To learn to find and apply average and instantaneous velocity and acceleration.

DISTANCE/POSITION FUNCTION:

- Often denoted by a function of the form *s*(*t*) where *s* is distance and *t* is time.

INSTANTANEOUS VELOCITY:

- Is the instantaneous rate of change of position with respect to time (found by finding the slope of a **tangent** line to *s(t)*)
- the first derivative of a position function *v(t) = s′(t)*
- Some key ideas:
  - The particle is at rest when velocity = 0
  - The maximum height occurs when velocity = 0 (on the way up)
  - When the particle hits the ground, height = 0

INSTANTANEOUS ACCELERATION:

- Is the rate of change of velocity with respect to time (found by finding the slope of a **tangent** line to *v(t)*)
- Is the second derivative of a position function and the first derivative of a velocity function *a(t) = v′(t) = s″(t)*
- Measured in units such as m/s^2, etc.

AVERAGE VELOCITY:

- Is the average rate of change of position with respect to time (found by finding the slope of a **secant** line to *s(t)*)
- $v_{average} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

AVERAGE ACCELERATION:

- Is the average rate of change of velocity with respect to time (found by finding the slope of a **secant** line to *v(t)*)
- $a_{average} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$

Ex #1: Suppose that an object moves along a straight line so that its position (directed distance) in metres from its starting point (origin) is given by the function $s(t) = \frac{1}{3}t^3 + 2t, t \geq 0$ where *t* is time measured in seconds.

a) Find the average velocity of the object between the times t = 9 and t = 15? https://www.desmos.com/calculator/ywplw7hpgi
(Note: You cannot determine an average velocity to simply averaging two velocities)

b) Find the instantaneous velocity when t = 9?

c) Find the average acceleration between the times t = 9 and t = 15

d) b) Find the instantaneous acceleration when t = 9?

Ex #2: The position of an object on a line is givenby the function $s(t) = 2t^3 - 15t^2 + 24t$ where *t* is measured in seconds and *s(t)* is measured in metres. Note that $t \geq 0$.

a) Find the velocity at any time t

b) Find v(3) and v(6)

c) Perform a sign analysis of the velocity function v(t)

d) Find the open interval(s) for which the object is moving to the right (in the positive direction) and to the left (in the negative direction)

e) Find the total distance the object has moved during the first 6 seconds.

f) Find the acceleration at any time, t.

g) During what open time interval(s) is the velocity increasing and decreasing?

Ex #2: In a diving competition, Ty's height above water, *t* seconds after leaving the diving board, can be approximated by the function $h(t) = -5t^2 + 5t + 10$.

a) How high above the water was Ty initially?

b) Find Ty's velocity and acceleration at any time, *t*.

c) When did Ty reach his maximum height?

d) What was his maximum height above the water?

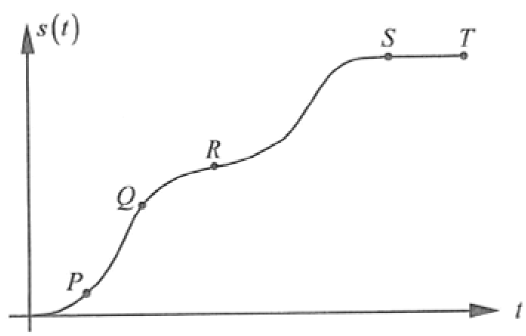
e) When did Ty hit the water?

f) What was his velocity when he hit the water?

Ex #3:

The graph at right shows a car's distance from town as it travelled along a straight stretch of highway. Answer each of the following questions explaining your conclusions.

(a) What was the initial velocity of the car?
(b) Was the car going faster at Q or at R?
(c) Was the car going faster at P or at Q?
(d) Was the car speeding up or slowing down at P?
(e) Was the car speeding up or slowing down at Q?
(f) Was the car speeding up or slowing down at R?
(g) What happened to the car between S and T?



OUTCOME 6 DAY 1 ASSIGNMENT (Section 3.1 & 3.2 in Text)
DUO TANG: P15

To be able to identify, set up, and solve optimization problems..

In this section we work on word problems where we try to find the maximum or minimum value of a variable. The solution is referred to as the **optimum** solution.

TIPS FOR SOLVING OPTOMIZATION PROBLEMS:

1. **UNDERSTAND THE PROBLEM:**
   Optimization problems often appear in the form of a word problem. It is most important that you read the problem over carefully until you have a clear idea as to what it is you are being asked to maximize or minimize. You may want to experiment with some hypothetical "answers" to bring clarification to the problem and assist you in creating the appropriate function.

