VIDEO LINKS FOR BACKGROUND REVIEW:

The following videos are helpful for reviewing concepts from PC 20 and PC 30 and/or methods and discussions that may have been missed during those courses. We will be briefly talking about these concepts in the future but a full review will not take place at that time.

Interval Notation:	a)	https://goo.gl/rrZNX9
	b)	https://goo.gl/rHJM6g
	c)	https://goo.gl/dWz6KF
Zeroes and Dividing:	a)	https://goo.gl/1CdVLM
	b)	https://goo.gl/FKdDj4
Solving Inequalities:	a)	https://goo.gl/hiaeeM
	b)	https://goo.gl/zz5KtD
Absolute Value:	b)	https://goo.gl/ryqURG
Function Notation:	a)	https://goo.gl/GBiQZj
	b)	https://goo.gl/DwzC3e
	c)	https://goo.gl/bVnyPo
Classifying Functions:	a)	https://goo.gl/FEmvGR
	b)	https://goo.gl/hUZtht
	c)	https://goo.gl/HegV5C
Piecewise Functions:	a)	https://goo.gl/avjvde
	b)	https://goo.gl/msZgJX
	c)	https://goo.gl/TBJTyi
Function Characteristics:	b)	https://goo.gl/uWgTVD
Function Transformations:	a)	https://goo.gl/m9qtXj
	b)	https://goo.gl/UikSV1
Domain & Range	b)	https://goo.gl/JmD9VV
Function Operations:	a)	https://goo.gl/tyFaLn
	b)	https://goo.gl/k1UQv9

VIDEO LINKS: a) https://goo.gl/P11Guu b) https://goo.gl/Pqg6DP

REVIEW: Types of Factoring

1) Polynomials of the form ax^2+bx+c

- Take out GCF
- Use the window/box method: https://goo.gl/dMqSeB or decomposition https://goo.gl/jg9P7e or guess and check

Ex: Factor the following

a) $2x^2 - 7x + 3$

2) Difference of Squares

Ex. Factor the following

a) $2x^2 - 8$ b) $2x^4 - 18x^2$ c) $x^4 - 16y^4$ d) $x^2 - 7$

3) Synthetic Division:

- Take out GCF (if possible)
- Use synthetic division to factor the remaining polynomial

Ex. Factor the following

$6x^3 - 15x^2 - 12x + 9$

FACTORING REVIEW ASSIGNMENT: AS FOLLOWS

1. Factor.

(a) $x^2 - x - 2$ (b) $x^2 - 9x + 14$

(c) $x^2 + 7x + 12$ (d) $2x^2 - x - 1$

(e) $5x^2 + 13x + 6$ (f) $6y^2 - 11y + 3$

(g) $t^3 + 2t^2 - 3t$ (h) $3x^4 + 7x^3 + 2x^2$

2. Factor.

(a) $4x^2 - 25$ (b) $x^3 - 1$

(c) $t^3 + 64$ (d) $y^3 - 9y$

(e) $8c^3 - 27d^3$ (f) $x^6 + 8$

(g) $x^4 - 16$ (h) $r^8 - 1$

3. Factor.

(a) $x^3 - x^2 - 16x + 16$ (b) $x^3 - 7x + 6$

(c) $x^3 + 5x^2 - 2x - 24$ (d) $x^3 + 2x^2 - 11x - 12$

(e) $4x^3 + 12x^2 + 5x - 6$ (f) $x^4 - 3x^3 - 7x^2 + 27x - 18$

SOLUTIONS

1. (a) $(x-2)(x+1)$ (b) $(x-2)(x-7)$
(c) $(x+3)(x+4)$ (d) $(2x+1)(x-1)$
(e) $(5x+3)(x+2)$ (f) $(3y-1)(2y-3)$
(g) $t(t-1)(t+3)$ (h) $x^2(3x+1)(x+2)$

2. (a) $(2x+5)(2x-5)$
(b) $(x-1)(x^2+x+1)$
(c) $(t+4)(t^2-4t+16)$
(d) $y(y+3)(y-3)$
(e) $(2c-3d)(4c^2+6cd+9d^2)$
(f) $(x^2+2)(x^4-2x^2+4)$
(g) $(x+2)(x-2)(x^2+4)$
(h) $(r+1)(r-1)(r^2+1)(r^4+1)$

3. (a) $(x+4)(x-4)(x-1)$
(b) $(x-1)(x+3)(x-2)$
(c) $(x-2)(x+3)(x+4)$
(d) $(x-3)(x+1)(x+4)$
(e) $(x+2)(2x-1)(2x+3)$
(f) $(x+3)(x-3)(x-2)(x-1)$

AP Calculus — New Prerequisite Skills A: Factoring

To factor using a GCF that has negative and rational exponents. To factor the sum and difference of cubes.

VIDEO LINKS: a) https://goo.gl/P11Guu b) https://goo.gl/Pqg6DP

REVIEW: Types of Factoring

4) GCF:

Always take out a Greatest Common Factor first. To do this see if all numbers can be divided by the same number. If there are the same variable in all of the terms, take out the lowest exponent:

Ex: Factor the following. Ensure that all coefficients inside the brackets are integers.

a) $-2x^2 + 12x - 4$ b) $12xyz - 24x^2y^3 + 3xy + 15x^5z^3$

c) $5c^4 + \frac{7}{3}c^2d$

5) Factoring with Rational or Negative Exponents

To take out the GCF when the exponents are fractions, take out the smallest exponents.

