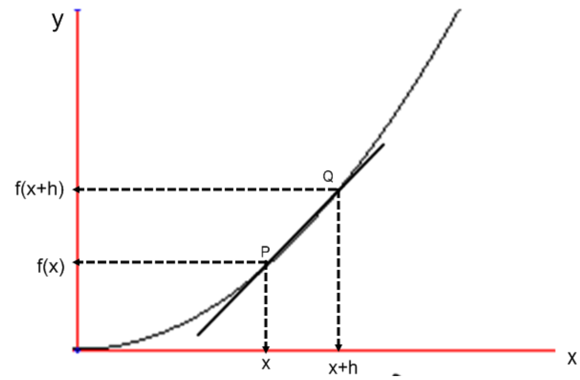
3.1 Day 1: The Derivative of a Function

I CAN DEFINE A DERIVATIVE AND UNDERSTAND ITS NOTATION.

Last chapter we learned to find the slope of a tangent line to a point on a graph by using a secant line where the distance between the two points of the secant line, h, approaches zero. This slope is called **THE DERIVATIVE.**

- Slopes of Tangents at a general point (x, f(x)) = Finding the derivative
- The slope of the SECANT line PQ is a value: $m = \frac{f(x+h)-f(x)}{h}$
- In order to turn secant PQ into a tangent line (going through just P), we continually move point Q closer to P until the distance between them, h, approaches zero. To find the numerical value of the slope of the tangent line we need to use limits in the above formula.



DERIVATIVE: The derivative of a function f(x) represents the slope of a line tangent to a curve at any point (x, f(x+h)) and is defined as follows:

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- When the above formula is used for any value of x, we leave the value of x in the formula. The answer we get will not be a numerical value for a specific slope, rather it will be a general formula that can be used to find the value of the slope at a specific value of x, x = a.

Sometimes the formula is modified to look like $f'(a) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

- This formula does not give us a general slope equation, rather it gives us the specific numerical value of the slope at a value of x = a on the graph.
- The original equation is very useful if we will be determining multiple slopes on a single equation as it gives us a simple formula that can be used easily with different values of x = a. The second equation is useful if we know for sure that we will only be computing one slope for the given function f(x)

Example 1: Use the definition of the derivative to find the slope of the tangent line to the function

a) $f(x) = x^2 \ at \ (2, f(2))$

b) $f(x) = x^2\ at\ (-3, f(-3))$

c) Find the function f'(x) that will allow us to easily find the derivative function at any point f'(a)

Example 2:

a) use the definition of the derivative to find $f'(x)$ if $f(x) = 2x^2 - x^3$

- ask yourself – is this asking for f'(a) or f'(x)? What is the difference between the two?

b) Find the coordinates of the two points on the curve $f(x) = 2x^2 - x^3$ at which the slope of the tangent line is 1.

https://www.desmos.com/calculator/0qqoxypp9j

You will remember that last chapter we had an alternate definition for the slope of a tangent line

SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (FORMULA 1) **(This formula will be used more in Ch 3):**

- The slope of a line tangent to a curve at a point (a, f(a)) is defined as follows:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- We can now expand this definition and call it another formula for finding **THE DERIVATIVE** of a function at a point x = a

$$f'(a) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example 3: Differentiate (find the derivative) of the function $f(x) = \sqrt{x}$ at x = a, and then at x = 4.

There are many ways to denote the derivative of a function $y = f(x)$.

Derivative Notation	Words describing derivative notation	Important points about the derivative notation
$f'(x)$		
$f'(a)$		
y'		
$y'(a)$		
$\frac{d}{dx}y = \frac{dy}{dx}$		
$\left.\frac{dy}{dx}\right\|_{x=a}$		
$\frac{df}{dx} = \frac{d}{dx}f(x)$		

$$\therefore f'(x) = y' = y'(x) = \frac{d}{dx}y = \frac{dy}{dx} = \frac{d}{dx}f(x) = \frac{df}{dx}$$

<u>**NOTE:**</u> $\frac{d}{dx}$ should NOT be thought of as a fraction or as itself a derivative; it should be thought of as an operator that instructs you to take the derivative and treat x as the variable.

Example 4: Complete the table below, stating what each of the indicated limits finds in terms of the derivative of a function, *f*(*x*).

Definition of the Derivative (slope function)	$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
Definition of the Derivative (slope value)	$f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$	
Alternate Form of the Definition of the Derivative	$f'(a) = \lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$	

3.1 Day 1 Assignment: Textbook P 105 #1-12, 17, 18
Duo Tang Page 15 (Ch 3) #1-5

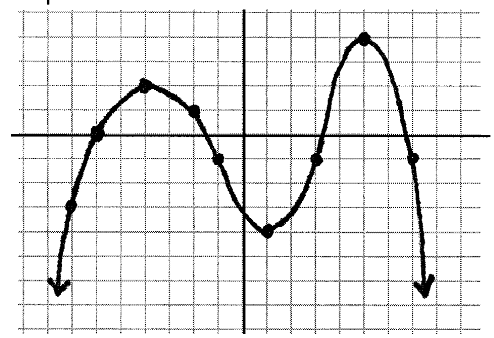
VIDEO LINKS: a) https://bit.ly/2Mp8ZI4 b) https://bit.ly/2LZs2wA

3.1 Day 2: The Derivative of a Function

I CAN DETERMINE THE RELATIONSHIPS BETWEEN THE GRAPH OF A FUNCTION AND ITS DERIVATIVE.

- Because we can think of the derivative at a point in graphical terms as slope, we can get a good idea of what the graph of the function *f′(x)* looks like by *estimating the slopes* at various points along the graph of *f(x)*.

- We estimate the slope of the graph of *f* in *y*-units per *x*-unit at frequent intervals. We then plot the estimates in a coordinate plane with the horizontal axis in *x*-units and the vertical axis in slope units.



Understanding the Derivative from a Graphical and Numerical Approach

So far, our understanding of the derivative is that it represents the slope of the tangent line drawn to a curve at a point.

Complete the table below, estimating the value of $f'(x)$ at the indicated *x* – values by drawing a tangent line and estimating its slope.

