6.1 Estimating with Finite Sums *(30L OUTCOME)*

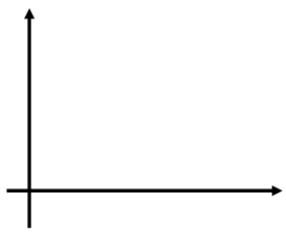
I CAN USE ESTIMATE THE AREA UNDER A CURVE WITH FINITE SUMS

VIDEO LINKS:

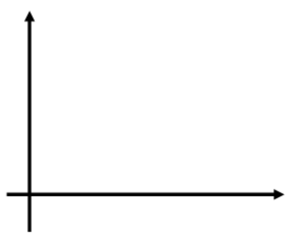
a) https://bit.ly/2T9tQCs b) https://bit.ly/2Ta91qE

EX #1:

a) A train moves along a track at a steady rate of 150 miles per hour from 2:00 A.M. to 5:00 A.M. What is the total distance traveled by the train?



a) What if the speed varies?



Review of course up to this beautiful moment in time ☺:

We have covered limits and their use in finding the slope of a tangent line to a curve. We applied this concept of a limit to define and find derivatives of several functions.

Now, we will work "backwards". If the derivative tells us the slope of a function, what would going backwards tell us? If we know that the given function IS the derivative of another function, what would the other function be compared to its derivative?

Derivative is about ________________________________ or __________________________

Anti-Derivative is about __________________________________ or __________________________________

RIEMANN SUM DESMOS (RRAM, LRAM, MRAM & TRAP) http://bit.ly/apcalcriemann or http://bit.ly/riemanncreate

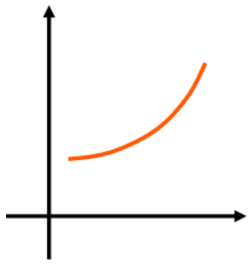
RIEMANN SUM Rectangular Approximation Method (RAM) – The method we use to find the area under a graph this when the graph is curved and not a straight line. There are three types: Right Ram (RRAM), Left Ram (LRAM) or Midpoint Ram (MRAM)

Assumptions for today:

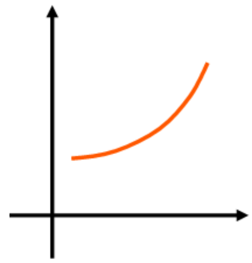
- *f(x)* is continuous
- *f(x)* is positive
- we are finding the area under the graph on the interval [a, b]

A. The function is INCREASING

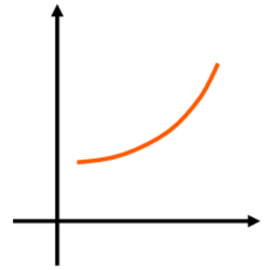
Using the Left-Hand Endpoint - **LRAM**



Using the Right-Hand Endpoint - **RRAM**

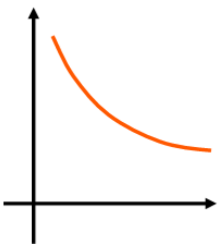


Using the MIDPOINT - **MRAM**

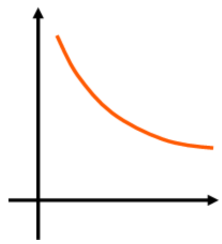


B. The function is DECREASING

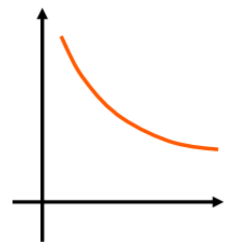
Using the Left-Hand Endpoint - **LRAM**



Using the Right-Hand Endpoint - **RRAM**

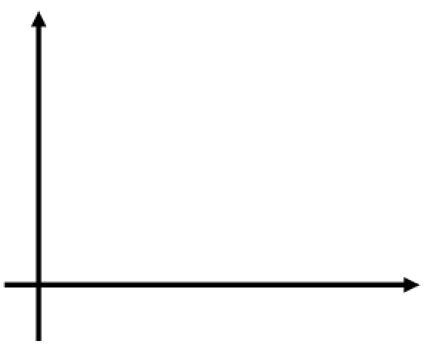


Using the MIDPOINT - **MRAM**

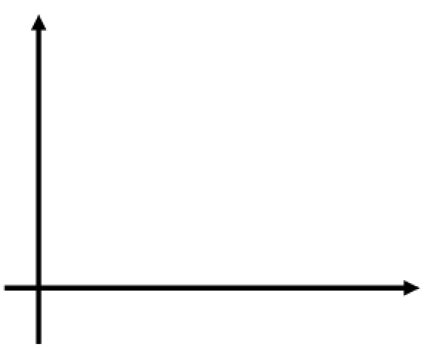


EX #2: A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 3$ for time $t \geq 0$. Approximate the area under the curve on [0, 4] using four rectangles of equal width and heights determined by

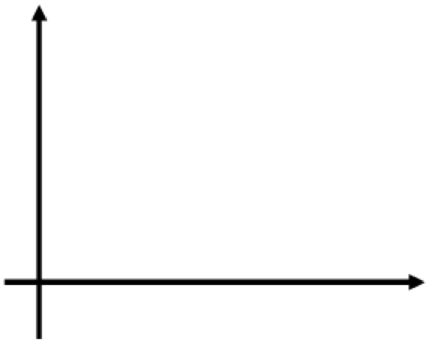
a) the left endpoints of the intervals (Left-Riemann Sum)



b) the right endpoints of the intervals (Right-Riemann Sum)



c) the midpoints of the intervals (Midpoint-Riemann Sum)



EX #3: Oil is leaking out of a tank. The rate of flow is measured every two hours for a 12-hour period, and the data is listed in the table below.

Time (hr)	0	2	4	6	8	10	12
Rate (gal/hr)	40	38	36	30	26	18	8

(a) Draw a possible graph for the data given in the table.

(b) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a left Riemann sum with three equal subintervals. Will this be an over or under approximation?

(c) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a right Riemann sum with three equal subintervals. Will this be an over or under approximation?

(d) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a midpoint Riemann sum with three equal subintervals. Will this be an over or under approximation?

EX #4: Consider problem #4 from the 2006 AP exam:

T (seconds)	0	10	20	30	40	50	60	70	80
v(t) (ft. per sec)	5	14	22	29	35	40	44	47	49

Using a midpoint Riemann sum with 3 subintervals of equal length, approximate the area of the curve of v(t) from 10 to 70. **Label your answer with correct units and explain what you just found in terms of this particular problem.**

EX #5: A hot cup of coffee is taken into a classroom and set on a desk to cool. When $t = 0$, the temperature of the coffee is 113° F. The rate at which the temperature of the coffee is dropping is modeled by a differentiable function R for $0 \le t \le 8$, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Values of $R(t)$ at selected values of time t are shown in the table below.

t (minutes)	0	3	5	8
$R(t)$ (° F/min.)	5.5	2.7	1.6	0.8

(a) Estimate the temperature of the coffee at $t = 8$ minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

(b) Estimate the temperature of the coffee at $t = 8$ minutes by using a right Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

6.1 Assignment TEXTBOOK P274 #3, 5, 6 (Answer is 1200 g), 15, 16, 17
DUO TANG PAGE 77 Questions 1-4 (30L)

6.5 The Trapezoidal Rule *(30L OUTCOME)*

I CAN CALCULATE AREAS UNDER THE CURVE USING THE TRAPEZOID RULE

VIDEO LINKS: a) https://bit.ly/2t2hrVN b) https://bit.ly/2RwqVlS

EX #1: An amusement park opens its doors at 8AM (t = 0). The rate that people enter the park is given below for various times during the day until the park closes at 10 PM (t = 14). Use the table for given values to answer the following questions.

