To review properties of logarithms and take the derivative of logarithmic functions.

Note: Transcendental functions are non algebraic functions (logarithmic functions, exponential functions and trigonometric functions are all examples of transcendental functions)

REVIEW:

- A logarithmic function is the inverse of an exponential function.
  - If $y=2^x$ is a given exponential function, it's inverse is the logarithmic function $x=2^y$. Although this last function is a logarithmic function, it is still written in exponential form and is very challenging to work with as we cannot use traditional algebraic techniques to solve for y. Therefore we rewrite the equation into an equivalent (but different looking) logarithmic form

    $x=2^y \quad \Leftrightarrow \quad y=\log_2 x$

RULES OF LOGARITHMS:

- $\log_c 1 = 0$ since in exponential form $c^0 = 1$.
- $\log_c c = 1$ since in exponential form $c^1 = c$
- $\log_c c^x = x$ since in exponential form $c^x = c^x$
- $c^{\log_c x} = x, x > 0$, since in logarithmic form $\log_c x = \log_c x$

PROPERTIES OF LOGARITHMS:

Property Name	In Symbols	In Words
1. **Product Property**	$\log_b(XY) = \log_b X + \log_b Y$	The logarithm of a product is the sum of the logarithms of the numbers in the product.
2. **Quotient Property**	$\log_b\left(\frac{X}{Y}\right) = \log_b X - \log_b Y$	The logarithm of a quotient is the difference between the logarithm of the numerator and the logarithm of the denominator.
3. **Power Property**	$\log_b\left(X^k\right) = k\log_b X$	The logarithm of a power is the exponent multiplied by the logarithm of the base.
4. **Change Of Base**	$\log_c X = \frac{\log X}{\log c}$	To find the logarithm of a number X in base c, divide the common logarithm of X by the common logarithm of c. (Applying this property would likely require a calculator.)

REVIEW Example #1: Evaluate the following logarithms:

a) $\log_2 8$

b) $\log_3 81$

c) $\log_9 9$

d) $\log_3 \frac{1}{9}$

e) $\log_5(-25)$

f) $\log_{10} 1000$

Example #3: Evaluate $\log_4 9$ to five decimal places

CHANGE OF BASE FORMULA

$\log_a x = \dfrac{\log_b x}{\log_b a}$ To change to base 10 use $\log_a x = \dfrac{\log x}{\log a}$

Example #2: Rewrite as a single log and evaluate.

a) $4\log_4 2 - 2\log_4 8$

b) $\log_6 8 + \log_6 9 - \log_6 2$

c) $3\log_{\frac{2}{3}} 2 - 3\log_{\frac{2}{3}} 3$

d) $\log_7 7\sqrt{7}$

The number **e** is a famous irrational number, and is one of the most important numbers in mathematics.

The first few digits are:

2.7182818284590452353602874713527 (and more ...)

*It is often called **Euler's number** after Leonhard Euler (pronounced "Oiler").*

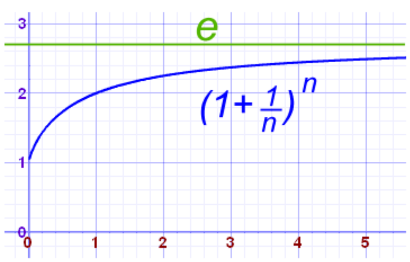
e is the base of the Natural Logarithms (invented by John Napier).

Calculating

There are many ways of calculating the value of **e**, but none of them ever give a totally exact answer, because **e** is irrational (not the ratio of two integers).

But it **is** known to over 1 trillion digits of accuracy!

For example, the value of $(1 + 1/n)^n$ approaches **e** as n gets bigger and bigger:



n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

The value of e can be calculated by finding $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n$. We can see in the diagram to the right that

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71827....$$

Another Calculation

The value of e is also equal to $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \ldots$ (etc)

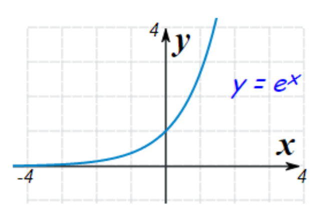
(Note: "!" means factorial)

The first few terms add up to: $1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = 2.718055556$

In fact Euler himself used this method to calculate **e** to 18 decimal places.

