CALCULUS 30: OUTCOME 8A DAY 1 – ANTIDERIVATIVES & INTEGRALS (TEXTBOOK: 9.1)

To find the equation of a function (by sight) given its derivative, and to learn the meaning of the terms ANTIDERIVATIVE and INTEGRAL and INTEGRATION.

Imagine seeing the following question

$f(x) =$ [redacted]

$f'(x) = 2x$

Can you work backwards to find the equation of f(x)?

- Kristin said the answer is $f(x)=x^2 + 5$, Ella said the answer is $f(x) = x^2 – 6$ and Anna said the answer is $f(x) = x^2$. Who is correct?

ANTIDERIVATIVES:

- If a function F'(x) is known to be the derivative of another function f(x), the **ANTIDERIVATIVE** of the function F'(x) will perform the backwards operation of the derivative and will give you the function in its original state before the derivative was found.
- **If F is any function such that F'(x) = f(x), then the general antiderivative of f(x) will be F(x) + C, where C is an arbitrary constant.**

  Ex: Let F'(x) = f(x) = 8x, then the antiderivative will be $F(x) = 4x^2 + C$

Ex #1: Find the antiderivative of f.

(Note: The function given is f(x) but it is also F'(x), the derivative of another function called F(x))

a) $f(x) = x^6$

b) $f(x) = \sqrt{x}$

c) $f(x) = x^3 - 6x^{-2}$

c) $f(x) = \frac{3}{x^5}$

d) $f(x) = 3x^2 + 2$

e) $f(x) = 6x^{-\frac{1}{2}} + 10\cos 2x$

Ex #2: Find a function f(x) for which $f'(x) = e^{3x}$

Ex #3: Find a function f(x) for which f'(x) = $2x^2 + x - 6$ and f(6) = -4.

INTEGRATION

- The process of finding the **ANTIDERIVATIVE** is called **INTEGRATION**
- The symbol for integration is $\int$, and it is the opposite of finding the derivative.

  EX: Finding a derivative would look like $\frac{d}{dx}\left(x^4\right) = 4x^3$ While finding an integral would look like $\int 4x^3 dx = x^4 + C$
- The notation $\int 4x^3 dx$ means "What function, if differentiated with respect to *x*, has $4x^3$ as its derivative?"
  - The answer, x^4 + C is called an **INDEFINITE INTEGRAL (the antiderivative)**
  - The C is called the **CONSTANT OF INTEGRATION**
- The x^4 is called the **INTEGRAND**

Differentiation	Integration
	The Sum/Difference Rule
1. $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$	1. $\int\left[f(x) \pm g(x)\right]dx = \int f(x)dx \pm \int g(x)dx$ The integral of a sum (difference) of functions is the sum (difference) of the integrals of the functions.
	The Constant Multiple Rule
2. $\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x)$	2. $\int cf(x)dx = c\int f(x)dx$ The integral of a constant times a function is the same as the constant times the integral of the function. You can "pull out" constant multipliers of a function from the integral sign.
	The Power Rule
3. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + C\right) = \frac{1}{n+1}(n+1)x^n = x^n$	3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, **provided that** $n \neq -1$. To integrate a power of x (other than -1) increase the exponent by 1, and divide by the new exponent.

4. $\frac{d}{dx}(kx + C) = k$, where k is a real number.	4. $\int k dx = kx + C$
5. $\frac{d}{dx}\left(\frac{1}{\ln b}b^x + C\right) = \frac{1}{\ln b}b^x \ln b = b^x$	5. $\int b^x dx = \frac{1}{\ln b}b^x + C$
6. $\frac{d}{dx}\left(e^x + C\right) = e^x$	6. $\int e^x dx = e^x + C$
7. $\frac{d}{dx}(\sin x + C) = \cos x$	7. $\int \cos x dx = \sin x + C$
8. $\frac{d}{dx}(-\cos x + C) = -[-\sin x] = \sin x$	8. $\int \sin x dx = -\cos x + C$
9. $\frac{d}{dx}\left[\ln\lvert x\rvert + C\right] = \frac{1}{x}$	9. $\int \frac{1}{x}dx = \ln\lvert x\rvert + C$