Time (hours)	0	2	5	10	14
p ‘ (t) (1000’s ppl/hour)	7	11	8	4	0

a) Use a left Riemann Sum with 4 subintervals to approximate the total number of people that entered the park during the day.

b) Use a right Riemann Sum with 4 subintervals to approximate the total number of people that entered the park during the day.

c) Find the average of the answers from questions 1 and 2.

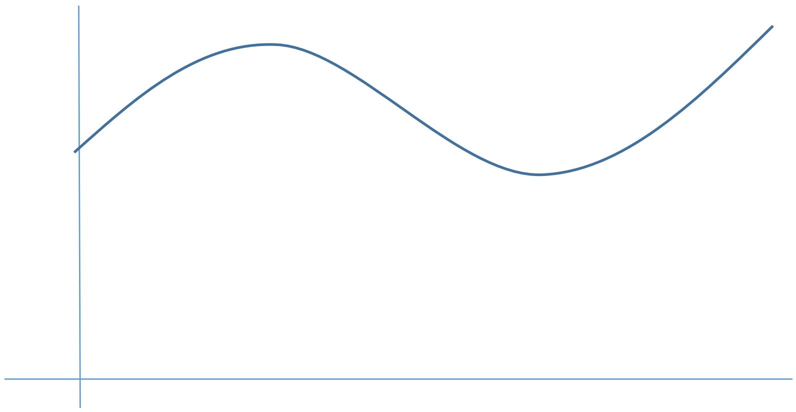
Trapezoidal Approximation

We used rectangles to approximate the area under a curve. However, this is not the only shape that we could have used to approximate the area. Rectangles are easy to calculate, which is what makes them convenient.

However, another shape that is relatively easy to use is the TRAPEZOID. Trapezoids are easy to use to find area and they are generally a more accurate approximation.

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Important note: When approximating area using trapezoids, the bases of the trapezoid are the vertical line segments (height of the function) and the height of the trapezoid is always on the x-axis. **http://bit.ly/apcalcriemann**



Ex #2: Given $f(x) = x^2 + 1$, use a trapezoidal approximation to find the area under the curve using 3 subintervals of equal length over the interval [0, 6].

Ex #3: Using the table from Example 1, approximate the total people entering the park using a trapezoidal approximation with 4 subintervals as given in the table.

Time (hours)	0	2	5	10	14
p ' (t) (1000's ppl/hour)	7	11	8	4	0

6.5 Part 1 Assignment: DUO TANG Page 78-79 #1 – 8

I CAN USE ESTIMATE THE AREA UNDER A CURVE WITH RIEMANN SUMS

VIDEO LINKS:

a) https://bit.ly/2AZy60y & https://bit.ly/2TceRbc b) https://bit.ly/2UaXcQX & https://bit.ly/2FU2QTP

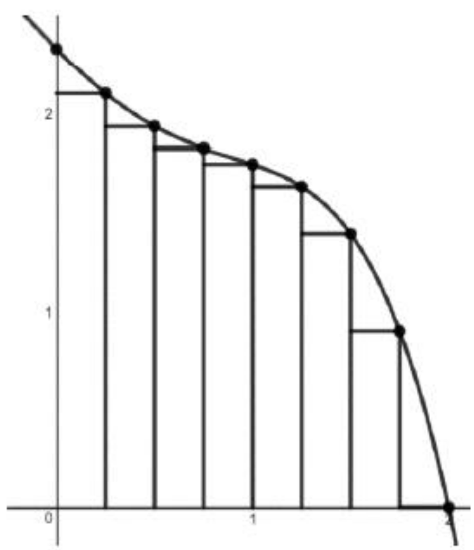
- LRAM, MRAM and RRAM are all Riemann (ree-mahn) sums and are used to estimate the area under a curve using rectangles. The formula we learned last day said that the sum of areas of those rectangles that approximates the formula can be given as:

$$S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$$

Definition of the Area of a Region in the Plane (Soon to be called the Definite Integral)

Let f(x) be continuous and non-negative (above the x axis) on [a, b]. The area of the region bounded by the graph of f(x), the x-axis and the vertical lines *x=a* and *x=b* is: $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ (This is basically just an infinite Riemann Sum)

To find the area of $f(x)$ on the interval $[a, b]$ using a right hand Riemann Sum.



Width of each rectangle: $\Delta x = \dfrac{b-a}{n}$

Each step on interval: $x_k = a + \Delta x \cdot k$($k$ is an integer $\in (1, n)$

Height of each rectangle: $f(x_k)$

Area of each rectangle: height·width $= f(x_k)\Delta x$

Approximation of area; sum of rectangles: $\sum_{k=1}^{n} f(x_k)\Delta x$

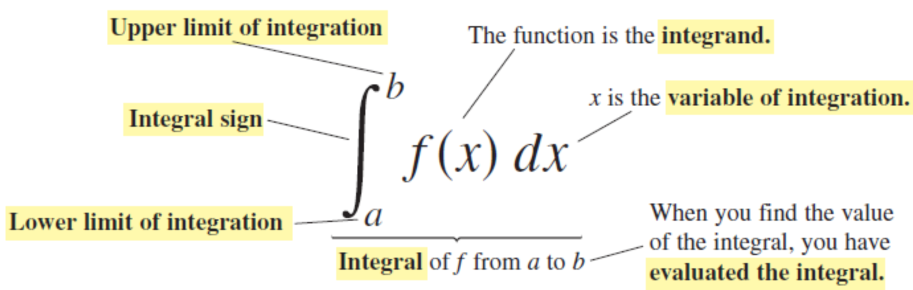
Actual area: $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)dx$

Note: You will NEED to know the components of the above info!!!

DEFINITE INTEGRAL & INTEGRAL NOTATION (Leibniz Notation)

DEFINITE INTEGRAL = $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n} f(x_k)\Delta x$ **=** $\int_a^b f(x)\,dx$

is read as "the integral from a to b of f of x dee x," or sometimes as "the integral from a to b of f of x with respect to x." The component parts also have names:



The value of the definite integral of a function over any particular interval depends on the function and not on the letter we choose to represent its independent variable. If we decide to use t or u instead of x, we simply write the integral as

$$\int_a^b f(t)\,dt \quad \text{or} \quad \int_a^b f(u)\,du \quad \text{instead of}$$

A Better Notation?

No matter how we represent the integral, it is the same *number*, defined as a limit of Riemann sums. Since it does not matter what letter we use to run from a to b, the variable of integration is called a **dummy variable.**

Informal Definition of Definite Integral

Suppose f is continuous for $a \le x \le b$. The definite integral from a to b is:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_{i-1})\Delta x \quad \text{OR} \quad \int_a^b f(x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$f(x)$ is called the integrand, a and b are called the limits of integration.

EX #1: Express the following sums as definite integrals. We will not be evaluating them yet, but if we could, what would that evaluation represent?

a) $\sum\limits_{k=1}^{n}(c_k)^3\,\Delta x$ on the interval [0, 10]

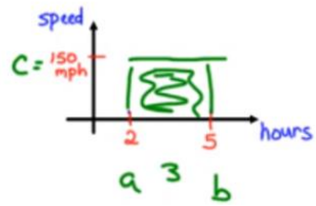
b) $\sum\limits_{k=1}^{n}(c_k)\sin(c_k)\Delta x$ on the interval [-3, 4]

EVALUATING INTEGRALS (Only learning how to evaluate SOME integrals for today…..)