Growth

e is used in the "**Natural**" Exponential Function:



Graph of **f(x) = e^x**

It has this wonderful property: "it's slope is it's value"

DIFFERENT BASES OF LOGARITHMS

- Up until this point, we have discussed logs with any base *b* of the form $y = \log_b u$
- From PC 30 you should remember that when we are speaking of a base 10 logarithm where *b*=10, it is called the common logarithm and we don't actually write the 10. Therefore a question such as $\log_{10}x$ will actually appear as logx
- When the number known as *e* is used as the base, where *e* = 2.718….., this is known as the natural logarithm $\log_e x$. Due to its importance, it actually has its own new form and we say that $\log_e x = \ln x$

Example #4: Evaluate the following to five decimal places.

a) ln 7

b) ln10000

c) ln1

Example #5: Use the definition of a logarithm to solve for x

a) $\log_3 x = -1$

b) lnx = 2

c) 2ln(x + 1) = -1

THE DERIVATIVE OF A LOG FUNCTION WITH BASE b: $y = \log_b u$

If $y = \log_b u$, then the derivative is $\frac{dy}{dx} = \left(\frac{1}{u}\right)(\log_b e)\left(\frac{du}{dx}\right)$

- In words this means that the derivative of the log of a function "*u*" is equal to the reciprocal of u, multiplied by the logarithm of the number *e* to the base b, multiplied by the derivative of the function "*u*"

Example #6: Find the derivative of the following logarithmic functions

a) $y = \log_5 x$

b) $y = \log_6 x^9$

c) $y = 12\log_7\left(x^3 + 4x\right)$

d) $y = \log_8\left(\frac{2x^3}{x^2 + 3}\right)$

END OF DAY 1 NOTES

Finding the derivative of lnx:

- What mathematical rule do we use to evaluate the following?

a) $\log_4 4$ b) $\log_7 7$ c) $\log 10$

- How can we use that same rule to evaluate $\log_e e$?

THE DERIVATIVE OF A LOG FUNCTION WITH BASE e: y = lnx (or y = log$_e$u)

- If $y = \log_e u = \ln x$, then the derivative is $\frac{dy}{dx} = \left(\frac{1}{u}\right)(\log_e e)\left(\frac{du}{dx}\right)$. Since we have established that $\log_e e = 1$, we are actually left with the following derivative formula :
- Given $y = \ln x$, $\frac{dy}{dx} = \left(\frac{1}{u}\right)\left(\frac{du}{dx}\right)$

Example #7: Find the derivative of each of the following functions

a) $f(x) = \ln(x^9)$ b) $y = 8\ln(4x - 5)$ c) $y = \ln 7$

d) $f(x) = \ln\left(\frac{5x^2}{2x-1}\right)$ e) $f(x) = \ln\sqrt[3]{6x-5}$

Example #8: Find the slope of the tangent line to the graph of the function y = 3lnx at the point x = 4

Example #9: Find the derivative of the function $f(x) = 2x^3 \ln\left(x^2 + 4\right)$

OUTCOME 7 DAY 1 & 2 ASSIGNMENT p 27 in Duo Tang

Day 1 Assignment:

- ***Review Questions from PC 30 1a-h, 2acegjl, 3acegik, 5a (Do as many as you need – you will need to know and use all this info)***
- **New Questions: 1ijkln, 2bdfhi, 3bdh, 4abcdefgj, 5bcd, 6acd, 7**

Day 2 Assignment : 8acfijlmoq, 9abcefghijkl, 12, 15

NOTE: This is Section 8.3 & 8.4 in Text

CALCULUS 30: OUTCOME 7 DAY 3 – DERIVATIVES OF EXPONENTIAL FUNCTIONS (TEXT: 8.1 & 8.2)

To review properties of logarithms and take the derivative of logarithmic functions.