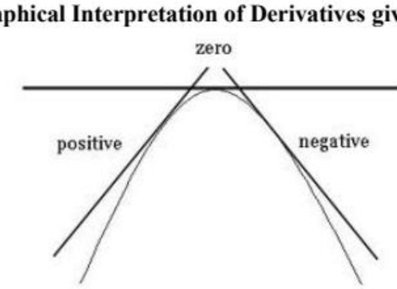
x – Value	Estimation of Derivative	Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum
–7		
–6		
–4		
–2		
–1		
1		
3		
5		
7		

Based on what you observed in the above table, what inferences can you make about the value of the derivative, $f'(x)$, and the behavior of the graph of the function, *f*(*x*)?

At a point $x = a$ on $f(x)$,

- If the tangent line has a positive slope, then the derivative $f'(a)$ is a positive value and is above the x-axis of $f'(x)$.
- If the tangent line has a negative slope, then the derivative $f'(a)$ is a negative value and is below the x-axis of $f'(x)$
- If the tangent line has slope zero (is horizontal), then the derivative $f'(a)$is zero and is on the x-axis of $f'(x)$
  https://www.intmath.com/differentiation/derivative-graphs.php

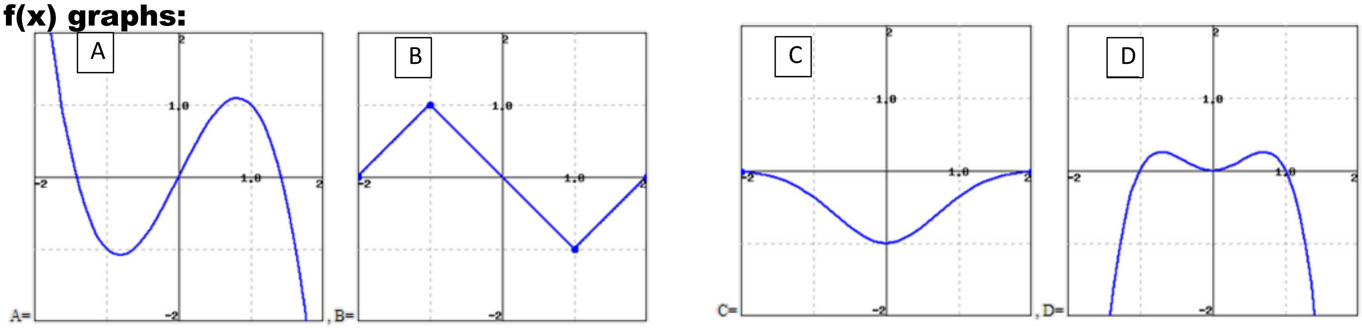
Graphical Interpretation of Derivatives given *f(x)*



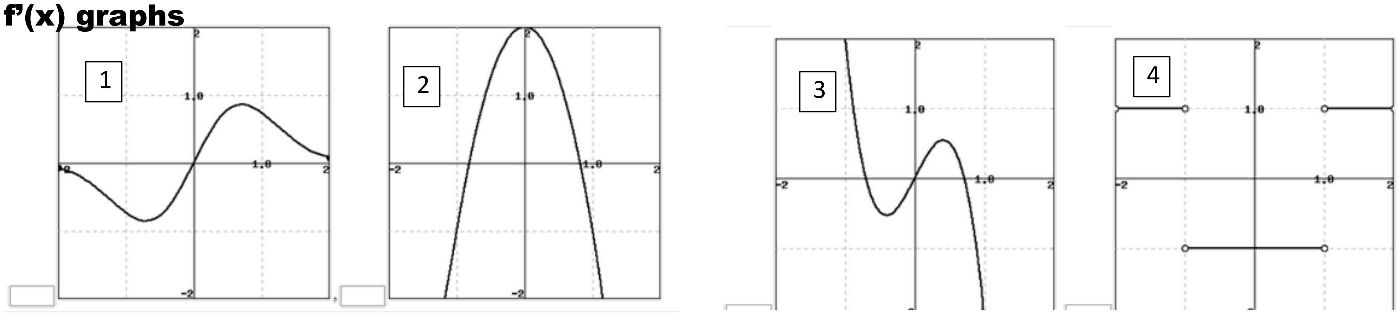
https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

Example 1: Match the graph of f(x) with its derivative, f'(x)

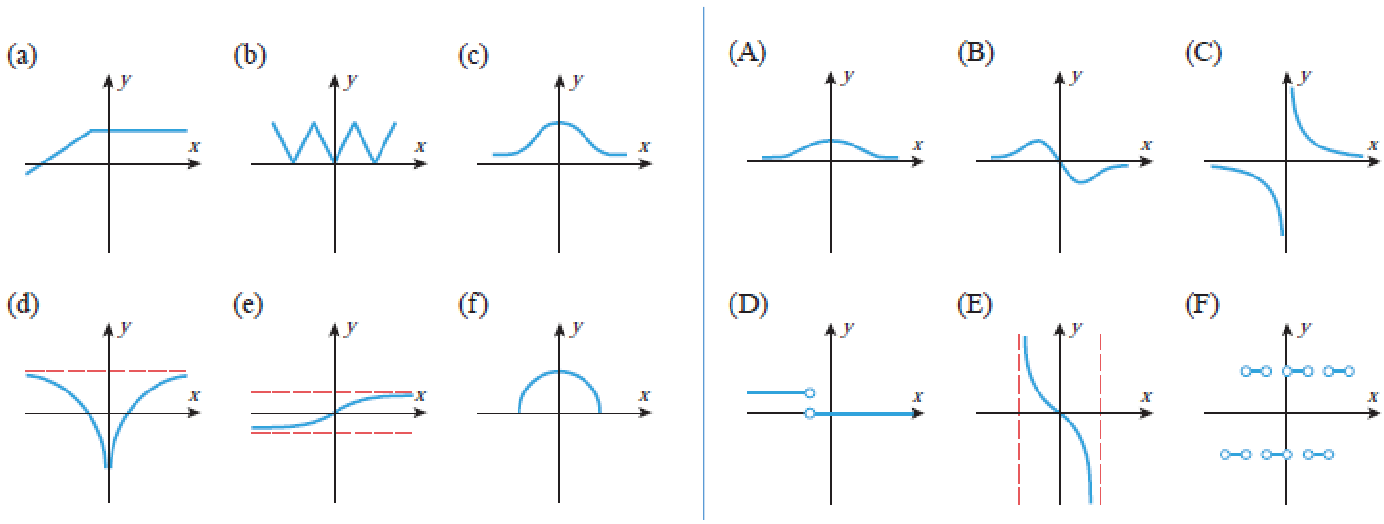
f(x) graphs:



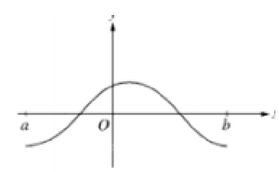
f'(x) graphs

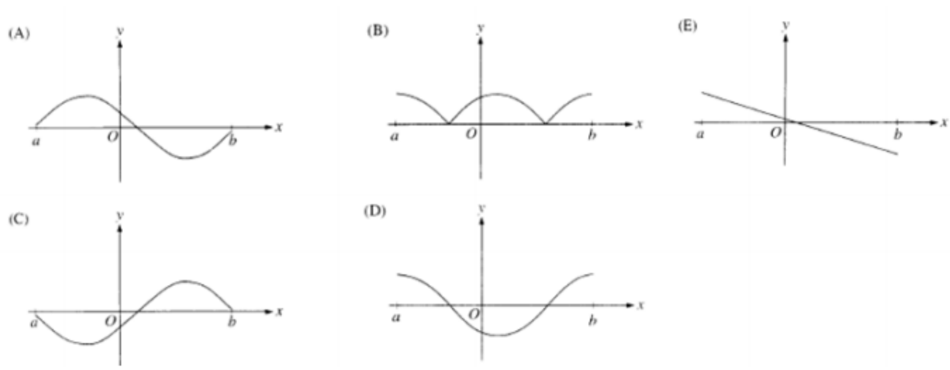


Example 2: Match the graph of the functions shown in a-f with the graphs of their derivatives in A-F

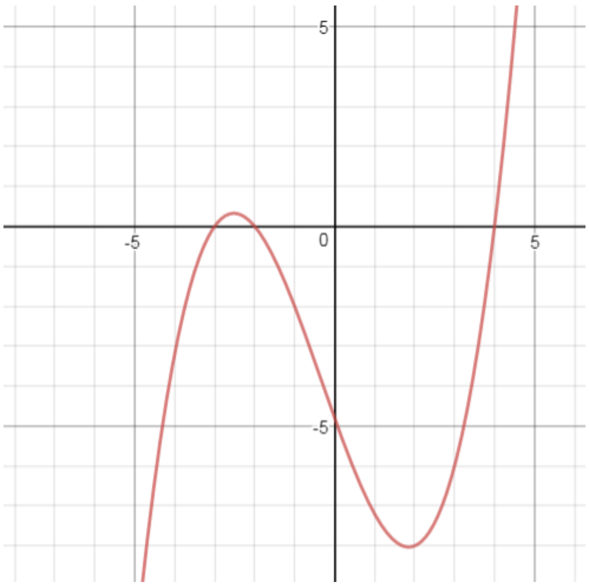


Example 3: The graph of *f* is shown in the figure to the right. Which of the graphs below could be the graph of the derivative of *f*?





Example 4: Given the following graph of f(x), sketch the graph of f'(x)



https://www.desmos.com/calculator/rmzuqwiyho

LOCAL LINEARITY – we say a function is LOCALLY LINEAR at $x = a$ if the graph looks more and more like a straight line as we zoom in on the point $(a, f(a))$.

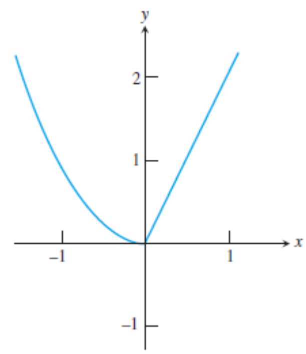
If $f'(a)$ exists, then f is locally linear at $x = a$.
If f is NOT locally linear at $x = a$, the $f'(a)$ does NOT exist. https://bit.ly/2MokIXq https://www.desmos.com/calculator/5t8cecojeu

According to Theorem 3 in section 2.1, we can conclude that a function has a TWO SIDED Derivative at a point if and only if the function's right hand and left hand derivatives are both defined and equal at that point.

- If we are dealing with a closed interval on a function that is differentiable at its endpoints, we have what are called ONE SIDED Derivatives at its endpoints

Example 5: Show that the following function has left-hand and right-hand derivatives at x = 0, but no derivative there.

$$y = \begin{cases} x^2, x \leq 0 \\ 2x, x > 0 \end{cases}$$



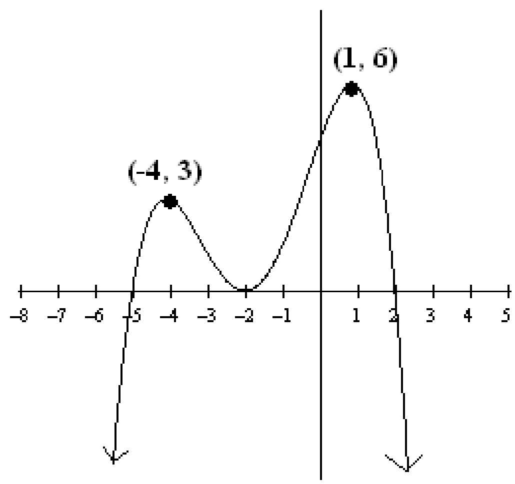
Example 6: Numerically, the value of the derivative at a point can be estimated by finding the slope of the secant line passing through two points on the graph on either side of the point for which the derivative is being estimated.

x	–3	0	1	4	6	10
***f*(*x*)**	2	1	–3	0	–7	2

***x* – Value**	**Estimation of Derivative**	**Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum**
0		
1		
4		
6		

Example 7: The graph of a function, *g*(*x*), is pictured to the right. Identify the following characteristics about the graph of the derivative, $g'(x)$. Give a reason for your answers.