1. Evaluating the Definite Integral of a Constant

This is like our first example in this unit: A train moves along a track at a steady rate of 150 miles per hour from 2:00 A.M. to 5:00 A.M. What is the total distance traveled by the train? Our answer represented the area under the curve (please note that we call all functions "curves" – even if they are linear) and an integral also represents the area under the curve.

- Therefore this question as an integral would look like $\int_2^5 150dx$.
- Since the answer to this question was the area under the rectangle

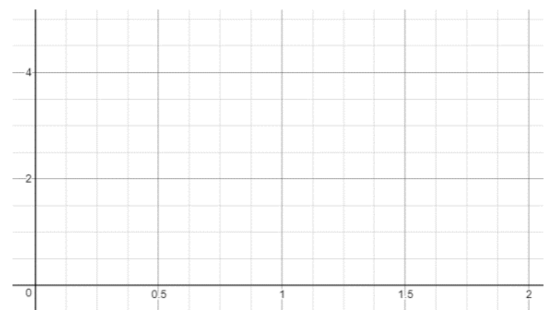


- We can say that area = width x height = c(b – a) = 150(5 - 2) = $\int_2^5 150dx$=450
- **In general, the integral of a constant will be:** $\int_a^b cdx = c(b-a)$

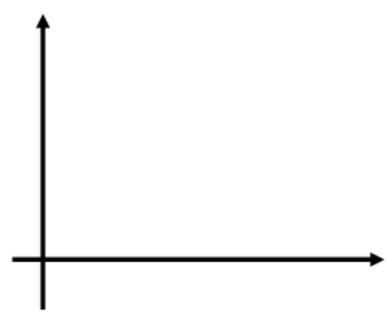
2. Evaluating the Definite Integral whose graph produces a common geometric shape

EX #2: Use your knowledge of area of geometric shapes to evaluate the following integrals. Sketch.

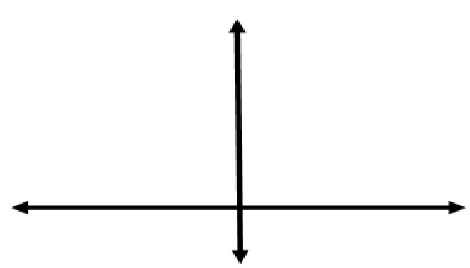
a) $\int_{1/2}^{3/2} (-2x+4)dx$



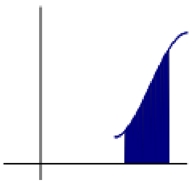
b) $\int_{\sqrt{2}}^{5\sqrt{2}} rdr$



c) $\int_{-2}^{2} \sqrt{4-x^2}dx$

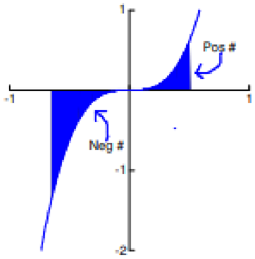


Up to this point, we have been looking a functions that were non-negative. This was because we were restricting our work with Riemann sums to finding area. We can integrate functions that go below the x-axis as long as we note:



When $f(x)$ is positive of [a, b], (where a<b) the definite integral represents the area between the x-axis, the function and the lines $x = a$ and

$$\int_a^b f(x)\,dx = shaded\ area$$



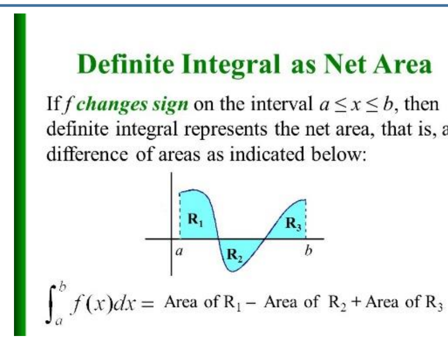
When $f(x)$ is positive for some x-values and negative for others (goes below the x-axis) on [a, b], (where a<b)

$\int_a^b f(x)\,dx$ is the sum of the area above the x-axis counted positively and the area below the x-axis counted negatively.

NET AREA:

- This is the area where negative area will "cancel out" positive area.
- It is the most common form of the integral that we will be calculating (when we learn to find the integral on a calculator, this will be the type of area that that it finds)

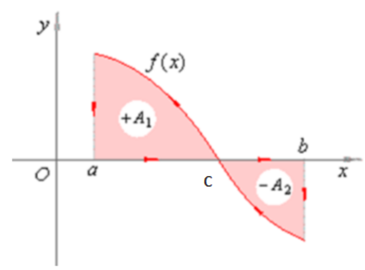
NET AREA= (area above the x axis) – (area below the x axis)



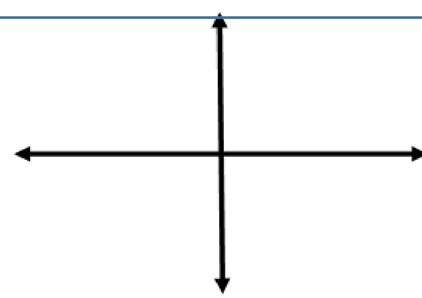
TOTAL AREA: (Not AS Commonly Used)

- This is the "total area space" combined that is between the curve and the x axis – in this situation we don't look at the area under the curve as negative, instead we make it positive and add it to the area above the x axis.
- **TOTAL AREA= (area above the x axis) + (area below the x axis)**

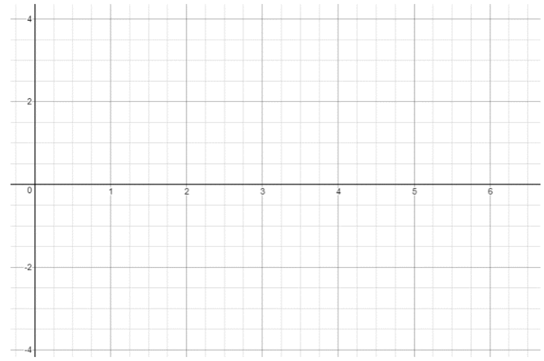
TOTAL AREA = $\int_a^b |f(x)|\,dx$



EX #3: Calculate the NET area for $\int_0^{2\pi} \sin(x)dx$. Sketch



EX #4: Calculate the NET and TOTAL area for $\int_0^6 (4-x)dx$. Sketch.

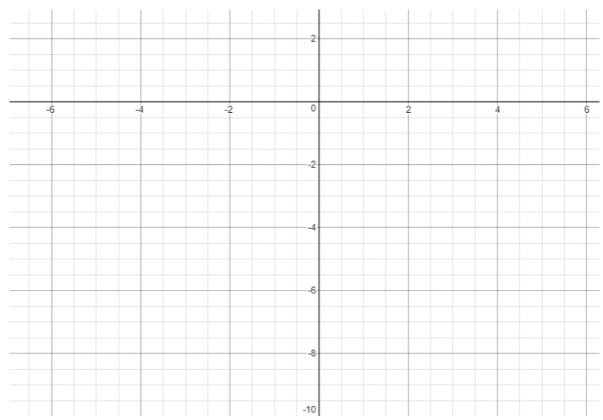


Continuity Implies Integrability

If a function is continuous on the closed interval [a, b], then it can be integrated on [a, b].