Ex. Factor the following

a) $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

b) $-5x^{-\frac{1}{2}} - 15x^{-\frac{3}{2}} + 10x^{-\frac{5}{2}}$

c) $\frac{1}{6}x^{-\frac{2}{5}} + \frac{7}{6}x^{\frac{3}{5}}$

d) $-\frac{5}{3}x^{\frac{1}{2}} + \frac{2}{9}x^{-\frac{3}{2}}$

NEW: SUM & DIFFERENCE OF CUBES

CHARACTERISTICS OF A SUM OR DIFFERENCE OF TERMS

- Two Terms
- The terms are separated by a + or a – sign
- Each term is a perfect cube

FORMULA FOR FACTORING A SUM OR DIFFERENCE OF CUBS

- $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Ex #1: Factor the following

a) $x^3 + t^3$

b) $x^3 + 125$

c) $8x^3 - 27$

d) $1000x^{12} + 343y^6$

e) $9x^4 - 9x$

f) $(x + 6)^3 - y^3$

g) $27 - (2a - 5)^3$

Ex #2: Factor the following.

a) $3(x - 3)^3 - 4(x - 3)^4$

b) $4 - x^{-2}$

Ex #2: a) Factor the following by grouping: $2x^3 - x^2 + 6x - 3$

b) Can you factor by grouping if you rearrange so the above question looks like: $2x^3 + 6x - x^2 - 3$?

c) Will factoring by grouping work for all four term polynomials? $2x^3 - 5x^2 - 4x + 3$

NEW: CRAZY GCF (This will be used a LOT this year

RULES FOR FACTORING:

- You need to factor out all coefficients that are fractions (all denominators must be factored out completely, use the GCF of the numerators and take it out – you may leave)
- If the first term is negative, you must factor it out
- Factor out the smallest exponent (the "coldest" exponent") of any variable that appears in each term OR any expression that appears in each term
- Leave all factors simplified (if you can add/subtract or multiply within a bracket you must do so)

Ex #2: Factor the following

a) $\left(x^3+2\right)^{1/3}+\left(x^3+2\right)^{-5/3}$

b) $-12x^3(3x+5)^3+3x^2(3x+5)^4$

c) $6x(x^2+1)^2(2-3x)^4-12(2-3x)^3(x^2+1)^3$

d) $2x^3(x-2)^{-1}(x+1)^{\frac{3}{4}}-4x^2(x-2)(x+1)^{-\frac{1}{4}}$

e) $\frac{5}{2}(2x^2+3)^2(5x-1)^{-\frac{1}{2}}+8x(5x-1)^{\frac{1}{2}}(2x^2+3)$

f) $\frac{3}{10}(x-1)^{-2}(2x+1)^{-\frac{3}{4}}-\frac{9}{10}(x-1)^{-2}(2x+1)^{\frac{1}{4}}$

RULES FOR FACTORING:

- You need to factor out all coefficients that are fractions
  - note that this means that you must completely factor out all denominators that appear in all coefficients of each term in your original question. Your final answer should not have any denominators (other than one) within the bracketed components in your answer
  - When it comes to the numerators of these coefficients, factor out their GCF (I tend to cover up the denominators with my hand and just factor out GCF's like you did in grade 10). You may end up with coefficients in the numerator (which will be over one) within the bracketed components in your answer
- If the first term is negative within a polynomial raised to the power of one, you must factor it out
- Factor out the smallest exponent (the "coldest" exponent") of any variable that appears in each term OR any expression that appears in each term
- You need to simplify the interior of each "bracketed" portion of your answer. If you end up with polynomials raised to a power of three or two within another set of brackets, you need to expand those interior polynomials and simplify the resulting terms.
  - ie: If you end up with the following as part of your answer $\left[(x-3)^2+(2x+1)^3\right]$, you would need to expand both the $(x - 3)^2$ and $(2x + 1)^3$, and then combine their terms to get $[9x^3 + 6x^2 + 18x - 7]$
- You may leave factors or variables with negative and/or rational exponents
- Each factor is written in descending order of powers

FACTORING ASSIGNMENT: AS FOLLOWS

1. Factor the following

a) $a^3 + b^3$	b) $m^3 - 8b^3$	c) $27t^3 + 1$	d) $54c^3 + 16d^3$
e) $x^4 - x$	f) $16xy^4 - 2x^4y$	g) $(x + 1)^3 + 1$	h) $(x + 2)^3 - 8$
i) $64x^6 + 27y^9$	j) $(a + 3)^3 - (a - 3)^3$	k) $(x^2 - 1)^3 - 27$	l) $8 + (x^2 - 6)^3$

2. Factor by grouping: $x^3 - x^2 - 16x + 16$

3. Create a quartic polynomial with four terms that can be factored using the grouping method (and is a different polynomial than any previous examples done in class). Factor your question.

4. Fully factor the following. Leave your answer in the proper form (following all guidelines talked about in class!)

5.

(a) $x^{\frac{5}{2}} - x^{\frac{1}{2}}$	(b) $x + 5 + 6x^{-1}$
(c) $x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 8x^{-\frac{1}{2}}$	(d) $2x^{\frac{7}{2}} - 2x^{\frac{1}{2}}$
(e) $1 + 2x^{-1} + x^{-2}$	(f) $(x^2 + 1)^{\frac{1}{2}} + 3(x^2 + 1)^{-\frac{1}{2}}$