Ex #3: Find the following integrals

a) $\int\left(x^4+x^{-3}-1\right)dx$

b) $\int\left(x^2+2\right)^2dx$

c) $\int\frac{3x^3-4x^2+5x-2}{x^2}\,dx$

d) $\int\sqrt{x}\left(x^2-3\right)dx$

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Note: This is section 9.1 in your Textbook

To find the integral of functions using *"u"* substitution.

When we are taking the integral of functions where the chain rule and/or the product rule was originally used in the derivative process, we can use *u* substation to help make taking the integral easier. The following formula's will be used:

Integration Formulas
(*u* represents a function)

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$	2. $\int e^u du = e^u + C$
3. $\int b^u du = \frac{1}{\ln b} b^u + C$	4. $\int \frac{1}{u} du = \ln\lvert u\rvert + C$
5. $\int \sin u du = -\cos u + C$	6. $\int \cos u du = \sin u + C$

Ex #1: Evaluate $\int (4x+3)^9 dx$.

Ex #2: Evaluate $\int x^2\left(x^3-5\right)^5\,dx$

Ex #3: Evaluate $\int (x+1)\sqrt[3]{x^2+2x+3}\,dx$

Ex #4: Evaluate $\int \frac{\sin\sqrt{5x+3}}{\sqrt{5x+3}}\,dx$

Ex #5: Evaluate $\int \frac{x^4-1}{x^5-5x}\,dx$

STEPS TO USING *U* SUBSTITUTION:

1. Choose your *u*. This will be the most complex function within the integral.
   - If u is a polynomial function is raised to an outside power, do NOT include the power in *u.*
   - If *u* is the inside function of a trig function, or the power of an exponential function, include any and all powers of that *u*
2. Find the derivative of *u*, $\frac{du}{dx}$
3. Cross multiply your answer in step 2 to solve for du
4. Compare your original integral to the integral that would contain *u* and du. Often there will be a constant term missing from the du part of the integral. If you are missing a constant, you can add that constant to your original integral as long as you multiply by 1 over that constant on the outside of the original integral.
5. Once you have all the components of *u* and du in your original integral, rewrite your integral change everything to *u* and du, making sure to add any power outside of *u*.
6. Use your basic integration rules to find the integral of step 5 (don't forget the C!)
7. Substitute *u* back to the expression that *u* originally was originally
8. You can quickly check your work by taking the derivative of your answer and checking to see that it matches your original integral

OUTCOME 8A DAY 2 ASSIGNMENT Page 36 in Duo Tang
Note: This is Section 11.3 in your Text

CALCULUS 30: OUTCOME 8A DAY 3 – DEFINITE INTEGRALS (TEXTBOOK: 11.2)

To calculate definite integrals.

In the last section we learned to evaluate INDEFINITE INTEGRALS. The answer to a definite integral is an expression ending in C. They are called indefinite because expressions, particularly with a C value, have an undetermined numerical value. Today we will learn about DEFINITE INTEGRALS. Definite integrals calculate the definite value of an integral on a defined interval– it will be a numerical answer.

DEFINITE INTEGRATION:

- Suppose that $f(x)$ is a continuous function on the closed interval [a, b], and that $\int f(x)dx = F(x)+C$.
  The numerical calculation of $F(b)-F(a)$ is called the **DEFINITE INTEGRAL OF** $f(x)$ **from a to b.**
- To show that we are taking the definite integral from a to b, we write this integral in the following format:
  - $\int_a^b f(x)dx = F(b)-F(a)$
  - We read this as "the integral from a to b of f(x) with respect to x is equal to F(b) – F(a)
  - The numbers *a* and *b* are termed the LOWER and UPPER bounds of integration, respectively.
  - The notation $\left[F(x)\right]_a^b$ is often used to represent $F(b)-F(a)$

Ex #1: Evaluate $\int_1^5 (4x+7)dx$

WHAT DOES THIS NUMBER REPRESENT? Let's think about the following…..

