UNIT 1 ASSIGNMENT #1

Duo-Tang: *Unit 1 Assignment 1* QUESTIONS 1-6 BELOW (NOTE: Question 6 will be handed in for marking)

1. Factor the following
   a) $a^3 + b^3$ b) $m^3 - 8b^3$ c) $27t^3 + 1$ d) $54c^3 + 16d^3$
   e) $x^4 - x$ f) $16xy^4 - 2x^4y$ g) $(x + 1)^3 + 1$ h) $(x + 2)^3 - 8$
   i) $64x^6 + 27y^9$ j) $(a + 3)^3 - (a - 3)^3$ k) $(x^2 - 1)^3 - 27$ l) $8 + (x^2 - 6)^3$
2. Factor by grouping: $x^3 - x^2 - 16x + 16$

3. Create a quartic polynomial with four terms that can be factored using the grouping method (and is a different polynomial than any previous examples done in class). Factor your question.

4. Fully factor the following. Leave your answer in the proper form (following all guidelines talked about in class!)
   (a) $x^{\frac{5}{2}} - x^{\frac{1}{2}}$ (b) $x + 5 + 6x^{-1}$
   (c) $x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 8x^{-\frac{1}{2}}$ (d) $2x^{\frac{7}{2}} - 2x^{\frac{1}{2}}$
   (e) $1 + 2x^{-1} + x^{-2}$ (f) $(x^2 + 1)^{\frac{1}{2}} + 3(x^2 + 1)^{-\frac{1}{2}}$

5. Fully simplify each factor and write using the proper notation.

a) $125x^3 + 64y^3z^6$
b) $8x^3 - (3y+2)^3$
c) $x^{\frac{5}{2}} + x^{\frac{3}{2}}$
d) $-20x^{\frac{-3}{4}} + 4x^{\frac{1}{4}}$
e) $12x^{\frac{-2}{3}} - 18x^{\frac{1}{3}}$
f) $(x^2-3)^{\frac{3}{2}} + (x^2-3)^{\frac{7}{2}}$
g) $(x+2)^{\frac{3}{2}} - (x+2)^{\frac{1}{2}}$
h) $(x^2+2)^{-\frac{5}{3}} + (x^2+2)^{-\frac{2}{3}}$
i) $(x+4)^{-\frac{1}{2}} - (x+4)^{-\frac{3}{2}}$
j) $(5x-1)^{\frac{1}{2}} - \frac{1}{3}(5x-1)^{\frac{3}{2}}$
k) $2x(3x-5)^{\frac{1}{2}} - \frac{4}{3}x^2(3x-5)^{\frac{-1}{2}}$
l) $-8(4x+3)^{-2} + 10(5x+1)(4x+3)^{-1}$
m) $-x^{3/2}(x^2+3)^{-3/2} + \frac{1}{3}x^{-1/2}(x^2+3)^{-1/2}$
n) $-\frac{5}{2}(1-2x)^{-1/2}(x-3)^{1/2} + \frac{1}{2}(x-3)^{-1/2}(1-2x)^{1/2}$

OUTCOME 1 DELTA MATH ASSIGNMENTS

Calc #1: Review of Factoring (Outcome 1)
Calc #2: Review of Trigonometry (Outcome 1)
Calc #3: Graph Analysis (Outcome 1)
Calc #4: Review of Special Functions (Outcome 1)

UNIT 1: ASSIGNMENT #2

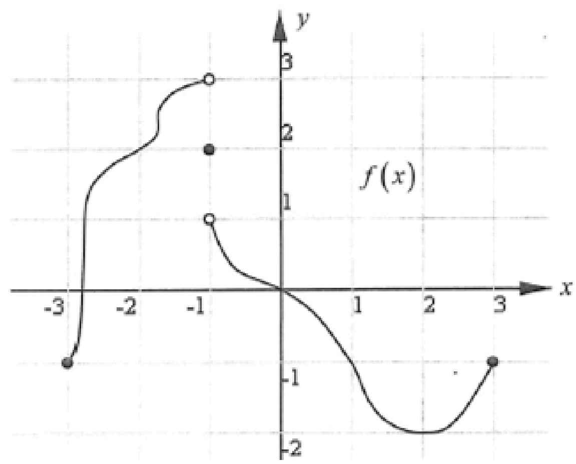
Text: P4 #1, 2

UNIT 1: ASSIGNMENT #3
Duo-Tang: QUESTIONS 3-14 BELOW

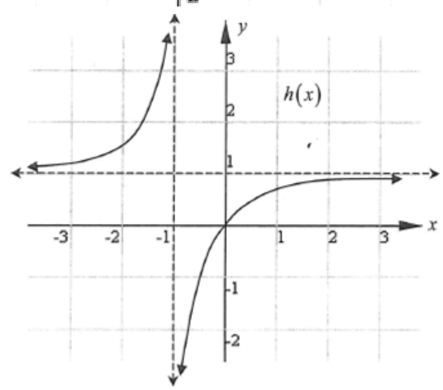
In questions 3 to 7, use your calculator and create a table of values to estimate the given limit, rounding your estimate to 5 decimal places where appropriate. Remember that to estimate $\lim\limits_{x\to b} f(x)$ you must include values of x in your table that approach b from both sides. For question 6, set your calculator to radians.