THE DERIVATIVE OF AN EXPONENTIAL FUNCTION y = b^u

- If $y = b^u$ where *u* is a function of *x* then the derivative is $\frac{dy}{dx} = \left(b^u\right)\left(\ln b\right)\left(\frac{du}{dx}\right)$

THE DERIVATIVE OF AN EXPONENTIAL FUNCTION y = e^u

- If $y = e^u$ where *u* is a function of *x* then the derivative is $\frac{dy}{dx} = \left(e^u\right)\left(\frac{du}{dx}\right)$

Example #1: Find the derivative of the following functions:

a) $y = 4^{5x}$

b) $y = 2^{x^2+5x}$

c) $y = x^3 \cdot 6^{2x-5}$

Example #2: Find the derivative of the following:

a) $y = e^{x^4}$ b) $y = 7e^{3x^2+5x}$ c) $f(x) = x^3 e^{x^2}$

d) $y = e^{\frac{5}{x}}$ e) $y = \dfrac{7e^{2x^2}}{x^2}$ f) $y = \left(e^x\right)^4$ g) $y = e^5$

PROPERTIES OF LOGARITHMS AND EXPONENTS

$\log_b b^u = u$ $\ln e^u = u \ \left(\text{or } \log_e e^u = u\right)$ $b^{\log_b u} = u$ $e^{\ln u} = u$

Example #3: Find the following derivatives

a) $y = \log_4 4^{7x+1}$ b) $f(x) = 2x\left(\log_5 5^{x^2-4x+3}\right)$

c) $y = \ln e^{4x}$ d) $y = x \ln e^{\sqrt{x}}$

e) $y = 2^{\log_2 (x+3)^4}$ f) $y = e^{\ln 4x}$

g) $y = e^{6\ln x}$

h) $f(x) = \left(\ln e^{x^4}\right)\left(e^{\ln x^6}\right)$

Example #4: Find the open interval(s) on which the function $f(x) = x^3 e^x$ is increasing and decreasing, and find the coordinates of any relative extrema.

OUTCOME 7 DAY 3 ASSIGNMENT P29 & 30 in Duo Tang
2odd, 4a, 5a, 6, 9, 14, 15, 17, 19, 21, 23, 25, 27, 28, 30, 31, 32, 35, 36, 38
Note: This is Section 8.3 & 8.4 in Text

CALCULUS 30: OUTCOME 7 DAY 4 – LIMITS INVOLVING TRIGONOMETRIC FUNCTIONS (TEXT: 7.1)

To take the limits of some specific trigonometric functions.

Using what is called **THE SQUEEZE THEOREM,** we can develop some rules that allow us s to take the limit of particular trig functions where substitution gives us indeterminate values, and where algebraic techniques don't apply.(Note: we do not have time in the course to develop the squeeze theorem and apply it to develop the following rules. If you know that you will be taking Calculus in University, I highly recommend you watch the following videos so that you have some idea of how this process works!)

- https://bit.ly/2KOR6Bs https://bit.ly/1t8dbj9

IMPORTANT LIMITS OF TRIGONOMETRIC FUNCTIONS:

a) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ **b)** $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$ **c)** $\lim_{x \to 0} \frac{\sin kx}{kx} = 1$ **d)** $\lim_{x \to 0} \frac{kx}{\sin kx} = 1$

e) $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$ **f)** $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta} = 0$

Ex #1: Use the above rules to assist in finding the following limits:

a) $\lim_{x \to 0} \dfrac{\sin 3x}{x}$

b) $\lim_{x \to 0} \dfrac{\sin 5x}{\sin 7x}$

c) $\lim_{x \to 0} \dfrac{\tan x}{2x}$

d) $\lim_{x \to 0} \dfrac{x^2}{\tan x}$

e) $\lim_{x \to 0} \dfrac{1 - \cos x}{\tan x}$

f) $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x}$

g) $\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x \cos x}$

OUTCOME 7 DAY 4 ASSIGNMENT (Section 7.1 in Text)

FA P 306 #7 – 11, 15, 17, 18, 21 – 24, 26 – 28, 31, 32, 34, 35

CALCULUS 30: OUTCOME 7 DAY 5 – DERIVATIVES OF SINE & COSINE (TEXTBOOK : 7.2)

To find and apply the derivatives of sine and cosine functions.