- **If you were given f(x) = 4x + 7, could you find f'(x)?**
  - What does f'(x) represent?
  - f'(x) is similar in nature to finding an indefinite integral. The answer is a general equation.
- **Could you find f'(5)?**
  - What does f'(5) represent?
  - Finding f'(5) is similar in nature to finding a definite integral. The answer is numerical.
- When you find $\int f(x)dx$, you are finding a general formula that, instead of being for slope, is for AREA. This formula will allow you to calculate the area between a curve and the x axis.
- When you find $\int_a^b f(x)dx$, you are finding the specific area of the region between the x = a and x = b.

https://www.desmos.com/calculator/c4v6myylfj

Ex #2: Evaluate $\int_{-5}^{-3} (3x^2-2x)dx$ https://www.desmos.com/calculator/jg3ohkddgp

Ex #3: Evaluate $\int_{\pi/6}^{\pi/2}(\sin^4 x\cos x)dx$ https://www.desmos.com/calculator/uktlcmivmw

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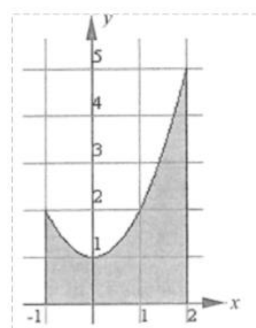
Note: This is Section 11.2 in your Textbook

CALCULUS 30: OUTCOME 8A DAY 4 – AREA UNDER A CURVE: THE FUNDAMENTAL THEOREM OF CALCULUS (TEXTBOOK: 11.2)

- **We have formula's that allow us to accurately calculate the area of certain shapes**

  Circles $A=\pi r^2$ **, rectangles** $A=(l)(w)$ **, triangles** $A=\frac{1}{2}bh$ **etc**
- **In the last lesson you learned that the answer to a definite integral represents the area between a function and the x axis.**
- **There is no known formula to calculate the shape of the shaded area of the figure to the left. If, however, you were told that the curve at the top of the shaded shape came from the equation y = x² + 1, could you write a definite integral whose answer would be the shaded area?**



THE FUNDAMENTAL THEOREM OF CALCULUS (a more formal look at what we did in the last lesson):

Suppose that $f(x)$ is a continuous function on the closed interval [a, b], and $f(x)\geq 0$ for all points in the interval (meaning that the entire graph of f(x) is located above the x axis). Then the area bounded by the x axis, the vertical lines x = a and x = b, and the function $y=f(x)$ is given by the integral $\int_a^b f(x)dx$

Ex #1: Find the area of the region bounded by the x-axis, the lines x = -3 and x = 4 and the function $f(x) = 2x^2 + 3$. Write the integral that corresponds to this question. https://www.desmos.com/calculator/czjuupyme3

Ex #2: Find the area of the region bounded by the x-axis, the lines $x = -\frac{\pi}{2}$ and $x = \pi$ and the function $y = \cos\frac{1}{2}x$ Write the integral that corresponds to this question. https://www.desmos.com/calculator/pl4ylkmrfn

Ex #3: Find the area of the region bounded by the x-axis, the lines x = -3 and x = 1 and the function $f(x) = \frac{1}{x^2}$.

Write the integral that corresponds to this question. https://www.desmos.com/calculator/emcylatutj

QUESTION: Is there a contradiction between these question and the definition of the Fundamental Theorem of Calculus?

Ex #4: Find the area trapped by the curve f(x) = $-x^2 + 2x + 3$ and the x axis.

https://www.desmos.com/calculator/emcylatutj

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Note: This is Section 11.2 in your Textbook

- **To date, the definite integrals we have dealt with were located ABOVE the x axis, and the integral gave us the area that is located betwe`en the curve and the x axis**
- **What if the curve is located BELOW the x axis?**

Ex #1: Find the area of the region bounded by the x-axis, the lines x = -1 and x = 3 and the function f(x) = x^2 - 2x - 5. Write the integral that corresponds to this question.