3. $\lim\limits_{x\to 2}\dfrac{x^2+x-6}{x^2-2x}$ 4. $\lim\limits_{x\to 0}\dfrac{3^x-1}{x}$ 5. $\lim\limits_{x\to 25}\dfrac{x-25}{\sqrt{x}-5}$ 6. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$ 7. $\lim\limits_{x\to 1}\dfrac{x^2-1}{|x-1|}$

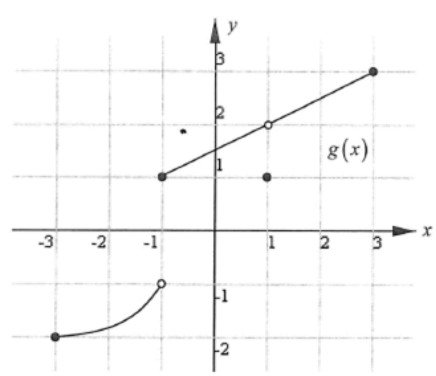
8. For the function $f(x)$ whose graph is shown, give the value of each quantity, if it exists. If it does not exist, explain why.
(a) $f(-3)$ (b) $f(0)$
(c) $f(2)$ (d) $f(-1)$
(e) $\lim\limits_{x\to -2} f(x)$ (f) $\lim\limits_{x\to 1} f(x)$
(g) $\lim\limits_{x\to 3^-} f(x)$ (h) $\lim\limits_{x\to -3^+} f(x)$
(i) $\lim\limits_{x\to 2^+} f(x)$ (j) $\lim\limits_{x\to 2^-} f(x)$
(k) $\lim\limits_{x\to 2} f(x)$ (l) $\lim\limits_{x\to -1^+} f(x)$
(m) $\lim\limits_{x\to -1^-} f(x)$ (n) $\lim\limits_{x\to -1} f(x)$



9. For the function $g(x)$ whose graph is shown, give the value of each quantity, if it exists. If it does not exist, explain why.
(a) $g(-1)$ (b) $g(1)$
(c) $\lim\limits_{x\to 0} g(x)$ (d) $\lim\limits_{x\to 1^+} g(x)$
(e) $\lim\limits_{x\to 1^-} g(x)$ (f) $\lim\limits_{x\to 1} g(x)$
(g) $\lim\limits_{x\to -3^+} g(x)$ (h) $\lim\limits_{x\to -1^+} g(x)$
(i) $\lim\limits_{x\to -1^-} g(x)$ (j) $\lim\limits_{x\to -1} g(x)$



10. For the function $h(x)$ whose graph is shown, find each of the following.
(a) $\lim\limits_{x\to 0} h(x)$ (b) $\lim\limits_{x\to -1^-} h(x)$
(c) $\lim\limits_{x\to -1^+} h(x)$ (d) $\lim\limits_{x\to -1} h(x)$
(e) $\lim\limits_{x\to \infty} h(x)$ (f) $\lim\limits_{x\to -\infty} h(x)$



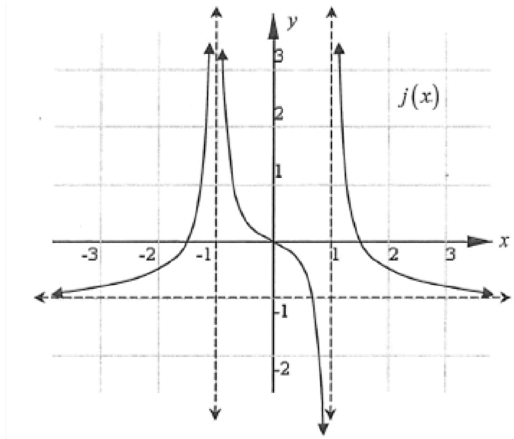
11. For the function $j(x)$ whose graph is shown, find each of the following.