The interval(s) where $g'(x) < 0$	
The interval(s) where $g'(x) > 0$	
The value(s) of *x* where $g'(x) = 0$	



3.1 Day 2 Assignment: Textbook P105 #13-16, 21, 22, 24, 25, 26, 31
Duo Tang Page 15 - 16(Ch 3) #1 - 9

VIDEO LINKS: a) https://bit.ly/2AYyMpt b) https://bit.ly/2vIoHac c) https://bit.ly/2LXo9YU

3.2: Differentiability

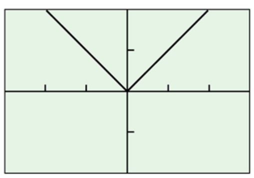
I CAN DETERMINE WHEN THE DERIVATIVE MIGHT FAIL TO EXIST.

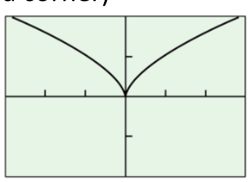
A function will not have a derivative at a point $P(a, f(a))$ where the slopes of the secant lines, $\frac{f(x)-f(a)}{x-a}$ fail to approach a limit as x approaches a.

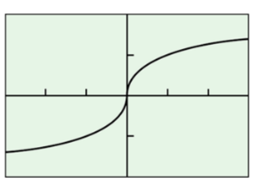
Differentiability Implies Continuity: If *f* is differentiable at *x=c*, then *f* is continuous at *x=c*.
(If a function is discontinuous at *x=c,* then it is nondifferentiable at *x=c.*)

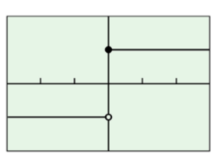
WARNING: Continuity DOES NOT guarantee differentiability.

The next figures illustrate four different instances where this occurs. For example, a function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has:

1. A corner where the one-sided derivatives differ.
   $f(x) = |x|$
   

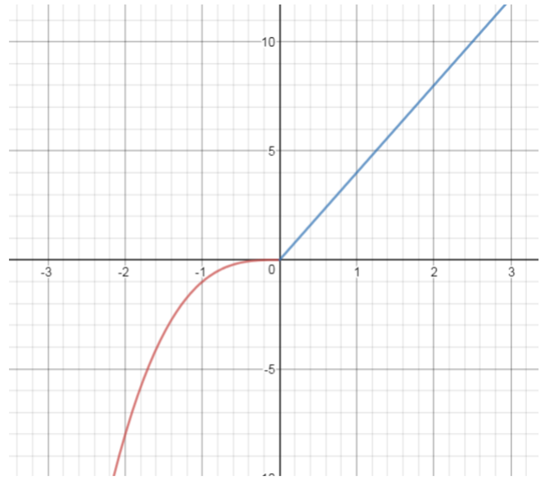
2. A cusp, where the slopes of the secant lines approach ∞ from one side and approach -∞ from the other (an extreme case of a corner)
   $f(x) = x^{\frac{2}{3}}$
   

3. A vertical tangent, where the slopes of the secant lines approach either ∞ or -∞ from both sides
   $f(x) = \sqrt[3]{x}$
   

4. A discontinuity (which will cause one or both of the one-sided derivatives to be nonexistent)
   $U(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$
   

Example 1: – Show that the function is not differentiable at x=0.

$$f(x)=\begin{cases} x^3, & x \le 0 \\ 4x, & x > 0 \end{cases}$$



Differentiability implies Local Linearity

Recall – this means a function that is differentiable at a closely resembles its own tangent line very close to x=a. (zoom in on your calculator they will look like they are right on top of each other)

NUMERICAL DERIVATIVE

Many graphing utilities can approximate derivatives numerically with good accuracy at most points of their domains. For small values of *h*, the difference quotient $\dfrac{f(a+h)-f(a)}{h}$ is often a good numerical approximation of f'(a). However the same value of *h* will usually yield a better approximation if we use the symmetric difference quotient $\dfrac{f(a+h)-f(a)}{2h}$. Our graphing calculations have a built in function to calculate NDER f(x), the Numerical Derivative of *f* at point *a*. Our textbook substitutes a value of h 0 0.001.

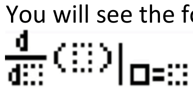
Example 2:

Find the numerical derivative of the function $f(x) = x^2 + 3$ at the point x = 2. Use a calculator with h=0.001.

STEPS:

CASE 1: Your Calculator is in the MATHPRINT MODE

1. [math], press 8 for nDeriv(
2. You will see the following screen



3. Fill in the boxes as follows:

$$\frac{d}{dx}\left(x^2+3\right)\Big|_{x=2}$$

4. [enter] to find the derivative

CASE 2: Your Calculator only has CLASSIC Mode

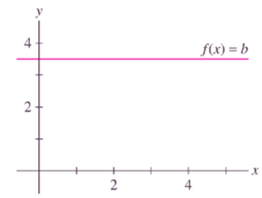
1. [math], press 8 for nDeriv(
2. Type in the following. Hit enter after you type "2"

The derivative is ____________.

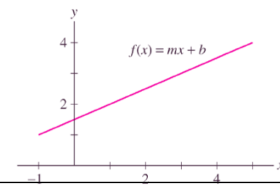
NOTE: You could also enter the function in the "y=" screen under y_1 or y_2 and then use that y function after $\frac{d}{dx}$

b) Calculate the actual derivative using the definition of the derivative.

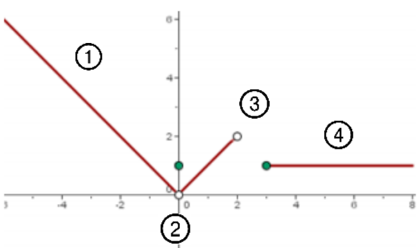
REMEMBER: If $f(x) = b$ is a constant function, then $f'(a) = 0$ for all a.

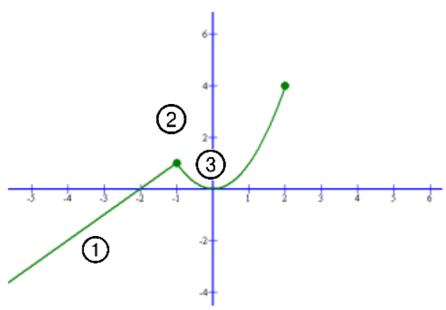


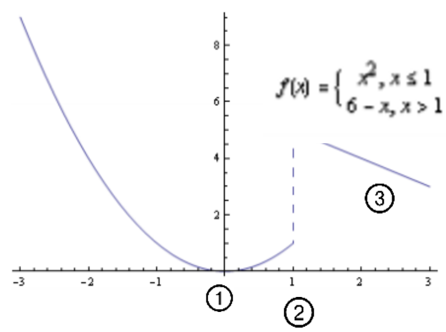
REMEMBER: If $f(x) = mx + b$ is a linear function, then $f'(a) = m$ for all a.