- From 2.1 we found that differentiability implied continuity
- Therefore, differentiability implies Integrability
- Beware : continuity does not imply differentiability and integrability does not imply continuity

EX #5: Calculate the NET area for the following $\int_{-5}^{6} \frac{9-x^2}{x-3}dx$. Sketch.



(Note: from now on, unless stated directly or implied in an application otherwise, integration will default to NET area)

EX #6: Use your graphing calculator to sketch the function contained within the following: $\int_{-1}^{1.5} x^2e^x dx$

https://www.desmos.com/calculator/9bw8ixvwgw

a) Do you expect the evaluation of this integral to be positive or negative?

b) Use Math 9 to calculate the integral. What does this answer represent?

EX #7: From the 2007 AP exam:

t (min)	0	2	5	7	11	12
r ‘ (t) (ft/min)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical balloon is expanding as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by the function r(t). The table above gives selected values of the rate of change, r ‘ (t), of the radius over the interval $0 \leq t \leq 12$.

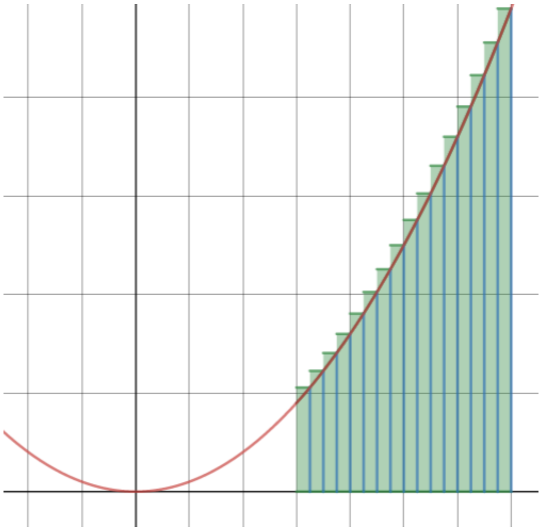
1. Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r\,'\,(t)dt$.

2. Using correct units, explain the meaning of $\int_0^{12} r\,'\,(t)dt$ in terms of the radius of the balloon.

EX #8: Note: Have your handout on How to write a Riemann Sum as a Limit handy!

Find a limit equal to $\int_3^7 x^2 dx$

To find the area of $f(x)$ on the interval $[a, b]$ using a right hand Riemann Sum.



Width of each rectangle: $\Delta x = \frac{b-a}{n}$

Each step on interval: $x_k = a + \Delta x \cdot k$ (k is an integer $\in (1, n)$)

Height of each rectangle: $f(x_k)$

Area of each rectangle: height·width $= f(x_k)\Delta x$

Approximation of area; sum of rectangles: $\sum_{k=1}^{n} f(x_k)\Delta x$

Actual area: $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)dx$

EX #9 : Find an integral expression equal to

a) $\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(\frac{10k}{n}\right)-5\right]\left(\frac{10}{n}\right)$

b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{\left(\frac{6k}{n}+3\right)+2}\right)\left(\frac{6}{n}\right)$

6.2 Assignment P286 #1—29 odd, CALC ACTIVE: 33-36, 41-46

AP Calculus

6.3 Day 1 Definite Integrals & Antiderivatives *(30 & 30L OUTCOME)*

I CAN USE PROPERTIES OF INTEGRALS AND THE MEAN VALUE THEOREM FOR INTEGRALS

VIDEO LINKS: a) https://bit.ly/2UybxaA b) https://bit.ly/2HHL9cL (up to the 22 minute mark)

Properties of Definite Integrals

1) If f is defined at $x=a$, then $\int_a^a f(x)\,dx = 0$

2) If f is integrable on [a, b], then $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

4) If f and g are integrable on [a, b] and k is a constant then the functions of $k\,f$ and $f \pm g$ are integrable on [a, b] and

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

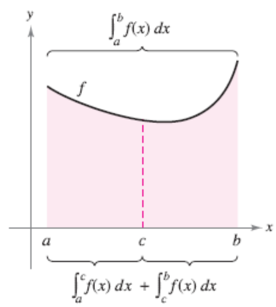
- **Explanation of Point 1** If f is defined at $x = a$, then $\int_a^a f(x)\,dx = 0$

- **Explanation of Point 2 above**

  In defining $\int_a^b f(x)$ as a limit of sums $\sum c_k\,\Delta x_k$, we moved from left to right across the interval $[a, b]$. What would happen if we integrated in the *opposite direction*? The integral would become $\int_b^a f(x)\,dx$—again a limit of sums of the form $\sum f(c_k)\Delta x_k$—but this time each of the Δx_k's would be negative as the x-values *decreased* from b to a. This would change the signs of all the terms in each Riemann sum, and ultimately the sign of the definite integral. This suggests the rule

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$$

- **Explanation of Point 3 above**



EX #1: Suppose

$$\int_{-1}^{1} f(x)\,dx = 5, \quad \int_{1}^{4} f(x)\,dx = -2, \quad \text{and} \quad \int_{-1}^{1} h(x)\,dx = 7.$$

Find each of the following integrals, if possible.

(a) $\int_4^1 f(x)\,dx$

(b) $\int_2^2 h(x)dx$

EX #2: Suppose

$$\int_{-1}^{1} f(x)\,dx = 5, \quad \int_{1}^{4} f(x)\,dx = -2, \quad \text{and} \quad \int_{-1}^{1} h(x)\,dx = 7.$$

Find each of the following integrals, if possible.

a) $\int_1^4 3f(x)dx$

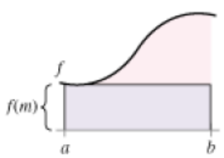
b) $\int_{-1}^1 [h(x) + f(x)]dx$

c) $\int_{-1}^{1} [2f(x) + 3h(x)]\,dx$

EX #3: Suppose $\int_{8}^{14} f(x)dx = -13$ and $\int_{11}^{14} f(x)dx = 9$, find $\int_{8}^{11} f(x)dx$

Mean Value Theorem for Integrals

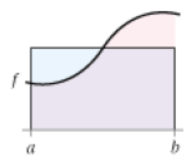
If f is continuous on the closed interval $[a,b]$, then there exists a number c in the closed interval $[a,b]$ such that $\int_{a}^{b} f(x)\,dx = f(c)(b-a)$.



Inscribed rectangle
(less than actual area)

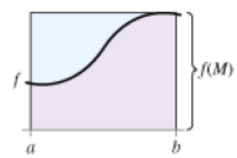
$\int_{a}^{b} f(m)\,dx = f(m)(b-a)$

$f(m)(b-a) < \int_{a}^{b} f(x)\,dx$



Mean Value Rectangle
(equal to actual area)

$\int_{a}^{b} f(x)\,dx = f(c)(b-a)$



Circumscribed Rectangle
(greater than actual area)

$\int_{a}^{b} f(M)\,dx = f(M)(b-a)$

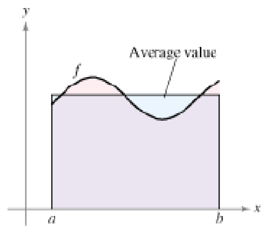
$f(M)(b-a) > \int_{a}^{b} f(x)\,dx$

- In other words, somewhere between the inscribed rectangle and the circumscribed rectangle, there is a rectangle whose area is precisely the area under the curve

EX #4: Show that $\int_{0}^{1} \sqrt{x+8}\,dx$ on [0, 1] lies between $2\sqrt{2}$ and 3

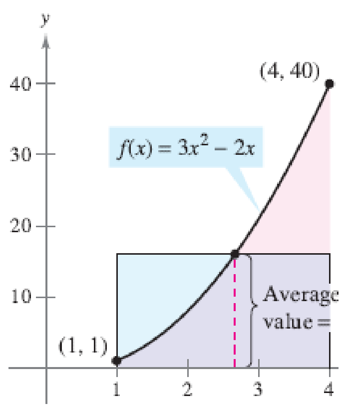
Average Value of a Function

If f is integrable on the closed interval $[a,b]$, then the **average value** of f on the interval is $\frac{1}{b-a}\int_a^b f(x)\,dx$.



