_{x\to 1}\dfrac{x^4-5x^2+1}{x+2}$

4. $\lim\limits_{x\to 4}\dfrac{x^2-16}{x-4}$

5. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}$

6. $\lim\limits_{h\to 0}\dfrac{(2+h)^2-4}{h}$

7. $\lim\limits_{x\to 0}\sqrt{x}$

8. $\lim\limits_{x\to 0^+}\sqrt{x}$

9. $f(x)=\begin{cases}-x-2 & \text{if } x\le -1\\ x & \text{if } -1\le x<1\\ x^2-2x & \text{if } x\ge 1\end{cases}$

 Find (a) $\lim\limits_{x\to -1} f(x)$

 (b) $\lim\limits_{x\to 1} f(x)$

10. $\lim\limits_{x\to 4}\dfrac{x+5}{x-4}$

11. $\lim\limits_{x\to 3}\dfrac{x+2}{(x-3)^2}$

12. $\lim\limits_{x\to 2^-}\dfrac{x^2-5x}{x^2-4}$

13. $\lim\limits_{x\to\infty}\dfrac{1}{x}$

14. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x}{3x^2+2}$

15. $\lim\limits_{x\to\infty}\dfrac{x^2+1}{2x-3x^3}$

16. $\lim\limits_{x\to\infty}\dfrac{4\lim\limits_{x\to\infty}\frac{4x^3+5}{8-3x^3}}{8-3x^3}$

17. $\lim\limits_{x\to -\infty}\dfrac{5x}{\sqrt{x^2+4}}$

18. $\lim\limits_{x\to 6}\dfrac{\sqrt{x-2}-2}{x-6}$

19. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{x+1}}{x}$

20. $\lim\limits_{x\to 3}\dfrac{|x^2-9|}{x-3}$

21. $\lim\limits_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

C. Find each derivative using the definition of the derivative: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

    1. $f(x) = 2x^2 - 3x + 4$ 2. $f(x) = \sqrt{2x - 1}$

D. Differentiate the following:

1. $y = 2x^2 - x$

2. $y = (x^8)^3$

3. $y = (2x + 1)^2$

4. $y = \dfrac{1}{4}x^4 + \dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 + x + 1$

5. $y = \dfrac{2}{x^3} + \dfrac{6}{\sqrt{x}}$

6. $y = (x^2 + 2x - 1)^2$

7. $y = (2x - 1)^{-1}(x + 2)^{-2}$

8. $y = (x + 3)\sqrt{x^2 + 2x}$

9. $y = \dfrac{3t^2 - 2t}{5 - 3t^2}$

10. $y = \dfrac{x^2 - x}{\sqrt{1 - x^2}}$

11. $y = \sqrt{27 - x^3}$

12. $y = \dfrac{1}{\sqrt[3]{3x^3 + 2x + 1}}$

13. $y = x^2 \sqrt[3]{2x^2 - 7}$

14. $y = \dfrac{2}{x - 6} + \dfrac{x + 6}{2x}$

15. $y = \dfrac{3x + 2}{(x^2 + 5)^2}$

16. $y^2 = 2xy - 3$

17. $2xy^2 - y^3 = x^2$

18. $\dfrac{2x}{x + y} = y$

19. Determine the slope and the equation of a tangent line at x = 3 for each of the following

a. #3 (above)   b. #6 (above)   c. #9 (above)

20 (Part 1). For each function BELOW find the following. Support all conclusions reached.

    (a) a sign analysis of $f'(x)$

    (b) the open intervals on which $f(x)$ is increasing and/or decreasing.

    (c) the critical numbers

    (d) the relative extrema

    (e) a sign analysis of $f''(x)$

    (f) the intervals on which $f(x)$ is concave up and concave down.

    (g) the coordinates of any inflection points.

    (h) the x and y intercepts.

    (i) the equations of any horizontal and vertical asymptotes.

    (j) a careful sketch of the function that supports all of the above features.