Example 3: Find the derivative graphically at the indicated points given f(x)







3.2 VIDEO LINKS: a) https://bit.ly/2M4I8UT b) https://bit.ly/2vKwkgd

3.2 Assignment: P114 #1-33 Odds, 41-45 Odds (Optional Twinning Activity by BP)

I CAN APPLY THE POWER AND SUM AND DIFFERENCE RULES TO FIND DERIVATIVES.

3.3 Day 1 VIDEO LINKS: a) https://goo.gl/FBSgsy c) https://goo.gl/rx6FpV

Do you see any patterns in the following questions between f(x) and the FINAL answer for f'(x)? Can we use this pattern to jump right to the answer without doing any of the "limit" work in between?

If f(x) = $3x^2$, find f `(x)

$$f'(x)=\lim_{h\to 0}\frac{\left[3(x+h)^2\right]-\left[3x^2\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{\left[3x^2+6xh+3h^2\right]-\left[3x^2\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{\left[3x^2+6xh+3h^2\right]-\left[3x^2\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{\left[3x^2+6xh+3h^2\right]-\left[3x^2\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{3h^2+6xh}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{h(3h+6x)}{h}$$
$$f'(x)=\lim_{h\to 0}3h+6x$$
$$f`(x)=6x$$

If f(x) = $5x^3$, find f `(x)

$$f'(x)=\lim_{h\to 0}\frac{\left[5(x+h)^3\right]-\left[5x^3\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{\left[5x^3+15x^2h+15xh^2+5h^3\right]-\left[5x^3\right]}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{15x^2h+15xh^2+5h^3}{h}$$
$$f'(x)=\lim_{h\to 0}\frac{h(15x^2+15xh+5h^2)}{h}$$
$$f'(x)=\lim_{h\to 0}15x^2+15xh+5h^2$$
$$f'(x)=15x^2$$

- So far, we have been using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find the slope of the tangent line of the curve $y=f(x)$ at the general point $(x, f(x))$, also called the derivative, or $f'(x)$ or $\frac{dy}{dx}$.
- This takes a lot of time and mistakes can be easily made if you are not careful. There is an easier way!!

DERIVATIVES: Do you notice the pattern in the following examples?

a) $s(t)=3t$
$s`(t)=3$

b) $f(x)=7x^2$
$f`(x)=14x$

c) $f(x)=-9x^5$
$f`(x)=-45x^4$

d) $f(x)=\frac{10}{x}$
$f(x)=10x^{-1}$
$f`(x)=-10x^{-2}$

THE POWER RULE (part 1):

- If $f(x)=x^n$, where n is a real number, then $f'(x)=nx^{n-1}$
- In Leibniz notation we say that $\frac{d}{dx}x^n=nx^{n-1}$.

THE POWER RULE (part 2):

- If $f(x)=cx^n$, where c is a constant and n is a real number, then $f'(x)=(c)(n)x^{n-1}$
- In Leibniz notation we say that $\frac{d}{dx}\left[cf(x)\right]=c\frac{d}{dx}f(x)$

Example 1: Find $f`(x)$ or $\frac{dy}{dx}$ of the following functions: (NOTE: Sometimes you will be asked to give answers without any negative or rational exponents. Sometimes you can leave negative and/or negative exponents. The AP Exam could give multiple choice answers in a variety of forms – be able to change between these forms!)

a) $f(x) = x^{15}$ b) $f(x) = \frac{1}{x^3}$ c) $f(x) = \sqrt{x}$ d) $f(x) = \frac{1}{\sqrt[3]{x^2}}$

NOTE: At this point we often negative exponents in the answers. We will sometimes leave radicals in the denominator and not rationalize the denominator.

Example 2: Find $f`(x)$ or $\frac{dy}{dx}$ of the following functions:

a) $f(x) = -4x^{15}$ b) $f(x) = (5x^4)^3$ c) $f(x) = \sqrt{7x}$

d) $f(x) = \sqrt[3]{\frac{4}{x^2}}$ e) $y = 3x^3\sqrt[3]{x}$

THE CONSTANT RULE: If $f(x) = c$, where c is a constant (#), then $f'(x) = 0$

Example 3: If $f(x) = -5$, determine $f'(x)$.

Example 4: Find the equation of the tangent line to the curve $y = x^5$ at the point $(2,32)$.

THE SUM/DIFFERENCE RULE:

If $f(x)$ is the sum of 2 differentiable functions $f(x) = g(x) \pm h(x)$) then $f'(x) = g'(x) \pm h'(x)$

Example 5: Find $f`(x)$ or $\frac{dy}{dx}$ of the following functions:

a) $f(x) = 2x^3 + 7x^6 + 7e$

b) $f(x) = (4x-3)^2$

c) $f(x) = \frac{\pi x^6}{2} + x - \frac{3}{x}$

d) $f(x) = \frac{(3x-5)(3x+5)}{x^5}$

e) $f(x) = \sqrt{\frac{x}{3}} - \frac{2}{x} + 6$

Question: Can you graph a function and then graph its derivative WITHOUT actually calculating the derivative function?

Example 6: At what point(s) on the curve $y=-x^2+3x+4$ does the tangent line have a slope of 5?

3.3 Day 1 Assignment: Duo Tang Page 16 - 18 (Ch 3) Part A: 1-5 odds, 7, 8, 11 Part B: 1- 5

3.3 Day 2: The Product Rule

I CAN APPLY THE PRODUCT RULE TO FIND DERIVATIVES.

VIDEO LINKS: a) https://bit.ly/2B1xv0T(Start at time 16:30) b) https://bit.ly/2OhiIko

THE PRODUCT RULE:

- When you are taking the derivative of the product of two expressions, the derivative will be

$$[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x)$$

In other words, the derivative of the product of two expressions will be:

(First expression)(Derivative of second expression) + (second expression)(derivative of first expression)

NOTE: It's very important to realize that the derivative of a product DOES NOT equal the product of the derivatives

$$[f(x)g(x)]' \neq f'(x)g'(x)$$

Example 1: Find the derivative, $\frac{dy}{dx}$ if $y=(2x^3+7)(3x^2-x)$. Use the product law.