- This is a result of the Mean Value Theorem of Integrals. If $\int_a^b f(x)\,dx = f(c)(b-a)$, then $f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$.
- $f(c)$ from the Mean Value Theorem is called the average value of f from $[a,b]$.

EX #4: At what point(s) in the interval [1, 4] does the function $f(x)=3x^2-2x$ assume it's average value? Use NINT (Math 9) to help you find your answer.



6.3 Day 1 Assignment P294 #1-7, 11-18

6.3 Day 2 Indefinite Integrals & Antiderivatives

I CAN FIND THE EQUATION OF A FUNCTION (by sight) GIVEN ITS DERIVATIVE, AND TO LEARN THE MEANING OF THE TERM ANTIDERIVATIVE

VIDEO LINKS: a) **https://bit.ly/2se4XKq** b) **https://bit.ly/2GXvzUu**

Imagine seeing the following question $f(x) =$ [redacted]

$$f'(x) = 2x$$

Can you work backwards to find the equation of f(x)?

- Kristin said the answer is $f(x)=x^2 + 5$, Ella said the answer is $f(x) = x^2 - 6$ and Anna said the answer is $f(x) = x^2$. Who is correct?

ANTIDERIVATIVES:

- If a function $F'(x)$ is known to be the derivative of another function $f(x)$, the **ANTIDERIVATIVE** of the function $F'(x)$ will perform the backwards operation of the derivative and will give you the function in its original state before the derivative was found.
- **If F is any function such that $F'(x) = f(x)$, then the general antiderivative of $f(x)$ will be $F(x) + C$, where C is an arbitrary constant.**

  Ex: Let $F'(x) = f(x) = 8x$, then the antiderivative will be $F(x) = 4x^2 + C$
- **The function F(x) is called the "general solution" of the differential equation (it is an equation that involves derivatives of a function)**

Ex #1: Find the antiderivative of f.

(Note: The function given is f(x) but it is also F'(x), the derivative of another function called F(x))

a) $f(x) = x^6$

b) $f(x) = \sqrt{x}$

c) $f(x) = x^3 - 6x^{-2}$

d) $f(x) = \frac{3}{x^5}$

e) f(x) = $3x^2 + 2$

f) $f(x) = 6x^{-\frac{1}{2}} + 10\cos 2x$

Ex #2: Find a function f(x) for which $f'(x) = e^{3x}$

Ex #3: Find a function f(x) for which $f'(x) = 2x^2 + x - 6$ and $f(6) = -4$.

INTEGRATION

- Up until today, we were finding DEFINITE INTEGRALS. Definite integrals find the area under a curve for an interval [a, b] $Area = \int_a^b f(x)dx$
- Today we are seeing INDEFINITE INTEGRALS for the first time. Indefinite integrals are an algebraic manipulation of a function to discover which original function has the new function as a derivative
- The process of finding the **ANTIDERIVATIVE** is called **INTEGRATION**
- The symbol for integration is $\int$, and it is the opposite of finding the derivative. Notice that there are no bounds of a or b in indefinite integrals.

  EX: Finding a derivative would look like $\frac{d}{dx}\left(x^4\right) = 4x^3$ While finding an integral would look like

  $\int 4x^3 dx = x^4 + C$
- The notation $\int 4x^3 dx$ means "What function, if differentiated with respect to *x*, has $4x^3$ as its derivative?"
  - The answer, x^4 + C is called an **INDEFINITE INTEGRAL (the antiderivative)**
  - The C is called the **CONSTANT OF INTEGRATION**
- The x^4 is called the **INTEGRAND**

Basic Integration Rules

<u>Differentiation Formula</u>	<u>Integration Formula</u>
$\frac{d}{dx}[C]=0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx]=k$	$\int k\,dx = k\,x + C$
$\frac{d}{dx}[k\,f(x)]=k\,f'(x)$	$\int k\,f(x)\,dx = k\int f(x)\,dx$
$\frac{d}{dx}[f(x)\pm g(x)]=f'(x)\pm g'(x)$	$\int [f(x)\pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C,\quad n\neq -1 \quad \textit{Power Rule}$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}[\cot x]=-\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\csc x]=-\csc x\cot x$	$\int \csc x\cot x\,dx = -\csc x + C$

Ex #3: Find the following integrals

a) $\int\left(x^4+x^{-3}-1\right)dx$

b) $\int\left(x^2+2\right)^2 dx$

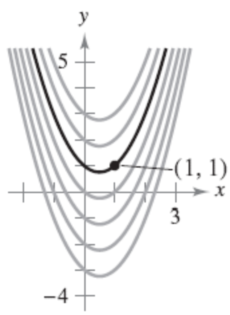
c) $\int \frac{3x^3 - 4x^2 + 5x - 2}{x^2}\, dx$

d) $\int \sqrt{x}\left(x^2 - 3\right) dx$

e) $\int \frac{\sin x}{1 - \sin^2 x} dx$

Ex #4: Find the equation of y given the derivative and the indicated point on the curve.

a) $\frac{dy}{dx} = 2x - 1$



b) $\frac{dy}{dx} = 2(x-1)$ (3, 2)

c) $\frac{dy}{dx} = -\frac{1}{x^2}$ (1, 3)

6.3 Day 2 Assignment: DUO TANG PAGE 83 #1-45 Odd

6.4 Day 1 Definite Integrals & Antiderivatives

I CAN FIND THE EQUATION OF A FUNCTION (by sight) GIVEN ITS DERIVATIVE, AND TO LEARN THE MEANING OF THE TERM ANTIDERIVATIVE

- What is the connection between antiderivatives we did yesterday using **INDEFINITE INTEGRALS** and the work we did previously when we found the answer to **DEFINITE INTEGRALS** by finding the area?
- We will not always be able to use geometry (or have access to our graphing calculator/Math 9) to determine the area under a curve to find the numerical area answer to a DEFINITE INTEGRAL. We will sometimes need an equation – these equations will be the answers to the INDEFINITE INTEGRALS we found yesterday.

CALCULATING DEFINITE INTEGRALS USING ANTIDERIVATIVES:

If f(t) is a function with respect to x and F(x) is an ANTIDERIVATIVE of a function with respect to x, then,

$$\int f(t)dt = F(x) + C$$

This is an INDEFINITE INTEGRAL.

- The answer will be an equation of a function
- The F(x) equation on the right is the antiderivative of the equation f(t) under the integral sign. For the moment just ignore the fact that the variable has changed (it is just a dummy variable).

$$\int_a^x f(t)dt = F(x) - F(a)$$

or

$$\int_a^b f(t)dt = F(b) - F(a)$$

This is a DEFINITE INTEGRAL.