6. FULLY factor the following. Leave your answer in the proper form (follow all guidelines talked about in class!) . **You WILL HAVE TO HAND IN THIS QUESTION!!!**

a) $125x^3 + 64y^3z^6$

b) $8x^3 - (3y + 2)^3$

c) $x^{\frac{5}{2}} + x^{\frac{3}{2}}$

d) $-20x^{\frac{-3}{4}} + 4x^{\frac{1}{4}}$

e) $12x^{\frac{-2}{3}} - 18x^{\frac{1}{3}}$

f) $(x^2 - 3)^{\frac{3}{2}} + (x^2 - 3)^{\frac{7}{2}}$

g) $(x + 2)^{\frac{3}{2}} - (x + 2)^{\frac{1}{2}}$

h) $(x^2 + 2)^{-\frac{5}{3}} + (x^2 + 2)^{-\frac{2}{3}}$

i) $(x + 4)^{-\frac{1}{2}} - (x + 4)^{-\frac{3}{2}}$

j) $(5x - 1)^{\frac{1}{2}} - \frac{1}{3}(5x - 1)^{\frac{3}{2}}$

k) $2x(3x - 5)^{\frac{1}{2}} - \frac{4}{3}x^2(3x - 5)^{\frac{-1}{2}}$

l) $-8(4x + 3)^{-2} + 10(5x + 1)(4x + 3)^{-1}$

m) $-x^{3/2}(x^2 + 3)^{-3/2} + \frac{1}{3}x^{-1/2}(x^2 + 3)^{-1/2}$

n) $-\frac{5}{2}(1 - 2x)^{-1/2}(x - 3)^{1/2} + \frac{1}{2}(x - 3)^{-1/2}(1 - 2x)^{1/2}$

To rationalize numerators or denominators of a given expression.

RATIONALIZING A NUMERATOR OR DENOMINATOR

- Will turn that numerator or denominator into a RATIONAL expression (will remove the roots)
- To rationalize the numerator or the denominator, multiply both the numerator and the denominator by the conjugate of the numerator or denominator that you are rationalizing
  - REMEMBER: The **CONJUGATE** of a binomial is a binomial that is identical to the original binomial but containing the opposite middle sign

Ex #1: State the conjugate of each of the following:

a) $\sqrt{a}-\sqrt{b}$

b) $\sqrt{x+4}+2$

Ex #2:

a) Rationalize the numerator of

$$\frac{\sqrt{x+4}-2}{x}$$

b) Rationalize the denominator of

$$\frac{5}{\sqrt{x+3}+\sqrt{x}}$$

RATIONALIZING ASSIGNMENT: AS FOLLOWS

1. Rationalize the numerator.

(a) $\dfrac{\sqrt{x}-3}{x-9}$

(b) $\dfrac{\frac{1}{\sqrt{x}}-1}{x-1}$

(c) $\dfrac{x\sqrt{x}-8}{x-4}$

(d) $\dfrac{\sqrt{2+h}+\sqrt{2-h}}{h}$

(e) $\sqrt{x^2+3x+4}-x$

(f) $\sqrt{x^2+x}-\sqrt{x^2-x}$

2. Rationalize the denominator.

(a) $\dfrac{1}{\sqrt{x+1}-1}$

(b) $\dfrac{4}{\sqrt{x+2}+\sqrt{x}}$

(c) $\dfrac{x}{\sqrt{x^2+1}+x}$

(d) $\dfrac{x^2}{\sqrt{x+1}-\sqrt{x-1}}$

SOLUTIONS

1. (a) $\dfrac{1}{\sqrt{x}+3}$ (b) $-\dfrac{1}{x+\sqrt{x}}$ (c) $\dfrac{x^2+4x+16}{x\sqrt{x}+8}$

(d) $\dfrac{2}{\sqrt{2+h}-\sqrt{2-h}}$

(e) $\dfrac{3x+4}{\sqrt{x^2+3x+4}+x}$

(f) $\dfrac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

2. (a) $\dfrac{\sqrt{x+1}+1}{x}$ (b) $2(\sqrt{x+2}-\sqrt{x})$

(c) $x(\sqrt{x^2+1}-x)$

(d) $\frac{1}{2}x^2(\sqrt{x+1}+\sqrt{x-1})$

REVIEW: SLOPE

$m = \frac{rise}{run}$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

- Parallel lines have the same slope
- Perpendicular lines have slopes that are negative reciprocals of each other.

REVIEW: Different ways to Write Equations of Lines:

Point-slope form

$y = m(x - x_1) + y_1$

NOTE: *This textbook uses a slightly different version for Point-Slope form than we used in gr 10. They use it as a transformation of a line as we learned in PC 30 and solve for y.*

Slope-Intercept Form

$y = mx + b$

General Linear Form: $Ax + By = C$ (A and B not both zero, A≥0, $A, B, C \in Z$ (integers))

NOTE: *This textbook uses different terminology than our grade 10 textbook did. In grade 10 we were told that GENERAL FORM looks like Ax + By + C = 0 and that STANDARD FORM looks like Ax + By = C*

Increments:

- The net change in x and y between two points.
- If a particle moves from the point (x_1, y_1) to the point (x_2, y_2), the **INCREMENTS** in its coordinates are
  $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Example 1: Write equations in point slope form for the line through point (-1, 2) that is a) parallel, and b) perpendicular to the line L: $y = 3x - 4$

Example 2: Find the increments for movement from (-1,0) to (4, -3)

Example 3: Write the equation for the line through (7, -2) and (-5, 8). Leave your answer in Point-Slope, Slope Intercept and General Form.

1.1 Review Assignment: Exercises P9 #5-11 odd, 17, 19, 27-37 odd, 42, 43

1.1 Assignment: Exercises P 9: #1, 3, 13, 15, 21, 23

1.2 Functions & Their Graphs

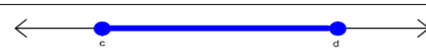
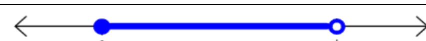
To review functions and their properties.

Function – from a set D to a set R = assigns a unique element in R to each element in D

This one is a function

Interval Notation:

- CLOSED intervals contain their boundary points
- OPEN intervals contain NO boundary points.

Open or closed?	Interval notation	Set notation	Graph
			
			
			
			
			
			
			
			
			

- **Union** $(A \cup B)$ consists of all elements that are in A or in B or in both.
- **Intersection** $(A \cap B)$ consists of all elements that are found in both A and B.