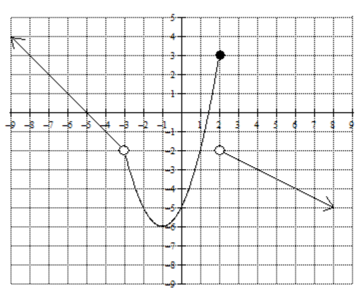
Example 2: Differentiate $f(x)=\sqrt{x}(2-3x)$ using the product law and simplify. Express your answer using a rationalized common denominator.

Example 3: Use the product law to find the equation of the tangent line to the graph of $f(x) = x^2(3x^2+2)(2x^3-1)$ when $x = 1$.

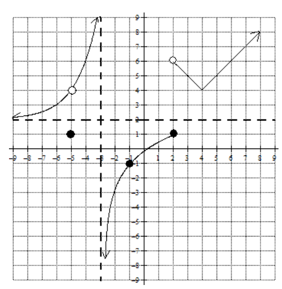
Example 4: Find the slope of $y = \left(4\sqrt{x} + \frac{2}{x^2}\right)\left(\sqrt[3]{x} - x^3\right)$ at the point x = 1

Example 5: Below are graphs of two functions—*f*(*x*) and *g*(*x*). Let $P(x) = f(x) \cdot g(x)$ and let $R(x) = x^2 \cdot g(x)$. Use the graphs to answer the questions that follow.

Graph of *f*(*x*)



Graph of *g*(*x*)



A) If $g'(-4) = 2$, what is the value of $P'(-4)$?

B) If $R'(-2)$ = 20, what is the value of $g'(-2)$?

c) Find the equation of the line tangent to the graph of *P*(*x*) when *x* = –4.

Example 5: Let $f(x)$ and $g(x)$ be differentiable functions such that the following values are true.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
4	1	7	2	−3
3	−2	−3	−4	2
−1	2	−2	1	−1

a) Estimate the value of $f'(3.5)$.

B) If $q(x)=2f(x)-4g(x)$, what is the value of $q'(4)$?

c) If $p(x)=-2f(x)g(x)$, what is the value of $p'(3)$?

D) Find the equation of the line tangent to the graph of $v(x)=x^3\cdot f(x)$ when $x=-1$.

E) If $k(x)=(2f(x)+3)(3-g(x))$, what is the value of $k'(3)$?

3.3 Day 2 Assignment: Duo Tang P 18-19 (Ch 3) Part A: 1-20 EVEN, 21-24 Part B: a, b

3.3 Day 3: The QUOTIENT Rule

I CAN APPLY THE QUOTIENT RULE TO FIND DERIVATIVES.

VIDEO LINKS: a) https://bit.ly/2M0393K b) https://bit.ly/2M8EE3x

THE QUOTIENT RULE:

- Given a function in the form of a quotient, $F(x)=\dfrac{f(x)}{g(x)}$, then $F'(x)=\dfrac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}$.

  (Note that we are using a capital F(x) for the quotient function)

- In other words, the derivative of the product of two expressions will be:
  [(bottom)(derivative of top) – (top)(derivative of bottom)] divided by (bottom squared)
- It is customary NOT to expand the expression in the denominator when applying the quotient rule
  Low d'High Minus High d 'Low, all over the square of what's below……..

Example 1: Differentiate $F(x)=\dfrac{x^2+2x-3}{x^3-1}$.

Example 2: Find $\dfrac{dy}{dx}$ if $y=\dfrac{\sqrt{x}}{1+2x}$.

Example 3: Find the coordinates of two points on the graph of the function $f(x)=\dfrac{10x}{x^2+1}$ at which the tangent line is horizontal.

Example 5: Let $f(x)$ and $g(x)$ be differentiable functions such that the following values are true.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	2	–1	9	–1
3	–5	–3	–4	6
4	1	7	8	–2

a) Estimate the value of $g'(2.5)$.

B) If $p(x)=\dfrac{g(x)}{f(x)}$, what is the value of $p'(4)$? What does this value say about the graph of $p(x)$ when $x = 4$? Give a reason for your answer.

If $q(x)=2x^2\left(\dfrac{f(x)}{g(x)}\right)$, what is the value of $q'(2)$?

3.3 Day 3 Assignment: Duo Tang P 20 (Ch 3) Part A: 1-14 Even, 17-19 Part B: a & b

AP Calculus

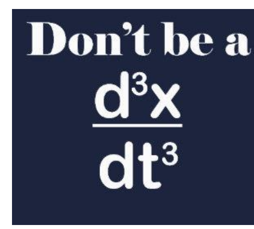
3.3 Day 4: Higher Order Derivatives

I CAN COMPUTE THE 2nd, 3rd and 4th DERIVATIVES.

VIDEO LINKS: a) https://bit.ly/2KArjvW b) https://bit.ly/2Mg5uqN

Higher Order Derivatives:

- We can take the derivative of a derivative function, and the derivative of that function and so on.
- A first derivative is written as $f'(x)$ or $\frac{dy}{dx}$
  - A first derivative represents the slope of a tangent line or rate of change (how the slope of the original function changes). A common example of the first derivative is that velocity is a first derivative of a distance function.
- A second derivative is written as $f''(x)$ or $\frac{d^2y}{dx^2}$
  - A second derivative measures how fast the first derivative function (often velocity) is changing, specifically how the rate of change/slope of the tangent line of the original function changes. A common example of the second derivative is acceleration in that acceleration is the second derivative of a distance function (but the first derivative of a velocity function)
- A third derivative is written as $f'''(x)$ or $\frac{d^3y}{dx^3}$.
  - An example of a third derivative measures how fast acceleration is changing with respect to time. In physics this can also be known as jerk/jolt/surge or lurch.