- When you have values for both the upper and lower limits of the integral, the answer will be numerical and will represent the area under the curve.
- The fact that we are using an x as the upper bound rather than a "b" is just a different notation.
- The F(x) equation that we use will be the same as we used for an indefinite integral but we don't need to include the C. Why? Answer after doing Ex #1.

Ex #1: Evaluate $\int_0^2 5x\,dx$ using the above method of DEFINITE INTEGRALS. Confirm your answer geometrically and using Math 9.

QUESTION: Could you evaluate this function by including the "C's" in F(x)?

THE FIRST FUNDAMENTAL THEOREM OF CALCULUS (FTC 1)

If a function *f(x)* is continuous on [a, b] and *F(x)* is an antiderivative of *f(x)* on the interval [a, b], then:

$$\int_a^b f(x)dx = F(b) - F(a)$$ or we can say that $$\int_a^b f(x)dx = \left. F(x) \right]_a^b = F(b) - F(a)$$

Ex #2: Evaluate the definite integral: $\int_2^5 (-3x+4)dx$

Ex #3: Evaluate the definite integral: $\int_{-1}^1 (x^3 - 9x)dx$

Ex #4: Evaluate the definite integral $\int_1^9 \frac{\sqrt{x}-1}{\sqrt{x}}dx$

Ex #5: Evaluate the definite integral: $\int_0^4 \left|x^2 - 4x + 3\right| dx$

Ex #6: Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$

https://www.desmos.com/calculator/otflvkyfty

Ex #7: Find the value of *c* guaranteed by the Mean Value Theorem for Integrals for the function *f(x)=cosx* over the interval $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$.

Ex #8: Pictured below is a table of values that shows the values of a function, *f*(*x*), and its first and second derivative for selected values of *x*. Use the information in the table to answer the questions that follow.

a) What is the value of $\int_{-3}^{1} f'(x)dx$.

x	−3	−1	1	3	5
$f(x)$	4	0	−2	1	3
$f'(x)$	−2	1	0	3	2
$f''(x)$	−1	0	2	−3	−1

b) What is the value of $\int_{-1}^{3} f'(x) + f''(x)dx$?

x	-3	-1	1	3	5
$f(x)$	4	0	-2	1	3
$f'(x)$	-2	1	0	3	2
$f''(x)$	-1	0	2	-3	-1

C) What is the value of $\int_{1}^{5} 3f''(x)dx$?

d) What is the value of $\int_{-3}^{3} \frac{1}{2}f'(x)+2f''(x)dx$?

e) What is the equation of the tangent line to the graph of $f(x)$ at $x = 3$?

f) Use the equation of the tangent line in #5 to approximate the value of $f(3.1)$. Is this an over or under approximation of $f(3.1)$? Give a reason for your answer.

6.4 Day 1 Assignment Duo Tang Page 84 #5-23 Odd, 27, 31, 45, 46

I CAN USE THE FUNDAMENTAL THEOREM OF CALCULUS

Ex #1: Find $\frac{dy}{dx}$ if $y=\int_{-\pi}^{x}(\cos t)dt$

THE SECOND FUNDAMENTAL THEOREM OF CALCULUS (FTC 1)

If a function *f(x)* is continuous on an open interval *I* containing *a*, then for every x in the interval,

$$\frac{d}{dx}\left[\int_{a}^{x} f(t)dt\right]=f(x)$$

In other words, where the integrand is continuous, the derivative of a definite integral with respect to its upper limit is equal to the integrand evaluated at the upper limit (please note that the lower limit must be a constant!)

Ex #2: Find $\frac{dy}{dx}$ $y=\int_{0}^{x}(t^2+t)dt$

Ex #3: Find $\frac{dy}{dx}$ $y=\int_{x}^{6}(5t\cos t)dt$

Ex #4: Use the Second Fundamental Theorem of Calculus to find *F'*(*x*)

a) $F(x)=\int_{1}^{x}\sqrt[4]{t}\,dt$

b) $F(x)=\int_{2}^{x^2}\frac{1}{t^3}\,dt$

RULE: When the bottom value of the integral is a number and the top is another function other than x, such as $y=\int_{a}^{g(x)}f(t)dt$,

Up here is a function other than just x

Then $\frac{dy}{dx}=\left[f(g(x))\right]\left[g'(x)\right]$.

In common language it means we evaluate f(t) by plugging in g(x) and then we multiply that by $g'(x)$

REDO b) from above using this shortcut:

c) $F(x)=\int_{0}^{x^2}\sin\left(\theta^2\right)d\theta$

Ex #5: Find $\frac{dy}{dx}$ if $y=\int_{2}^{-2x}\frac{dt}{5-e^t}$

Ex #6: Find $\frac{dy}{dx}$ if $y = \int_{x^2}^{x^4} (\ln\sqrt{2t+3})dt$

Ex #7: Construct a function of the form $y = \int_a^x f(t)dt + C$ that satisfies the given conditions.

a) $\frac{dy}{dx} = \cos x$, and y = 0 when x = 2

b) $\frac{dy}{dx} = \ln x$, and y = 2 when x = 5

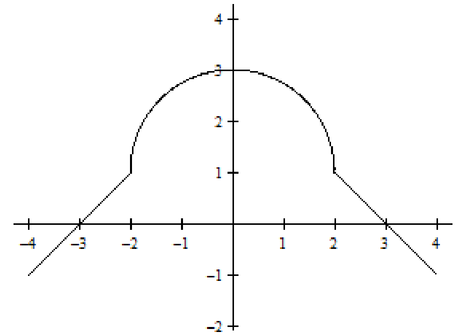
Ex #8: $f(x)=\dfrac{x^2+1}{x^2}$ $\left[\dfrac{1}{2},\ 2\right]$

a) Find the average value of the function over the interval.

b) Find all values of x in the interval for which the function equals its average value. **(Graph →)**

Ex #9: Find the average value of the function $y=\sec(x)\tan(x)$ on $\left[0,\dfrac{\pi}{3}\right]$ using antiderivatives. This question is to be done without technology – the graph has been included only as a visual learning tool for this example.
https://www.desmos.com/calculator/hg8pmjuoai

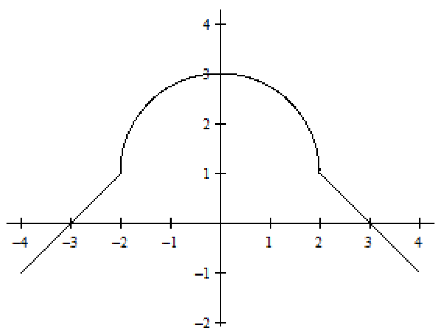
Ex #8: Pictured to the right is the graph of $g(t)$ and the function $f(x)$ is defined to be $f(x)=\int_{-4}^{2x} g(t)dt$.



1. Find the value of $f(0)$.

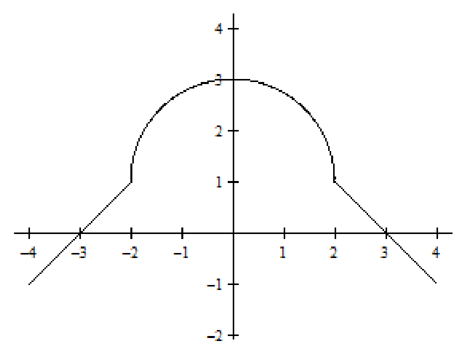
2. Find the value of $f(2)$.