- A function of x is **<u>even</u>** if it is symmetrical about the **y-axis**. $f(-x) = f(x)$

- A function of x is **<u>odd</u>** if it is symmetrical about the **origin.** $f(-x) = -f(x)$

Piecewise Functions: Equations that need to be described by more than one equation. Following is the form of a piecewise equation:

$$y = \begin{cases} First\ Equation\ Piece, followed\ by\ its\ domain \\ Second\ Equation\ Piece, followed\ by\ its\ domain \\ Additional\ Pieces\ of\ Equation,\ their\ domain \end{cases}$$

Absolute value functions defined as a piecewise function: $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

Composite functions:

- $f(g(x)) = fog$ - this means that you plug all of g(x) into each x value of the function f(x)
  - Please note that $f \circ g \neq f \bullet g$

Example 1: Identify the domain of the following. Use the notation stated.

a) $3x - 1 \leq 5x + 3$ (Interval Notation) b) $|x - 3| \leq 4$ (Set Notation) https://www.desmos.com/calculator/fphc1zaxfv

b) $x^2 < 16$ (Set and Interval Notation)

Example 2: Find all real solutions to the equation $f(x) = x^2 - 5$

a) $f(x) = 4$

b) $f(x) = -6$

Example 3: Find all real solutions to the equation $f(x) = \sqrt{x+7}$ when $f(x) = 4$

Example 4: Identify the domain and range and roughly sketch the function $y = \sqrt{x^2 - 4}$

Example 5: Determine whether $y = x^3 - x$ is odd, even or neither. https://www.desmos.com/calculator/q1fs0jxr0h

STEPS TO GRAPHING PIECEWISE FUNCTIONS ON A TI-84

1 Press [ALPHA][Y=][ENTER] to insert the n/d fraction template in the Y= editor.

2 Enter the function piece in the numerator and enter the corresponding interval in the denominator.

To enter the first function piece in Y_1, enter (**X + 8)** in the numerator and (**X < –1)** in the denominator.

Press [2nd][MATH] to insert an inequality from the Test menu. Press

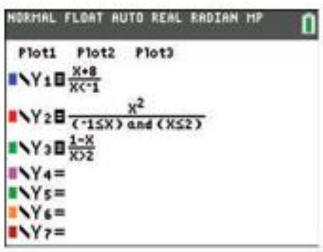
[2nd][MATH][►]

to insert "**and**" from the Logic menu.

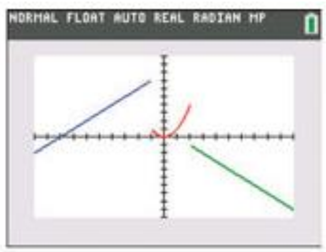
Your calculator can't evaluate a sandwiched inequality like this one: (**–1 < X < 2). Fortunately,** (**–1 < X < 2)** can also be written as a compound inequality:

$(-1 \le X)$ and $(X \le 2)$

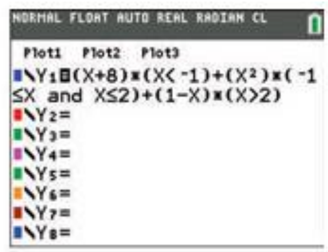
See the first screen.



Using division



Piecewise graph



Using multiplication

3 Press [GRAPH] to graph the function pieces.

See the second screen.

ALTERNATE METHOD (WORKS ON 84 or 83) (Third image above)

1 In the Y= editor, enter the first function piece using parentheses and multiply by the corresponding interval (also in parentheses).

Don't press [ENTER] yet!

2 Press [+] after each piece and repeat until finished.

Refer to the third screen. In order for you to see the whole equation, the calculator was temporarily switched to Classic mode.

Example 6: Use your TI to graph the following piecewise function. What is the domain and range?

$$f(x) = \begin{cases} 2x - 1, & x \leq 0 \\ x^2 + 3, & x > 0 \end{cases}$$

https://www.desmos.com/calculator/8fot5hwjlc

Example 7: Composite functions.

Given $f(x) = 2x - 3 \text{ } and \text{ } g(x) = 5x, find \text{ } fog.$

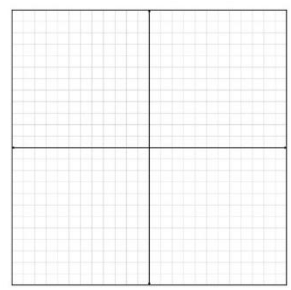
1.2 Assignment: P19 #1-27 odds & 31, 35, 36, 37, 39, 42, 45

REVIEW: 1.3 Exponential Functions

To review exponential functions and to learn about Euler's number *e*.

EXPONENTIAL FUNCTIONS:

$f(x) = a^x$ is the exponential function with a base a, $a > 0, a \neq 1$



- Domain $f(x) = a^x \text{ } is \text{ } (-\infty, \infty)$
- Range $f(x) = a^x \text{ } is \text{ } (0, \infty)$
- If x is positive the function is increasing.
- If x is negative the function is decreasing.
- The function $y = ka^x, k > 0$ is a growth function if $a > 1$
- The function $y = ka^x, k > 0$ is a decay function if $0 < a < 1$

Recall – the inverse of an exponential function is a logarithmic function.

Exponent Rules: If $a > 0 \text{ } and \text{ } b > 0$ then the following hold.

1. $a^x \cdot a^y = a^{x+y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = (a^y)^x = a^{xy}$
4. $a^x b^x = (ab)^x$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

The number **e** is a famous irrational number, and is one of the most important numbers in mathematics.

The first few digits are:

2.7182818284590452353602874713527 (and more ...)