A. $f(x) = x^3 + 3x^2 - 24x$ B. $f(x) = \dfrac{x^2 + 2x - 4}{x - 2}$ C. $f(x) = 36x^{1/3} - 9x^{4/3}$

20 (Part 2). **Use the second derivative test** to determine the local extrema for the following functions:

    a) $f(x) = \dfrac{1}{3}x^3 - \dfrac{3}{2}x^2 - 10x + 2$ b) $f(x) = (x^2 - 3x + 2)^2$ c) $f(x) = \dfrac{1}{3}x^3 + 2$

21. What number exceeds its square by the greatest amount? What is that amount?

22. Sadie has 60 m of fencing that she plans to use to enclose a rectangular garden plot. One side of the garden will be against the barn, so she does not need to put a fence on that side. Find the dimensions of the plot that will maximize the area. What is the maximum area?

23. What are the dimensions of the largest rectangle that can be inscribed in a semi-circle of radius 60 m?

24. A rectangular box-shaped garbage can with a square base and an open top is to be constructed using exactly 2700 cm^2 of material. Find the dimensions of the box that will provide the greatest possible volume.

25. An oil well has been discovered offshore at W, 200 m from S, the nearest point on the shoreline. Town T is located 1000 m along the shore from point S. A pipeline must be installed underwater from W to V and then along the shoreline from V to T. If it costs $500/m to run the pipe underwater and $200/m to run the pipe along the shore, how far from S should V be located to minimize the total cost of the pipeline?
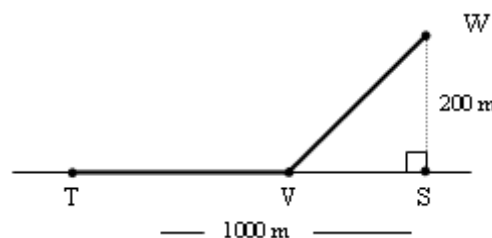
26. Determine the velocity and acceleration functions for the given position functions.

(a) $s(t)=2t^2+5t$

(b) $s(t)=\sqrt{3t+1}$

(c) $s(t)=\dfrac{4t}{t^2+1}$

27. A particle moves along the x-axis so that its position in metres after t seconds is given by the function $s(t)=\dfrac{1}{3}t^3+\dfrac{5}{2}t^2$. Find:

    (a) The velocity and acceleration at any time t.
    (b) The velocity when $t=4$.
    (c) The acceleration when $t=3$.
    (d) The position of the particle when the velocity is 66 m/s.
    (e) The velocity of the particle when the acceleration is 13 m/s^2.

28.) Find the derivative of each of the following functions.

    (a) $f(x)=\log_3(2x-5)$

    (b) $f(x)=12\log(x^3-3x)$

    (c) $f(x)=\log_{11}\left(\dfrac{x+1}{x-1}\right)$

    (d) $f(x)=\dfrac{\log_3 x}{x^3}$

29.) Find the derivative of each of the following functions.

    a. $f(x)=\ln(2x^3-5x+1)$

    b. $f(x)=\ln\sqrt{1-6x}$

    c. $f(x)=\ln\left(\dfrac{x^2-2}{x+2}\right)$

    d. $f(x)=\left[\ln(x^3-1)\right]^3$

    e. $f(x)=\ln\left[\ln(x^3+9x)\right]$

30.) A salesperson at a car dealership began work in an unfamiliar city. The number of people whose names the salesperson could remember after working x weeks is given by the function $f(x) = 12\ln(x+1) + 1$.

    (a) How many people did the salesperson know initially?
    (b) How many people did the salesperson know after 4 weeks? Round to the nearest integer.
    (c) Find $f'(x)$.
    (d) Find $f'(4)$ and $f'(9)$. Interpret your results.
    (e) As x increases, does $f'(x)$ increase or decrease? What might be the reason?