3. Find the value of $f'(1)$.



4. Find the value of $f'(-2)$.

5. Find the value of $f''(\frac{3}{2})$.



6.4 Day 1 Assignment TEXTBOOK P294 11& 13 (CALC ACTIVE), 31, 35, 45-50
TEXTBOOK Page 306 #1-39 Odd
DUO TANG Page 85 #1 & AP Calc 2004 Question

AP Calculus

6.4 Day 3 APPLICATIONS OF INTEGRATION *(30 & 30L OUTCOME)*

I CAN USE THE FUNDAMENTAL THEOREM OF CALCULUS

VIDEO LINKS: a) https://bit.ly/2Si9LNi b) https://bit.ly/2DPMtWR

RECALL THE FOLLOWING:

NET AREA:

- This is the area where negative area will "cancel out" positive area.
- It is the most common form of the integral that we will be calculating. Math 9 automatically finds NET area

NET AREA= (area above the x axis) – (area below the x axis)= $\int_a^b f(x)dx$

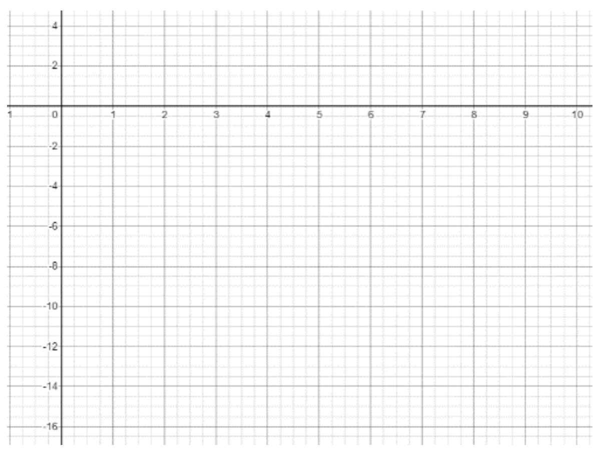
TOTAL AREA:

- This is the "total area space" combined that is between the curve and the x axis – in this situation we don't look at the area under the curve as negative, instead we make it positive and add it to the area above the x axis.
- **TOTAL AREA= (area above the x axis) + (area below the x axis) =** $\int_a^b |f(x)|dx$
- To find total area, you have to divide the graph into separate areas where the function is above the x axis and areas where it is below the x axis and integrate those separately and add them. Areas that are below the x axis require that you use the absolute value function on that integral.

Ex #1: Find the area of the region under the curve. Note: If the question asks and uses the word AREA, this means the TOTAL area. When it uses the word INTEGRAL, it means NET area.

$y = 8 - 4x,\ \ [1,6]$

https://www.desmos.com/calculator/rlrd6tdezk



Ex #2: Find the area under the curve $y = \frac{1}{x} - 1,\ 0.5 \le x \le 2$ https://www.desmos.com/calculator/tdrpfu7llb

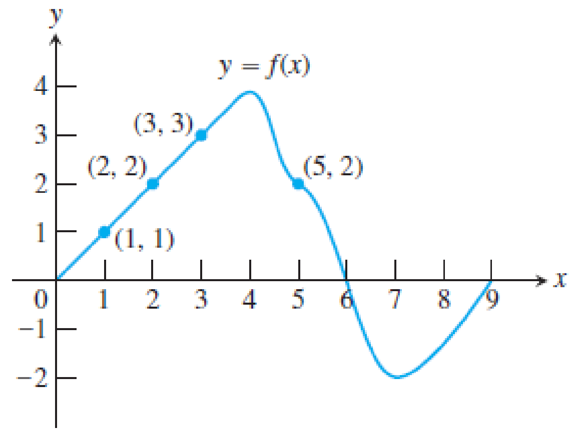
PHYSICAL APPLICATIONS: (SUPER IMPORTANT)
S = displacement/distance, v = velocity, a = acceleration

DISTANCE IS THE AREA UNDER THE VELOCITY CURVE: $s(t) = \int v(t)dt$

VELOCITY IS THE AREA UNDER THE ACCELERATION CURVE $v(t) = \int a(t)dt$

NOTE: You will particularly need to be able to "read between the lines" for AP questions in these situations. If for example they give you a situation such as $s = \int_0^t f(x)dx$, you would be expected to be able to recognize that f(x) is in fact the velocity function without it being explicitly stated.

Ex #3: The following function is a differentiable function whose position at time *t(sec)* is given by $s = \int_0^t f(x)dx$. Use the graph to answer the following questions and give reasons for your answers.



a) What is the particle's velocity at time t = 5?

b) Is the acceleration of the particle at time t = 5 positive or negative?

c) What is the particle's position at time t = 3?

d) At what time during the first 9 seconds does *s* have its largest value?

e) Approximately when is the acceleration zero?

f) When is the particle moving toward the origin? Away from the origin?

g) On which side of the origin does the particle lie at time t = 9?

Three Things to Always Keep In Mind:

(1) $\int_a^b v(t)dt = p(b) - p(a)$, where $v(t)$ represents the velocity and $p(t)$ represents the position.

(2) $\int_a^b v(t)dt =$ The Net Distance the particle travels on the interval from $t = a$ to $t =$ b. If $v(t) > 0$ on the interval (a, b), then it also represents the Total Distance.

(3) $\int_a^b |v(t)|dt =$ The Total Distance the particle travels on the interval (a, b), whether or not $v(t) > 0$. To be safe, always do this integral when asked to find total distance when given velocity.

Ex #3: The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 3t^2 + 6$ (NO CALC)

a. If the position of the particle at $t = 4$ is 72, what is the position when $t = 2$?

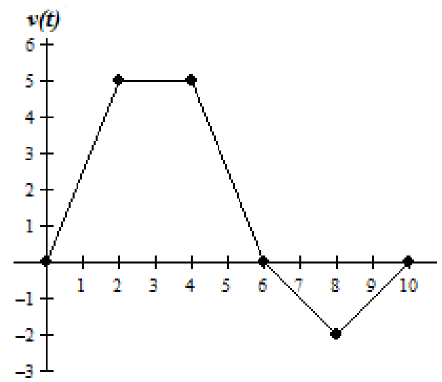
b. What is the total distance the particle travels on the interval $t = 0$ to $t = 7$?

Ex #4: The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 0.5e^t(t-2)^3$. (CALC ACTIVE)

a. If the position of the particle at t = 1.5 is 2.551, what is the position when t = 3.5?

b. What is the total distance that the object travels on the interval t = 1 to t = 5?

Ex #5: The graph of the velocity, measured in feet per second, of a particle moving along the x – axis is pictured below. The position, s(t), of the particle at t = 8 is 12. Use the graph of $v(t)$ to answer the questions that follow.



a. What is the position of the particle at t = 3?

b. What is the acceleration when t = 5?

c. What is the net distance the particle travels from t = 0 to t = 10?

d. What is the total distance the particle travels from t = 0 to t = 10?

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

Ex #6: The table above shows values of the velocity, $V(t)$ in meters per second, of a particle moving along the x – axis at selected values of time, t seconds.

a. What does the value of $\int_0^{18} V(t)dt$ represent?

b. Using a left Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

c. Using a right Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

d. Using a midpoint Riemann sum of 3 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

e. Using a trapezoidal sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

f. Find the average acceleration of the particle from t = 3 to t = 9. For what value of t, in the table, is this average acceleration approximately equal to $v'(t)$? Explain your reasoning.