*It is often called **Euler's number** after Leonhard Euler (pronounced "Oiler").*

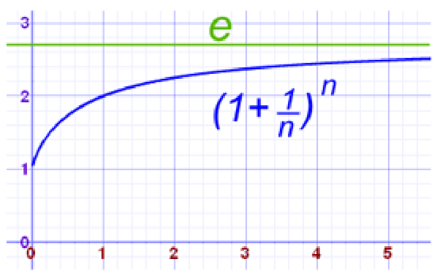
e is the base of the Natural Logarithms (invented by John Napier).

Calculating

There are many ways of calculating the value of **e**, but none of them ever give a totally exact answer, because **e** is irrational (not the ratio of two integers).

But it **is** known to over 1 trillion digits of accuracy!

For example, the value of $(1 + 1/n)^n$ approaches **e** as n gets bigger and bigger:



n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

The value of e can be calculated by finding $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. We can see in the diagram to the right that

$$e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71827....$$

Another Calculation

The value of e is also equal to $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+ ...$ (etc)

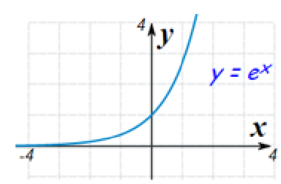
(Note: "!" means factorial)

The first few terms add up to: $1 + 1 + \frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120} = 2.718055556$

In fact Euler himself used this method to calculate **e** to 18 decimal places.

Growth

e is used in the "**Natural**" Exponential Function:



Graph of $\mathbf{f(x) = e^x}$

It has this wonderful property: "it's slope is it's value"

$y = e^x$ and $y = e^{-x}$ are used frequently as models of exponential growth and decay.

- Interest compounded continuously uses the model$y = Pe^{rt}$, where P is the initial investment, r is the interest rate as a decimal, and t is the time in years.

Example 1: The approximate number of fruit flies in an experimental population after t hours is given by $Q = 20e^{0.03t}, t \geq 0$

https://www.desmos.com/calculator/k20nnrg5gx

a) Find the initial number of fruit flies in the population
b) How large is the populations of fruit flies after 72 hours?
c) Use your TI to graph the function Q.

Example 2: find the zero of $f(x) = 5 - 2.5^x$

Example 3: Population of a province for several years

Year	*Population (thousands)*
1998	6901
1999	7000
2000	7078
2001	7193
2002	7288
2003	7386

a) Find the ratios of population in one year by the ratio of population in the previous year.

b) Based on a), create an exponential model for the population.

Example 4: Population growth – The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

a) Estimate the population in 1915 and 1940.

b) Approximately when did the population reach 50000?

COMMONLY USED EXPONENTIAL EQUATIONS:

Exponential Decay/Half Life $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{n}} = A_0(2)^{-\frac{t}{n}}$

Population Growth $P(t) = P_0(1+r)^t$

Interest $A(n) = A_0(1+i)^n$

1.3 Assignment: P26 #3, 6, 9, 12-18, 24, 30, 31, 35, 36, 39, 42, 45

To review logarithms and to learn about common logarithms.

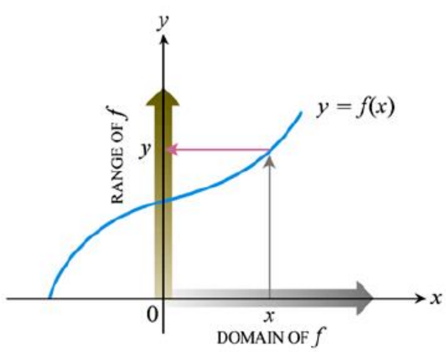
One-to-one function

- A function where each output value of a function is associated with exactly one input value
- Graphically, one-to-one functions must pass both the vertical and horizontal line tests

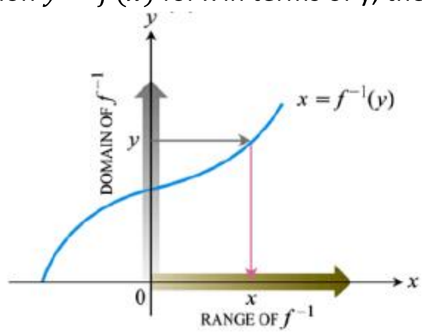
---

Inverses

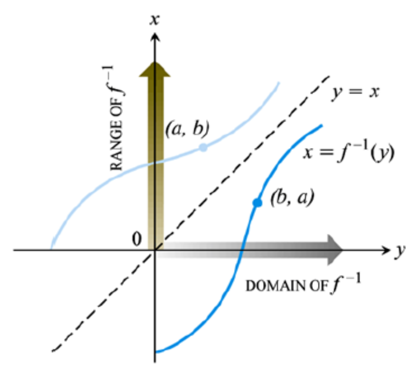
- The symbol for the inverse of f is f^{-1}, read f inverse. It is NOT a negative exponent.
- If a function is not one-to-one, its inverse will not be a function
- To find the inverse, solve the equation $y = f(x)$ for x in terms of y, then interchange x and y.



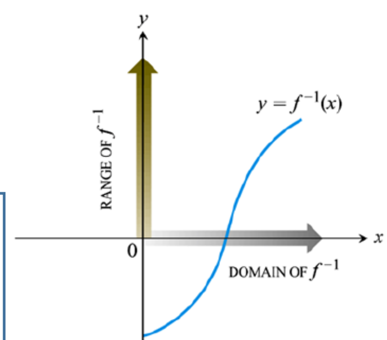
(a) To find the value of f at x, we start at x, go up to the curve, and then over to the y-axis.



(b) The graph of f is also the graph of f^{-1}. To find the x that gave y, we start at y and go over to the curve and down to the x-axis. The domain of f^{-1} is the range of f. The range of f^{-1} is the domain of f.