31.) Find the intervals in which the function is concave up and concave down.
    a. $f(x) = x^2 \ln x$

32.) What is the domain and range of the following function? Determine these results without the use of a graphing calculator. Confirm with a graphing calculator.
    a. $f(x) = e^{|x|}$

33.) Differentiate each of the following functions.
    a. $f(x) = 7^{x^3}$ b. $f(x) = 5^{x^2 + 4x + 3}$
    c. $f(x) = x \cdot 11^x$ (Use the product rule)   d. $f(x) = \sqrt{x-1}\left(4^{5x^4}\right)$

34.) Find the derivative of each of the following functions.
    a. $f(x) = e^{2x-5}$ b. $f(x) = \left(e^{3x+1}\right)^2$ c. $f(x) = x^2 e^x$ d. $f(x) = \dfrac{x+1}{e^{x^2-1}}$

35.) Suppose that the temperature of a cup of coffee in degrees Celsius, t minutes after it is poured, is given by the function $T(t) = 80e^{-0.1t} + 20$.

    (a) What was the initial temperature of the coffee?
    (b) When was the coffee $80°\,C$?
    (c) What was the temperature after 11 minutes?
    (d) Find $T'(t)$.
    (e) Find $T'(5)$ and interpret the result.
    (f) Find $\lim_{t \to \infty} T(t)$ and interpret the result.

36. Find the derivative of each of the following functions.
a. $f(x) = 3\sin 5x$ b. $f(x) = \sin(3^x)$ c. $f(x) = \sin^4 x$ d. $f(x) = \sqrt{\sin\sqrt{x}}$
e. $f(x) = \ln(\sin 3x)$ f. $f(x) = \sqrt{x+1}\sin 5x^2$ g. $f(x) = \sin\left[\sin(\sin 3x)\right]$

37.) Find the derivative of each of the following functions.

 a. $f(x) = -\cos\left(\dfrac{2}{3}x\right)$ b. $f(x) = \cos e^{11x}$ c. $f(x) = 12\sqrt{\cos 10x}$

 d. $f(x) = \sin x \cos 8x$ e. $f(x) = \dfrac{\sin(1-2x)}{2x}$

38.) Use implicit differentiation to find $\dfrac{dy}{dx}$.

 a. $x^2 + \cos^2 y = 2$ b. $y = \dfrac{\sin x}{\cos y}$

E. Integration – find the following definite and indefinite integrals

1. $F'(x) = x^2 - x^5$, $F(2) = 7$ Find $f(x)$

2. $\displaystyle\int \dfrac{1}{\sqrt[3]{x^2}}\,dx$

3. $\displaystyle\int \dfrac{x^3 + 5x^2 - 3x}{2x}\,dx$

4. $\displaystyle\int_{-1}^{2} x^3\,dx$

5. $\displaystyle\int \left(x^2 + 2x\right)dx$

6. $\displaystyle\int 2x\sqrt{x^2 + 1}\,dx$

7. $\displaystyle\int_{0}^{1} \sqrt[4]{x^5}\,dx$

8. $\displaystyle\int \dfrac{x-5}{2\sqrt[4]{x}}\,dx$

9. $\displaystyle\int \left(x^2 - 5\right)^8 2x\,dx$

10. $\displaystyle\int \dfrac{x^2}{\sqrt{1-x^3}}\,dx$

F Area

1. Find the area under $y = x^2 + 2$ from $x = 2$ to $x = 4$

2. Find the area below $y = 4 - x^2$ and above the $x-$axis

3. Find the area between $y = x^2$ and $y = 2x - x^2$

4. Find the area between $y = 2x$ and $y = x^2$

G. Outcome 7 – Transcendental Function

1. Evaluate each of the following limits.

(a) $\lim\limits_{x \to 0} \dfrac{6x}{\sin x}$

(b) $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x}$

(c) $\lim\limits_{x \to 0} \dfrac{\sin 9x}{\sin 7x}$

(d) $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x}$

(e) $\lim\limits_{x \to 0} \dfrac{\tan 4x}{\sin 3x}$

(f) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{4 - \cos^2 x}$

(g) $\lim\limits_{x \to 0} \dfrac{2x^2}{\tan^2 x}$

H. Outcome 8 (continued)

a) $\displaystyle\int \sin \dfrac{1}{4} x \, dx$

b) $\displaystyle\int e^{x/2} \, dx$

c) $\displaystyle\int e^{2x} \cos e^{2x} \, dx$

d) $\displaystyle\int \sin^2 x \cos x \, dx$

e) $\displaystyle\int \dfrac{\sqrt{\ln x}}{x} \, dx$