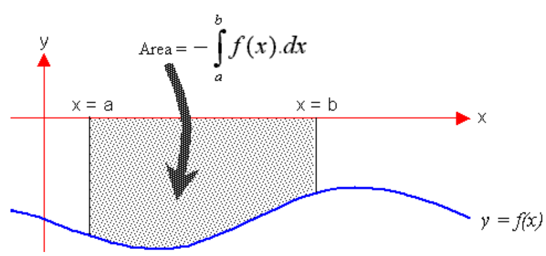
https://www.desmos.com/calculator/fwjqxkqqvg

- What is the problem with this answer?
- When you look at the graph, can you explain why the answer is negative?
- How could I adjust the formula for the Fundamental Theorem of Calculus to deal with area's that are below the x axis?

THE FUNDAMENTAL THEOREM OF CALCULUS: When the curve is BELOW the x axis

Suppose that $f(x)$ is a continuous function on the closed interval [a, b], and $f(x) \le 0$ for all points in the interval (meaning that the entire graph of the interval f(x) is located below the x axis). Then the area bounded by the x axis, the vertical lines x = a and x = b, and the function $y = f(x)$ is given by the integral

$$\int_a^b -f(x)dx \quad \text{or} \quad -\int_a^b f(x)dx$$

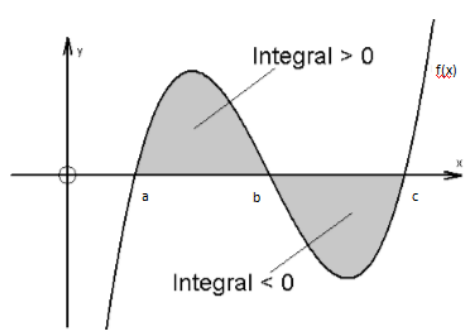


THE FUNDAMENTAL THEOREM OF CALCULUS:

When the curve is ABOVE AND BELOW the x axis

Suppose that $f(x)$ is a continuous function on the closed interval [a, c] where a<b<c and where $f(x) \ge 0$ on [a,b) and $f(x) \le 0$ on (b, c]. Then the area bounded by the x axis, the vertical lines x = a and x = c, and the function $y = f(x)$ is given by the sum of the integrals

$$Area = \int_a^c f(x)dx = \int_a^b f(x)dx - \int_b^c f(x)dx$$



Ex #2: Find the area of the region trapped by the x-axis and the curve $f(x) = x^3 - x^2 - 6x$. Find your intervals algebraically.

OUTCOME 8A DAY 5 ASSIGNMENT Page 38 in Duo Tang
Note: This is Section 11.2 in the Textbook

CALCULUS 30: OUTCOME 8A DAY 6 – AREA BETWEEN TWO CURVES TEXTBOOK 10.2

Ex #1: Find the area of the region bounded by $f(x)=x^2 -2x + 3$, $g(x) = -1- x^2$, and the lines $x = -1$ and $x = 2$

Take a look at the graph and decide what to do: https://www.desmos.com/calculator/xkfa6jjvsy

Ex #2: Find the area of the between $y = x^2 + 1$ and $y = x$ from $x = 1$ to $x = 3$.
https://www.desmos.com/calculator/grylnkquqf

Ex #3: Find the area trapped between the curves $f(x) = x - 4$ and $g(x) = x^3 - 4x^2$. Use algebraic techniques to find the interval(s). https://www.desmos.com/calculator/kjdky1spcu

OUTCOME 8A DAY 6 ASSIGNMENT Page 39 in Duo Tang
Note: This is Section 10.2 in your Textbook

OUTCOME 8A DAY REVIEW ASSIGNMENT - Page 40 & 41 in Duo Tang