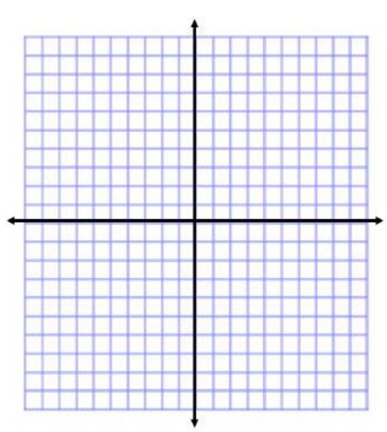


(c) To draw the graph of f^{-1} in the usual way, we reflect the system across the line $y = x$.



(d) Then we interchange the letters x and y. We now have a normal-looking graph of f^{-1} as a function of x.

Example #1: Given that $y = 4x - 12$ is one-to-one, find its inverse. Graph the function and its inverse.



Logarithms

- Given an exponential function $y = a^x$, its inverse function (still written as an exponential function) is $x = a^y$
  - Since we can not use traditional algebraic techniques to solve the inverse function for y, we rewrite $x = a^y$ into its equivilent logarithmic form $y = \log_a x$ (read as y equals the log of x to the base a)

- The domain of $y = \log_a x$ is (0,∞) *(Note that this is the range of the inverse function $y = a^x$)*
- The range of $y = \log_a x$ is (-∞,∞) *(Note that this is the the domain of the inverse function $y = a^x$)*

Base of a Log Function:

- When we consider the original exponential function $y = a^x$, are there any restrictions on the value of "a"?

- What are the restrictions on the value of "a" in logarithmic functions of the form $y = \log_a x$?

Logarithms with base e and base 10 will be the most used commonly bases in Calculus (e being the most common in AP Calculus)

- $y = \log_e x = \ln x$ is called the **Natural Logarithmic** function
  - When the number known as *e* is used as the base, where *e* = 2.718….., this is known as the natural logarithm $\log_e x$. Due to its importance, it actually has its own new form and we say that $\log_e x = \ln x$
- $y = \log_{10} x = \log x$ is often called the **Common Logarithmic** function
  - When 1=10, it is called the common logarithm and we don't actually write the 10. Therefore a question such as $\log_{10} x$ will actually appear as logx

$$x = 2^y \quad \Leftrightarrow \quad y = \log_2 x$$

RULES OF LOGARITHMS:

- $\log_c 1 = 0$ since in exponential form $c^0 = 1$.
- $\log_c c = 1$ since in exponential form $c^1 = c$
- $\log_c c^x = x$ since in exponential form $c^x = c^x$
- $c^{\log_c x} = x,\ x > 0$, since in logarithmic form $\log_c x = \log_c x$

PROPERTIES OF LOGARITHMS:

Property Name	In Symbols	In Words
1. Product Property	$\log_b(XY) = \log_b X + \log_b Y$	The logarithm of a product is the sum of the logarithms of the numbers in the product.
2. Quotient Property	$\log_b\left(\frac{X}{Y}\right) = \log_b X - \log_b Y$	The logarithm of a quotient is the difference between the logarithm of the numerator and the logarithm of the denominator.
3. Power Property	$\log_b\left(X^k\right) = k\log_b X$	The logarithm of a power is the exponent multiplied by the logarithm of the base.
4. Change Of Base	$\log_c X = \frac{\log X}{\log c}$	To find the logarithm of a number X in base c, divide the common logarithm of X by the common logarithm of c. (Applying this property would likely require a calculator.)

Example #1: Evaluate the following logarithms:

a) $\log_2 8$

b) $\log_3 81$

c) $\log_9 9$

d) $\log_3 \frac{1}{9}$

e) $\log_5(-25)$

f) $\log_{10} 1000$

Example #2: Evaluate $\log_4 9$ to five decimal places

CHANGE OF BASE FORMULA

$\log_a x = \dfrac{\log_b x}{\log_b a}$ | To change to base 10 use $\log_a x = \dfrac{\log x}{\log a}$

Example #3: Rewrite as a single log and evaluate.

a) $4\log_4 2 - 2\log_4 8$

b) $\log_6 8 + \log_6 9 - \log_6 2$

c) $3\log_{\frac{2}{3}} 2 - 3\log_{\frac{2}{3}} 3$

d) $\log_7 7\sqrt{7}$

Example #4: Evaluate the following to five decimal places.

a) ln 7

b) ln10000

c) ln1

Example #5: Use the definition of a logarithm to solve for x

a) $\log_3 x = -1$

b) $\ln x = 2$

c) $2\ln(x + 1) = -1$

Example #7: Solve the following for x:

a) $2^x = 12$

b) $e^x + 5 = 60$

Example #8: The population of a city is given by $P = 105300e^{0.015t}$ where t=0 represents 1990. According to this model, when will the population reach 150000?

Example #9: Solve for x

a) $e^x + e^{-x} = 3$

b) $2^x + 2^{-x} = 5$

1.5 Assignment: P42 #1-6, 7-12 (use Graphing Calc if necessary), 13, 16, 22, 33-39, 46

NOTE: For extra review you can visit my Pre-Calculus 30 page on my website and watch the videos on logarithms from last year https://carignanmath.weebly.com/

1.6 Trigonometric Functions

To review Trigonometric Functions and to learn about Inverse Trigonometric Functions.