- If a distance formula y = s(t), then
  y' = v(t) and
  y'' = a(t).
  - If, however, the initial function y = v(t) then it's first derivative y' = a(t)

Example 1: Find $\frac{d^2y}{dx^2}$ if $y = x^6$

Example 2: Find the second derivative of $f(x) = 5x^2 + \sqrt{x}$

Example 3: For the function: $f(x) = x^6 + 5x^4 - 3x^3 + x$. Find:

a) $f'(x)$

b) $f''(x)$

c) $f'''(x)$

d) $f^4(x)$

e) $f^5(x)$

Example 4: Some ice cubes were added to a cup of boiling water. The temperature of the water in degrees Celsius, t minutes after the ice cubes were added, can be approximated by the function: $T(t) = \frac{20t^2+100t+200}{t^2+t+2}$
https://www.desmos.com/calculator/gzqa4tgeps

(a) Find T(0), T(1), and T(5).

(b) Find $T'(t)$

(c) Find $T'(1) and\ T''(5)$.

(d) ind $\lim_{t\to\infty} \frac{20t^2+100t+200}{t^2+t+2}$

3.3 Day 4 Assignment: Textbook P 124 #24, 25, 30, 32-36, 39

3.4: Velocity and Other Rates of Change

I CAN UNDERSTAND RATES OF CHANGE INCLUDING VELOCITY

VIDEO LINKS: a) https://bit.ly/2M3osBi and https://bit.ly/2OOYcIZ
b) https://bit.ly/2MadrO5 and https://bit.ly/2MsUJ0Q

Definition: Instantaneous Rate of Change
The instantaneous rate of change of *f* with respect to *x* at *a* is the derivative:

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, provided the limit exists.

RECALL: What is AVERAGE rate of change? How do you find it?

Example 1: (Enlarging circles) https://www.youtube.com/watch?v=T9QwiBFN9gI

a) Find the rate of change of the area A of a circle with respect to its radius r.

b) Evaluate the rate of change of A at r=5cm and at r=10cm.

Motion along a line

If an object is moving along an axis, we may know its position *s,* on that line as a function of time t: s(t).

Definition: Instantaneous Velocity

The instantaneous velocity is the derivative of the position function s(t) with respect to time. At time *t* the velocity is:

$$v(t) = s'(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

Definition: Speed

Speed is the absolute value of velocity, positive or negative direction is not important.

$$Speed = |v(t)| = \left|\frac{ds}{dt}\right| = |s'(t)|$$

Definition: Instantaneous Acceleration

Acceleration is the derivative of velocity with respect to time. If a body's velocity at time t is $v(t) = \frac{ds}{dt}$, then the body's acceleration at time t is:

$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ or $a(t) = v'(t) = s''(t)$

Free-fall Constants (earth) – review chapter 1

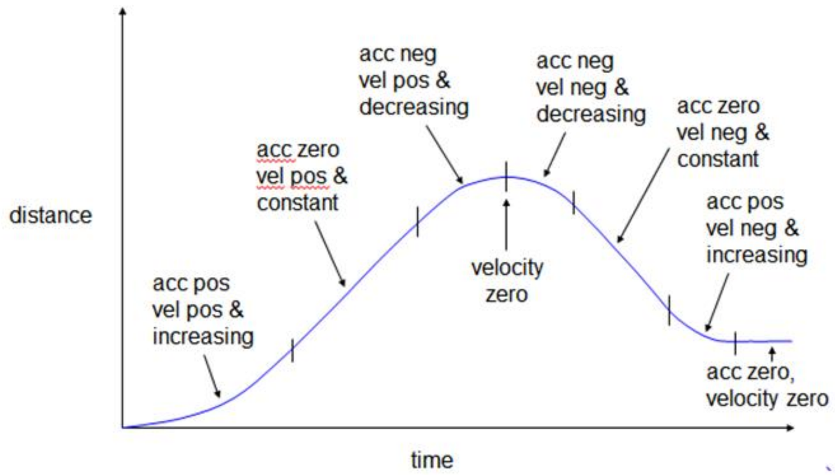
English units: $g = 32 ft/\sec^2$ $s = \frac{1}{2}(32)t^2 = 16t^2$

Metric units: $g = 9.8\ m/sec^2$ $s = \frac{1}{2}(9.8)t^2 = 4.9t^2$

In summary, let's correlate the concepts of position, velocity, and acceleration to what we already know about a function and its first and second derivative.

	corresponds with	
	corresponds with	
	corresponds with	

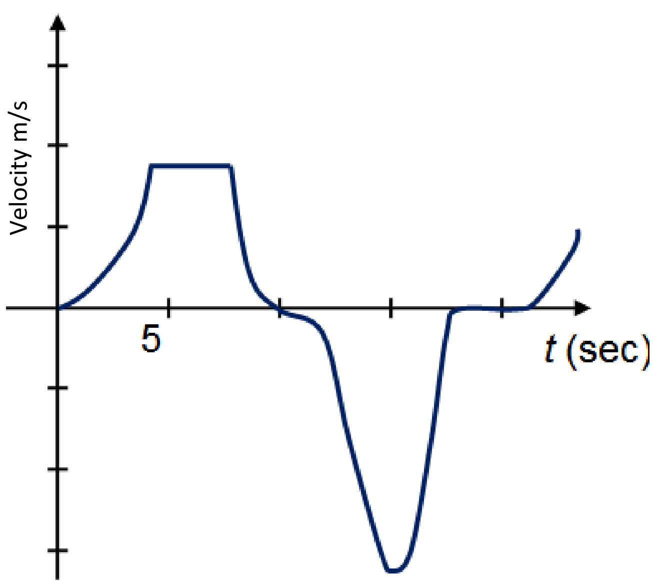
Reading a distance graph



Reading a velocity graph

A particle moves along an axis, and its velocity is shown in the graph below. Describe the motion, and state when speed is at a maximum.

v (m/sec)

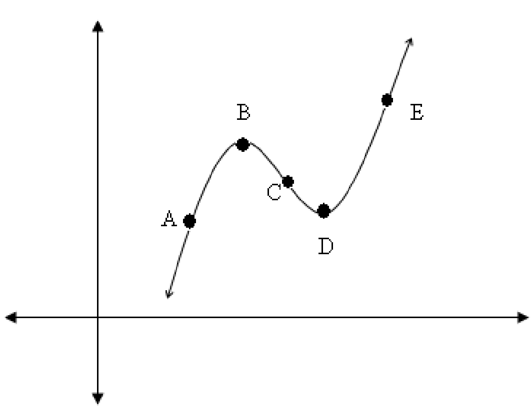


Let's summarize our relationships between position, velocity and acceleration below.