(a) $\lim\limits_{x\to -1^+} j(x)$ (b) $\lim\limits_{x\to -1^-} j(x)$

(c) $\lim\limits_{x\to -1} j(x)$ (d) $\lim\limits_{x\to 1^+} j(x)$

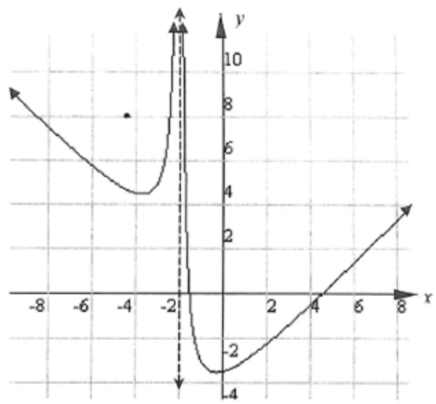
(e) $\lim\limits_{x\to 1^-} j(x)$ (f) $\lim\limits_{x\to 1} j(x)$

(g) $\lim\limits_{x\to \infty} j(x)$ (h) $\lim\limits_{x\to -\infty} j(x)$

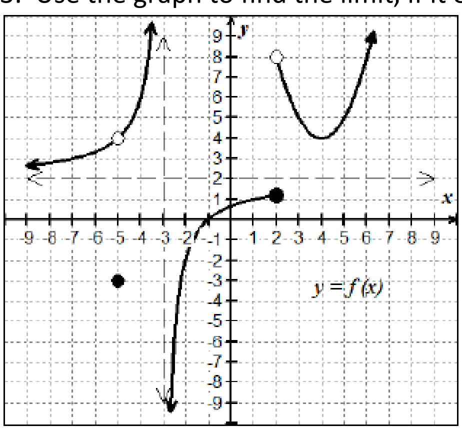


12. Shown at right is the graph of the function $f(x)=\dfrac{x^2-1}{|x+2|}-3$. Examine the graph and determine each of the following.

(a) $\lim\limits_{x\to -2} f(x)$ (b) $\lim\limits_{x\to -\infty} f(x)$ (c) $\lim\limits_{x\to \infty} f(x)$



13. Use the graph to find the limit, if it exists. If the limit does not exist, explain why.



A. $\lim\limits_{x\to -3^-} f(x)$ **B.** $\lim\limits_{x\to -3^+} f(x)$

C. $\lim\limits_{x\to 2^-} f(x)$ **D.** $\lim\limits_{x\to 2^+} f(x)$

E. $\lim\limits_{x\to \infty} f(x)$ **F.** $\lim\limits_{x\to -\infty} f(x)$

G. $\lim\limits_{x\to -3} f(x)$ **H.** $\lim\limits_{x\to 2} f(x)$

14. Sketch a graph of a function that satisfies each of the following conditions.

a) $\lim\limits_{x\to 1^-} f(x) = 2$ $\lim\limits_{x\to 1+} f(x) = -3$

$f(1) = 4$

b) $\lim\limits_{x\to -3^-} f(x) = 1$ $\lim\limits_{x\to -3+} f(x) = -4$

$\lim\limits_{x\to 2^-} f(x) = \infty$ $\lim\limits_{x\to 2^+} f(x) = -\infty$

DELTA MATH (Summative) : Calc 30 #5: Limits with Graphs

UNIT 1 ASSIGNMENT #4:

Text: P19 #2, 3, 4, 5c, 6, 7, 9, 8, 10, 11, 12, 13

UNIT 1: ASSIGNMENT #5

Duo- Tang : Questions #40-51 Below

40. $\lim\limits_{x\to\infty}\dfrac{6}{3x-2}$

41. $\lim\limits_{x\to\infty}\dfrac{2x+5}{x+1}$

42. $\lim\limits_{x\to\infty}\dfrac{6x^2-1}{2x^2+3x}$

43. $\lim\limits_{x\to-\infty}\dfrac{-4x^3}{x^3-2x^2}$

44. $\lim\limits_{x\to-\infty}\dfrac{(x+2)(2x-1)}{x^2+4x+1}$

45. $\lim\limits_{x\to\infty}\dfrac{2x^2}{x-1}$

46. $\lim\limits_{x\to-\infty}\dfrac{2x^2}{x-1}$

47. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{4x^2-x}}{x-2}$

48. $\lim\limits_{g\to\infty}\dfrac{2g+5}{\sqrt{g^2+6g}}$

49. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-4x+4}}{5-x}$

50. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2-2x}}{x^2}$

51. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)$

UNIT 1: ASSIGNMENT #6

Duo- Tang: Questions #52-63 Below

52. $\lim\limits_{x\to0^+}\dfrac{|x|}{x}$

53. $\lim\limits_{x\to0^-}\dfrac{|x|}{x}$

54. $\lim\limits_{x\to0}\dfrac{|x|}{x}$

Hint: rationalize the numerator.