VIDEO LINKS : https://bit.ly/2fdS1NJ followed by https://bit.ly/2Nicapn followed by https://bit.ly/2xrvJB7

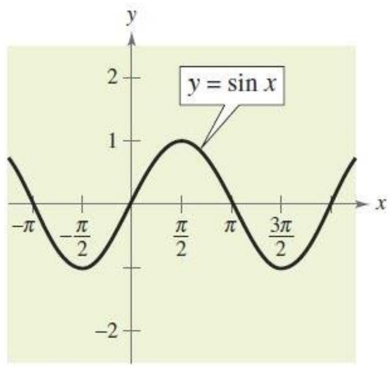
Arc length Formula: $a = \theta r$

Trigonometric Functions:

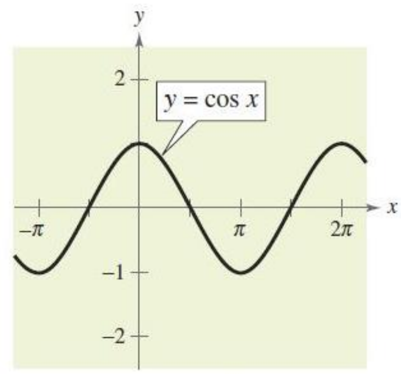
$\sin\theta = \dfrac{y}{r}$ $\quad \csc\theta = \dfrac{r}{y}$ $\quad \cos\theta = \dfrac{x}{r}$ $\quad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$

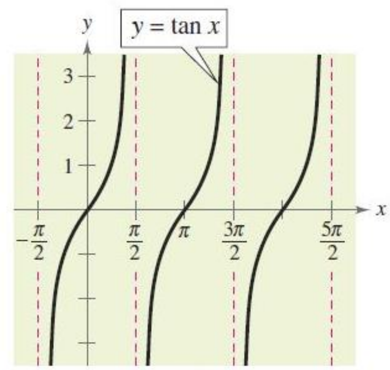
The graphs of the 6 trigonometric functions



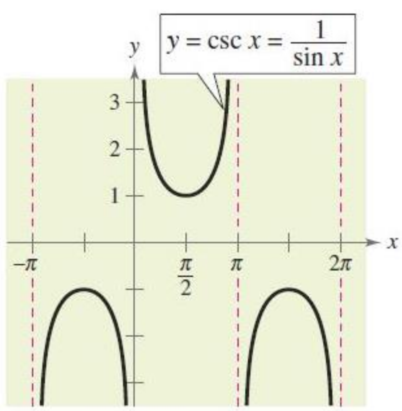
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



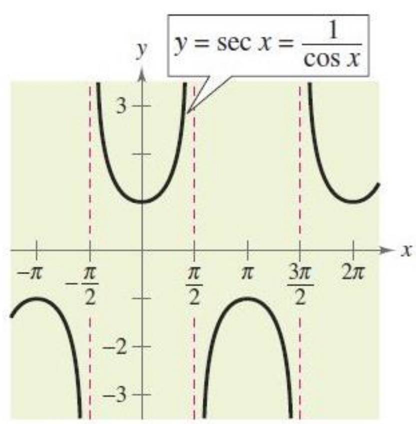
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



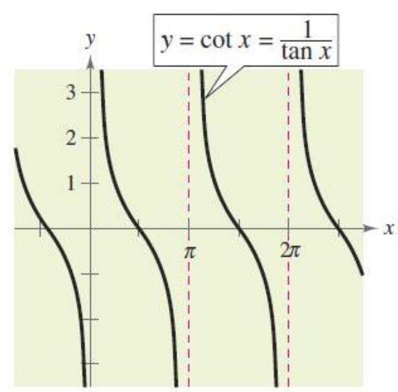
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π

Transformations of trigonometric functions

$y = af\big(b(x+c)\big) + d$ where:

$a = (amplitude)\ \ vertical\ stretch\ or\ compression$

$b = (period)\ horizontal\ stretch\ or\ compression$

$c = horizontal\ shift$

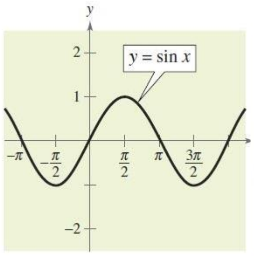
$d = vertical\ shift$

Inverse Trigonometric Functions

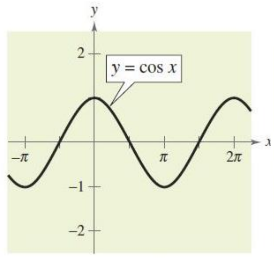
**** Note – none of the trig functions above are one-to-one. Those functions do not have inverses. However we can restrict the domain to produce a new function that does have an inverse.

- Given an initial trig function such as $y = \sin x$, we can find the inverse by switching the x and y in the traditional way. This would give you $x = \sin y$. How would you solve this for the variable y?

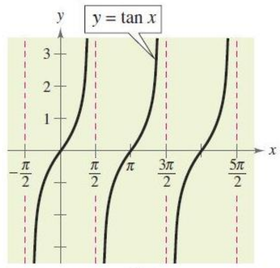
Note: For all of the following, $n \in Z$



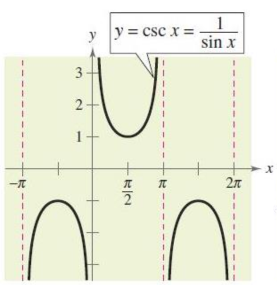
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



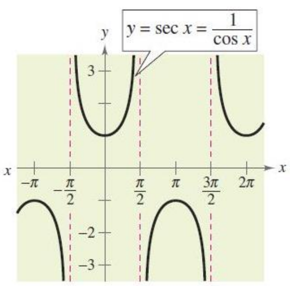
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



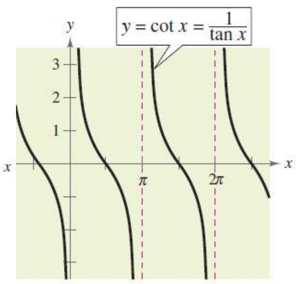
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