Velocity	***Position (Motion of the Particle)***
Is = 0 or is undefined	
Is > 0	
Is < 0	
Changes from positive to negative	
Changes from negative to positive	

Acceleration	***Velocity***
Is = 0 or is undefined	
Is > 0	
Is < 0	
Changes from positive to negative	
Changes from negative to positive	

The graph below represents the position, $s(t)$, of a particle which is moving along the x axis. Answer the given questions. Please note that later in the course you will always be expected to JUSTIFY these type of answers!



- At which point(s) is the velocity equal to zero?
- At which point(s) does the acceleration equal zero?
- On what interval(s) is the particle's velocity positive?
- On what interval(s) is the particle's velocity negative?
- On what interval(s) is the particle's acceleration positive?
- On what interval(s) is the particle's acceleration negative?

Five Commandments of Horizontal Particle Motion

1. **If the velocity is positive, the object is moving to the right.**
2. **If the velocity is negative, the object is moving to the left.**
3. **The speed is increasing when the signs of velocity and acceleration are the same.**
4. **The speed is decreasing when the signs of velocity and acceleration are opposite.**
5. **If the velocity is equal to 0 but the acceleration is not equal to 0, the object is momentarily stopped and changing directions.**

Example 2: (Modeling vertical motion. Use CALCULUS to find your answers)
A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph) It reaches a height of $s(t) = 160t - 16t^2 ft$ after t seconds.

a) How high does the rock go?

b) What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

c) What is the acceleration of the rock at any time t during its flight?

d) When does the rock hit the ground?

Example 3: (A moving Particle)

A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 3t + 2$, where s is measured in meters and t is measured in seconds.

a) Find the displacement during the first 5 seconds

b) Find the average velocity during the first 5 seconds

c) Find the instantaneous velocity when t=4

d) Find the acceleration of the particle when t=4

e) At what values of t does the particle change direction?

f) Where is the particle when s is a minimum?

3.4 Assignment: Textbook P 135 #1, 8, 9, 10a, 13, 15, 16, 19, 24, 34 & Duo Tang P21 (Ch 3) #1 & 2

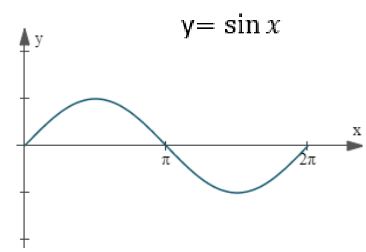
3.5: Derivatives of Trigonometric Functions

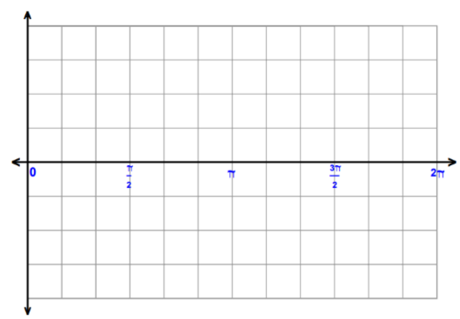
I CAN TAKE THE DERIVATIVE OF A TRIGONOMETRIC FUNCTION.

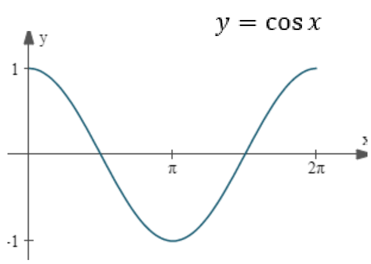
VIDEO LINKS: a) https://bit.ly/2Olzu1M and https://bit.ly/2Ok0Sgi
b) https://bit.ly/2KGIbRQ and https://bit.ly/2M2PwAY

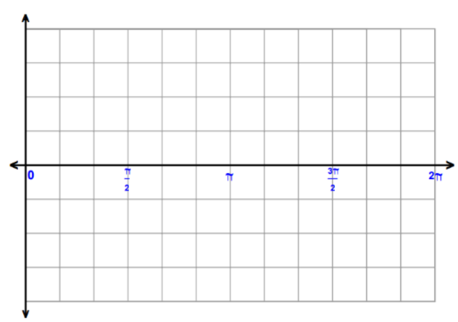
ACTIVITY:

1. Given the graph of y = sinx, draw points at all maximums, minimums and x intercepts. Write the coordinates (x, y) beside each point.
2. Using a different colour, draw the tangent line to each point. Estimate the slope of each point. On the side, write a list of a new set of coordinates that are of the form (x, slope)
3. On the graph of y = sinx, graph the set of ordered pairs from step 2. Join the points – this will be the graph of the derivative of y = sinx.
4. What does the graph of the derivative of y = sinx look like?
5. REPEAT THE PROCESS FOR THE GRAPH OF y = cosx









DERIVATIVES OF SINE AND COSINE (Valid only when the angle is measured in radians:

These need to be memorized!

If $y = \sin x$ then $\frac{dy}{dx} =$

If $y = \cos x$ then $\frac{dy}{dx} =$

Example 1: Find the derivative of $y = \frac{\sin x}{(\cos x - 2)}$

Example 2: Find the derivative of $y = x^2 \sin x$

Example 3: Find $y'' if\ y = \sec x$

Example 4:

An equation to the tangent line to the graph $y = x + \cos x$ at the point (0, 1) is:

a) $y = 2x + 1$ b) $y = x + 1$

b) c) $y = x$ d) $y = x - 1$

c) e) $y = 0$

YOU NEED TO COMMIT THESE TO MEMORY!

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

3.5 Assignment: P146 #1, 2, 4, 6, 9, 10, 21, 29, 35, 37, 46-48 & Duo Tang Page 22 (Ch3) #1 - 3

Ch 3 Review Assignment:
P105 #2, 7
P114 #13
P124 # 24b, 33
P 149 #1, 3, 4, 7, 32, 33, 34, 45, 46, 53, 57, 59, 61, 72, 81 and the following

Given $y=3x^2-7$, calculate y′ using the DEFINITION OF THE DERIVATIVE method.