6.4 Day 3 Assignment Duo Tang Page 86 & 87 #1-12 & AP Question

$$\frac{d}{dx}[\text{AMOUNT}] = \textbf{The rate at which that amount is changing}$$

For example, if water is being drained from a swimming pool and $R(t)$ represents the amount of water, measured in cubic feet, that is in a swimming pool at any given time, measured in hours, then $R'(t)$ would represent the rate at which the amount of water is changing.

$\frac{d}{dx}[R(t)] = R'(t)$ What would the units of $R'(t)$ be?___________________________

$$\int_a^b \text{RATE} = \text{AMOUNT OF CHANGE}$$

In the context of the example situation above, explain what this value represents: $\int_a^b R'(t)dt = R(b) - R(a)$.

EX #1: The table given below represents the velocity of a particle at given values of t, where t is measure in minutes.

t **minutes**	0	5	10	15	20	25	30
***v*(*t*)** **ft/minute**	0	1.6	2.7	3.1	2.4	1.6	0

a. Approximate the value of $\int_0^{30} v(t)dt$ using a midpoint Riemann Sum with 3 equal intervals. Using correct units of measure, explain what this value represents.

b. What is the value of $\int_5^{25} a(t)dt$, and using correct units, explain what this value represents.

EX #2: The temperature of water in a tub at time *t* is modeled by a strictly increasing, twice differentiable function, *W*, where *W*(*t*) is measured in degrees Fahrenheit and *t* is measured in minutes (Calc Active)

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

a) Using the data in the table, estimate the value of $W'(12)$. Using correct units, interpret the meaning of this value in the context of this problem.

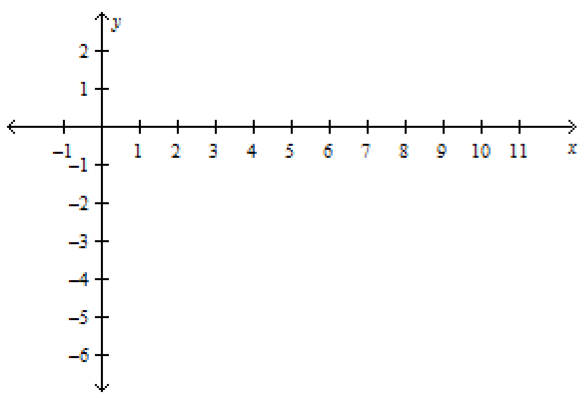
b) Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of this integral in the context of this problem.

c) For $20 \le t \le 25$, the function *W* that models the water temperature has a first derivative given by the function $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on this model, what is the temperature of the water at time
t = 25?

EX #3: A pan of biscuits is removed from an oven at which point in time, $t = 0$ minutes, the temperature of the biscuits is 100°C. The rate at which the temperature of the biscuits is changing is modeled by the function $B'(t) = -13.84e^{-0.173t}$.(Calc Active)

a) Find the value of $B'(3)$. Using correct units, explain the meaning of this value in the context of the problem.

b) Sketch the graph of $B'(t)$ on the axes below. Explain in the context of the problem why the graph makes sense.



c) At time $t = 10$, what is the temperature of the biscuits? Show your work.

EX #4: A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. During the first 5 days of a 60-day period, 3 millimeters of rainfall had been collected. The height of water in the can is modeled by the function, *S*, where *S*(*t*) is measured in millimeters and *t* is measured in days for $5 \le t \le 60$. The rate at which the height of the water is rising is given by the function $S'(t) = 2\sin(0.03t) + 1.5$.(Calc Active)

a) Find the value of $\int_{10}^{15} S'(t)dt$. Using correct units, explain the meaning of this value in the context of this problem.

b) At the end of the 60-day period, what is the volume of water that had accumulated in the can? Show your work.

EX #5: The rate at which people enter an auditorium for a concert is modeled by the function *R* given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; *R*(*t*) is measured in people per hour. V.I.P. tickets were sold to 100 people who are already in the auditorium when the when the doors open at $t = 0$ for general admission ticket holders to enter. The doors close and the concert begins at $t = 2$.

If all of the V.I.P. ticket holders stayed for the start of the concert, how many people are in the auditorium when the concert begins?

6.4 Day 4 Assignment: Duo – Tang Page 89&90 #1-7 & AP Question

CHAPTER 6 FORMULA'S TO KNOW

AP CALC: DERIVATIVES	AP CALC: INTEGRALS
$\frac{d}{dx}\left(x^n\right)=nx^{n-1}dx$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C$
$\frac{d}{dx}\left(e^u\right)=e^u\frac{du}{dx}$	$\int e^x dx=e^x+C$
$\frac{d}{dx}(\ln u)=\frac{1}{u}*\frac{du}{dx}$	$\int\frac{1}{x}dx=\ln\lvert x\rvert+C$
$\frac{d}{dx}\left(a^u\right)=(\ln a)a^u\frac{du}{dx}$	$\int a^x dx=\frac{a^x}{\ln a}+C$
$\frac{d}{dx}(\log_a u)=\frac{1}{(\ln a)u}*\frac{du}{dx}$	$\int\frac{1}{x\ln a}dx=\log_a x+C$
$\frac{d}{dx}(\sin u)=(\cos u)\frac{du}{dx}$	$\int\sin xdx=-\cos x+C$
$\frac{d}{dx}(\cos u)=-(\sin u)\frac{du}{dx}$	$\int\cos xdx=\sin x+C$
$\frac{d}{dx}(\tan u)=\left(\sec^2 u\right)\frac{du}{dx}$	$\int\sec^2 xdx=\tan x+C$
$\frac{d}{dx}(\sec u)=(\sec u\tan u)\frac{du}{dx}$	$\int\sec x\tan xdx=\sec x+C$
$\frac{d}{dx}(\cot u)=-\left(\csc^2 u\right)\frac{du}{dx}$	$\int\csc^2 xdx=-\cot x+C$
$\frac{d}{dx}(\csc u)=-(\csc u\cot u)\frac{du}{dx}$	$\int\csc x\cot xdx=-\csc x+C$
$\frac{d}{dx}\left(\sin^{-1}u\right)=\frac{1}{\sqrt{1-u^2}}*\frac{du}{dx}$	$\int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+C$
$\frac{d}{dx}\left(\cos^{-1}u\right)=-\frac{1}{\sqrt{1-u^2}}*\frac{du}{dx}$	
$\frac{d}{dx}\left(\tan^{-1}u\right)=\frac{1}{1+u^2}*\frac{du}{dx}$	$\int\frac{1}{1+x^2}dx=\tan^{-1}x+C$
$\frac{d}{dx}\left(\cot^{-1}u\right)=-\frac{1}{1+u^2}*\frac{du}{dx}$	
$\frac{d}{dx}\left(\sec^{-1}u\right)=\frac{1}{\lvert u\rvert\sqrt{u^2-1}}*\frac{du}{dx}$	$\int\frac{1}{\lvert x\rvert\sqrt{x^2-1}}dx=\sec^{-1}x+C$
$\frac{d}{dx}\left(\csc^{-1}u\right)=-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}*\frac{du}{dx}$	

- **Definition of the Derivative (both versions)**
- **Intermediate Value Theorem**
- **Mean Value Theorem for Derivatives**
- **Extreme Value Theorem**
- **Mean Value Theorem for Integrals**
- **Average Value of a Function**
- **First Fundamental Theorem of Calculus**
- **Second Fundamental Theorem of Calculus**
- **Total Area Vs Net Area**