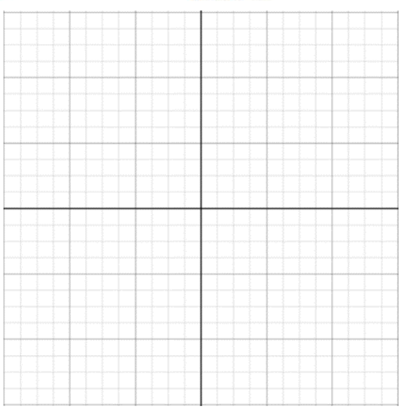
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π

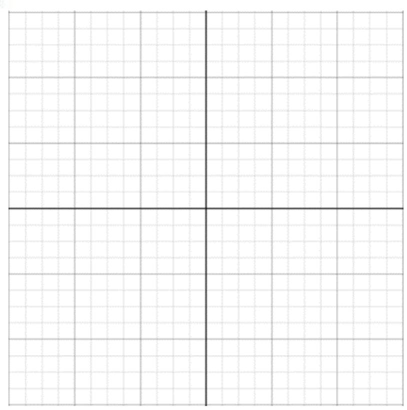


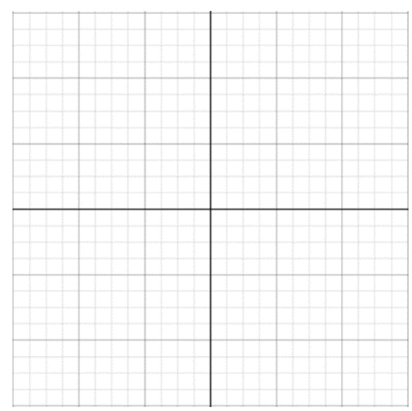
DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π

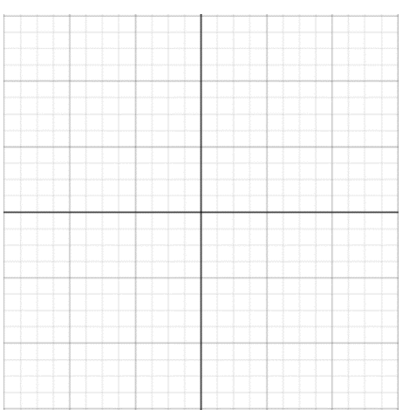
Inverse Trig Function	New Domain that makes the inverse trig function one-to-one (Old Range)	New Range that makes the inverse trig function one-to-one (Old domain)
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \sec^{-1} x$ NOTE: To graph you use $y = \cos^{-1}\left(\frac{1}{x}\right)$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi, y \neq \frac{\pi}{2}$
$y = \csc^{-1} x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \neq 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 \le y \le \pi$

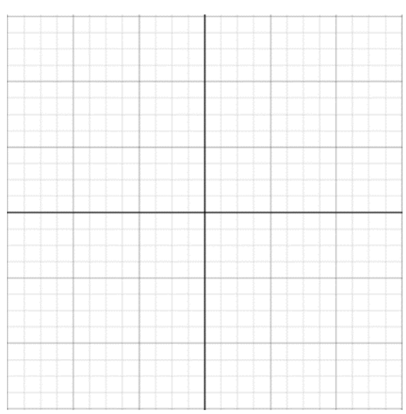
https://www.desmos.com/calculator/hnuoonuugx

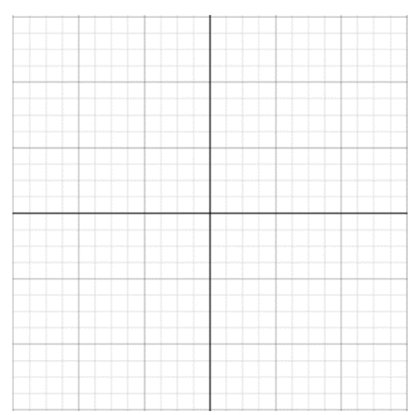












Example #1: Find the measure of $\sin^{-1}\left(-\frac{1}{2}\right)$ in degrees and radians.

Example #2: Find all the trigonometric values of θ if $\sin\theta = -\frac{3}{5}$, and $\tan\theta < 0$

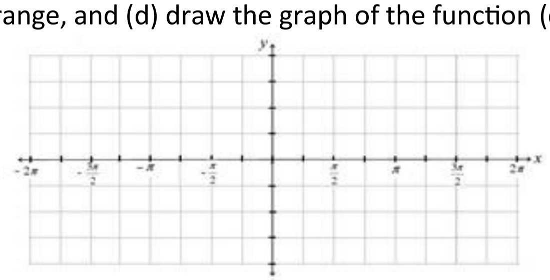
Example #3: Solve for x:

(a) $\sin x = 0.7\ in\ the\ domain\ 0 \le x < 2\pi$

(b) $\tan x = -2\ in\ domain -\infty < x < \infty\ in\ degrees$

Example #4: Find the length of an arc subtended on a circle of radius 3 by a central angle that measures $\frac{2\pi}{3}$.

Example #5: Determine the (a) period, (b) domain, (c) range, and (d) draw the graph of the function (one period)
$y = 3\cos(2x - \pi) + 1$



NOTE: For extra review you can visit my Pre-Calculus 30 page on my website and watch the videos on trigonometry from last year https://carignanmath.weebly.com/

1.6 Assignment: P52 #1-21 Odd, 23 (use table 1.20), 31-35 Odd, 37-41 Odd, 49, 50-55

REVIEW QUESTIONS FOR CHAPTER 1(No in class time will be given – pick and choose questions to do as appropriate for your learning needs)

REVIEW ASSIGNMENT: P55 #1, 4, 5, 12, 15, 18, 19, 23, 25, 26, 38, 40, 41, 43, 53, 55, 57, 58, 63, 67-70