DERIVATIVES OF SINE AND COSINE (Valid only when the angle is measured in radians:

- If $y = \sin u$ then $\frac{dy}{dx} = \cos u \frac{du}{dx}$
- If If $y = \cos u$ then $\frac{dy}{dx} = -\sin u \frac{du}{dx}$

Ex #1: Find the derivative of each of the following:

a) $y = \sin(5x+2)$

b) $y = -7\sin(x^3)$

c) $y = -4\cos^2 6x$

d) $y = 3\sin\left(5e^{2x}\right)$

e) $y = \ln\left[\cos\left(\frac{\pi}{3}\right)\right]$

Ex #2: Find the slope of the tangent line to y = sinx at x= $\frac{5\pi}{4}$

Ex #3: Find the derivative of the following:

a) $y = 8\sin^4 6x$

b) $y = \sqrt{\sin 5x}$

c) $y = \left(4x^3 + 5\right)\sin 8x$

d) $f(x) = 2\sin 3x \cos 4x$

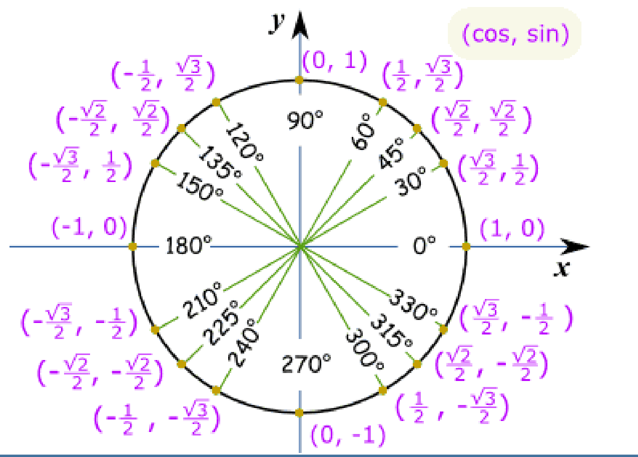
e) $y = \cos^3\left(\sin 3x^2\right)$

f) $y = \frac{2^{\cos 7x^2}}{e^{3x}}$

Ex #4: If $\sin x + \sin y = 1$, use implicit differentiation to find the derivative of y with respect to x.

Ex #5: Find the coordinates of any relative extrema in the interval $[0, 2\pi]$ for the function $f(x) = x + 2\sin x$

REMINDER (You need to know this info by heart!)



OUTCOME 7 DAY 5 ASSIGNMENT (Section 7.2 in Text)

FA P 313 #1a-r, 2ac (use implicit differentiation) , 3a, 4a, 5a PLUS THE FOLLOWING:

1. $f(x) = e^{\cos 3x}$
2. $f(x) = 4\ln(\cos x)$
3) $f(x) = \ln(\sin^2 x)$

OUTCOME 7 REVIEW ASSIGNMENT - Duo Tang P 30 & 31

VIDEO LINKS:

OUTCOME 7 DAY 1 & 2 Derivatives of Logarithms

Review of Logarithms:

[***https://bit.ly/2x3DhwJ***](https://bit.ly/2x3DhwJ)
[***https://bit.ly/2GH8bKQ***](https://bit.ly/2GH8bKQ)
[***https://bit.ly/2khsB4j***](https://bit.ly/2khsB4j)

Derivatives of Logarithms

Mr. S

[**https://bit.ly/2s67CoE**](https://bit.ly/2s67CoE)
[**https://bit.ly/2IHrmpO**](https://bit.ly/2IHrmpO)

CC

[**https://bit.ly/2LlWWLH**](https://bit.ly/2LlWWLH)

OUTCOME 7 DAY 3 – Derivatives of Exponents

Mr. S [**https://bit.ly/2IKqzbW**](https://bit.ly/2IKqzbW)
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DAY 4 – Limits of Trig Functions

Mr. S [**https://bit.ly/2IGfwjK**](https://bit.ly/2IGfwjK)
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DAY 5 – Derivatives of Sin and Cos

Mr. S [**https://bit.ly/2kiuD4I**](https://bit.ly/2kiuD4I)
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