2. **DRAW A DIAGRAM:**
   Many optimization problems require a diagram. Label your diagram carefully with all the given dimensions. Use one or more variables to label dimensions that you do not know. Using meaningful variables, such as *h* for height and *r* for radius is helpful.

3. **DETERMINE THE FUNCTION:**
   Set up a function that gives an expression for the quantity you are trying to optimize in terms of the constants and variables you established in step 2. Initially this function may be in terms of more than one variable. Read your problem carefully and re-examine your diagram to see how the variables are interrelated. The Pythagorean Theorem can also be helpful! *You must write your optimization function in terms of only one variable before you can proceed.*

4. **FIND THE DOMAIN OF THE FUNCTION:**
   Read the question carefully and determine the domain. It will be relevant in step 5.

5. **DETERMINE THE GLOBAL MAXIMUM OR MINIMUM:**
   Assuming the interval is closed, follow these steps:
   a) Find the derivative of the function
   b) Determine the critical numbers
   c) Evaluate the function at each critical number
   d) Evaluate the function at each endpoint of the interval
   e) Determine your global max or min from the points in steps c) and d) above.
      - *If your domain is an open interval, you may need to use limits to determine what happens to the optimization function as the variable approaches the boundaries of the open interval.*

6. BE **SURE YOU HAVE ANSWERED THE ACTUAL QUESTION!**

Ex #1: What positive number, when added to its reciprocal, gives the least possible sum? What is that sum?

Ex #2: Two nonnegative numbers have a sum of 10. Find these numbers if the sum of their cubes is to be a minimum.

Ex #3: A farmer has 1000 m of fencing and wants to enclose a rectangular pasture bordering a straight, swift river. The farmer thinks that the cattle will not wander into the river and thus a fence will not be needed along that side. Find the dimensions that will enclose the maximum area, and find that area.

Ex #4: A rectangular sheet of paper of dimensions 18cm x 25cm is to be made into an open box by cutting out equal squares from the corners and bending up the flaps. Find the length of the cut out squares so that the volume of the box will be a maximum. What will the maximum volume be?

Ex #5: A rectangular piece of metal 60 cm wide and several hundred metres long is to be folded along the centre of its length in order to make a long trough for irrigation. What should be the width of the trough at the top in order to maximize its carrying capacity? You may assume that the carrying capacity will be maximized if the area of the triangular cross section of the trough is maximized.

Ex #6: A can of pop is to hold 400ml (400 cm^3). Find the radius and height of the can if the amount of metal used is to be minimized. Ignore the thickness of the metal.

OUTCOME 6 DAY 2 ASSIGNMENT (Section 3.1 & 3.2 in Text)
DUO TANG: P16 & 17

To be able to identify and solve related rates problems by using implicit differentiation.

RECALL:

- The derivative can be considered the rate at which something occurs.
  - $\frac{dy}{dx}$ is the rate of change of y with respect to x.
- If the equation describes distance with respect to time, the derivative would then represent the **velocity**. This would be measured with units such as cm/s, m/s, km/h, etc.
  - where t represents time, $\frac{dy}{dt}$ represents the rate of change of the length of y relative to time.

Ex #1: Differentiate each expression relative to time, t.

a) z^2 b) 10 c) 10x d) $x^2 + y^2$ e) $3x^2y$

Today we are going to use this prior knowledge along with the Pythagorean Theorem to help us find how fast two objects are distancing or approaching each other if they are moving at right angles to or from each other.

TIPS FOR SOLVING RELATED RATES PROBLEMS:

1. **UNDERSTAND THE PROBLEM**
2. **DRAW A DIAGRAM:** Draw the triangle representing the problem.
   - Layer 1: Label all side with x, y and z (hypotenuse).
   - Layer 2: Write the length (distance) of all known sides.
   - Layer 3: Write all known velocities occurring at each side. Label them $\frac{dx}{dt}, \frac{dy}{dt}$ and $\frac{dz}{dt}$.
     - Note : *If the length of the side is decreasing, the velocity will be negative.*
       It is possible that one of the velocities may be 0 if there is no movement.
3. **DETERMINE ANY MISSING DISTANCES:**
   - Using the Pythagorean Theorem, determine the missing values of x, y or z.
   - Write these in your diagram. You may need to create a new one.
4. **IMPLICITLY DIFFERENTIATE THE PREVIOUS EQUATION (PYTHAGOREAN THEOREM) relative to time.**
5. **SIMPLIFY AND SUBSTITUTE ANY KNOWN VALUES INTO THE DIFFERENTATIATED EQUATION:**
   - When substituting, write the units of measurement.
   - You should now have a single unknown variable, which is the one you are looking for.
6. **ANSWER THE QUESTION:** Include units of measurement.

Ex #2: Colin boarded a "see-through" elevator, which climbed the outside of a building at a rate of 3m/s. His friend, Avram, stood on the sidewalk 24 m from the base of the elevator and watched Colin ascend. At what rate was the distance between them (their toes) changing when Colin was 45 m above the sidewalk?

Ex #3: A car is approaching an intersection travelling east at a rate of 60 km/h. A truck is moving away from the same intersection travelling south at a rate of 50 km/h. How is the distance between the car and the truck changing when the car is 21 m from the intersection and the truck is 20 m from the intersection?

OUTCOME 6 DAY 3 ASSIGNMENT (Section 3.5 in Text)
DUO TANG: P 18

To be able to identify and solve related rates problems (including area and volume) by using implicit differentiation.

Today we are going to use the same concept as we used in the last lesson, but will be applying it to determine how the area or volume of a geometric shape changes based on the change of another variable. Due to the increase in the number of variables, we will sometimes have to develop two or more equations per question and substitute to reduce the number variables before we take the derivative.

TIPS FOR SOLVING RELATED RATES PROBLEMS (with area and volume):

1. **UNDERSTAND THE PROBLEM**
2. **CREATE THE WHENEVER DIAGRAM AND EQUATION**
   - Draw and label your diagram with all of the timeless facts:
     - Variables, constants and rates of change.
   - Write the equation that links these variables and constants. The formulas are provided below.
   - Identify what variable or rate you are trying to find.
3. **A) IMPLICITLY DIFFERENTIATE THE PREVIOUS EQUATION RELATIVE TO TIME, then…**
   - Add your "when" information to your diagram.
   - Substitute all known rates of change and all "when" information into your new equation.
   - You should be left with only the one unknown, which should represent the answer to your question.
3. **B) IF THE IMPLICIT DIFFERENTIATION WOULD LEAVE YOU WITH TOO MANY UNKNOWNS…**
   - Before differentiating, use the geometry of the situation to eliminate the unknown variables by substitution. (ex: ratio)
   - Now, implicitly differentiate the equation with respect to t.
   - Substitute all known rates of change and all "when" information into your new equation.
   - You should be left with only the one unknown, which should represent the answer to your question.
4. **ISOLATE YOUR REMAINING UNKNOWN**
5. **ANSWER THE QUESTION WITH A FINAL STATEMENT**
   - Include units of measurement.
   - In your statement, your velocities should be expressed at positives. Use the terms increase and decrease.

FORMULAS

Area: Triangle: $A=\frac{bh}{2}$ Trapezoid: $A=h\frac{(a+b)}{2}$ Circle: $A=\pi r^2$

Perimeter: Square: $P=4l$ Rectangle: $P=2l+2w$ Circle: $C=2\pi r$

Surface Area:

Closed rectangular box: $S=2lw+2lh+2wh$ Right circular cylinder (open at top and bottom): $S=2\pi rh$

Sphere: $S=4\pi r^2$ Right circular cone with open base: $S=\pi rl$ where $l=$ slant height $=\sqrt{r^2+h^2}$

Volume: Prism or cylinder: $V=(\mathit{area}\text{ of base})h$ Sphere: $V=\frac{4}{3}\pi r^3$ Cone: $V=\frac{1}{3}\pi r^2 h$

Ex #1: A sphere is expanding and the measured rate of the increase of its radius is 6 cm/min. At what rate is its volume increasing when the radius is 10 cm?

Ex #2: The base of a triangle is growing at a rate of 6 cm/s while its height is shrinking at a rate of 5 cm/s. What is happening to the area of the triangle when the base is 20 cm and the height is 16 cm?

Ex #3: A conical vase is being filled at a rate of 10 cm^3/s. The vase is 30 cm high and has a radius of 4 cm at the top. Find the rate at which the water is rising when the depth of the water is 20 cm.

OUTCOME 6 DAY 4 ASSIGNMENT (Section in Text 3.5)
DUO TANG: P 19 & 20

OUTCOME 6 REVIEW
DUO TANG: P 21 to 24