55. $\lim\limits_{x\to2^+}\dfrac{|x-2|}{4-x^2}$

56. $\lim\limits_{x\to-1^-}\dfrac{|x^2-x-2|}{x+1}$

57. $\lim\limits_{x\to1^-}\dfrac{|x^5-1|}{x-1}$

58. $\lim\limits_{x\to3^-}\dfrac{x^3-27}{|x-3|}$

59. $\lim\limits_{x\to3^+}\dfrac{x^3-27}{|x-3|}$

60. $\lim\limits_{x\to3}\dfrac{x^3-27}{|x-3|}$

61. If $f(x)=\begin{cases}(x+1)2^{x-1}, \text{ if } x\ge0\\ 2^{1-x}, \text{ if } x<0\end{cases}$, find each of the following limits:

(a) $\lim\limits_{x\to3}f(x)$ (b) $\lim\limits_{x\to-3}f(x)$ (c) $\lim\limits_{x\to0^+}f(x)$ (d) $\lim\limits_{x\to0^-}f(x)$ (e) $\lim\limits_{x\to0}f(x)$

62. If $g(x)=\begin{cases}\dfrac{x^2-4}{x+2}, \text{ if } x\ne-2\\ -5, \text{ if } x=-2\end{cases}$, find each of the following limits:

(a) $\lim\limits_{x\to3}g(x)$ (b) $\lim\limits_{x\to-2}f(x)$

63. If $h(x)=\begin{cases}2, \text{ if } x<0\\ x+2, \text{ if } 0\le x\le4\\ x^2-11, \text{ if } x>4\end{cases}$, find each of the following limits:

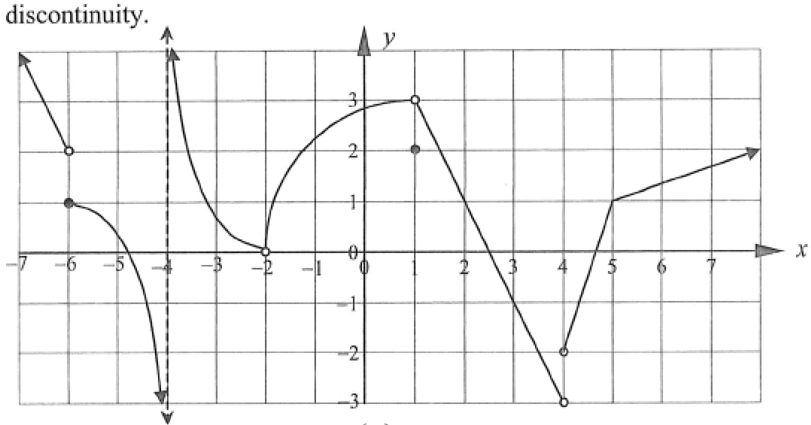
(a) $\lim\limits_{x\to-6}h(x)$ (b) $\lim\limits_{x\to0^-}h(x)$ (c) $\lim\limits_{x\to0^+}h(x)$ (d) $\lim\limits_{x\to0}h(x)$ (e) $\lim\limits_{x\to3}h(x)$

(f) $\lim\limits_{x\to4^-}h(x)$ (g) $\lim\limits_{x\to4^+}h(x)$ (h) $\lim\limits_{x\to4}h(x)$ (i) $\lim\limits_{x\to5}h(x)$

DELTA MATH (Summative) : Calc #6. Limits Analytically

UNIT 1: ASSIGNMENT #7
Duo- Tang: Questions #1-11 Below

1. Give the values of x at which the function $g(x)$, shown below, is discontinuous. Taking the conditions of the test for continuity at a point in order, what is the first condition not satisfied at each x-value? Classify each discontinuity.



$g(x)$

2. Using the test for continuity at a point, explain why each function is discontinuous at the given x-value. Classify each discontinuity.

(a) $f(x)=\dfrac{12}{x+3}$; at $x=-3$

(b) $g(x)=\dfrac{x-\pi}{x^2-\pi^2}$; at $x=\pi$

(c) $h(x)=\begin{cases} x^2+2x, & x\le 3 \\ 4x+5, & x>3 \end{cases}$; at $x=3$

(d) $i(x)=\begin{cases} \dfrac{x^2+5x+4}{x^3+1}, & x\ne -1 \\ 2, & x=-1 \end{cases}$; at $x=-1$

(e) $j(x)=\dfrac{|x+4|}{x^2+3x-4}$; at $x=-4$

3. Each of the following functions has a removable discontinuity at $x=3$. How should $f(3)$ be defined in order to remove the discontinuity?

(a) $f(x)=\dfrac{2x-6}{x^2-4x+3}$

(b) $f(x)=\begin{cases} \dfrac{x^2-9}{x^3-27}, & x\ne 3 \\ 1, & x=3 \end{cases}$

(c) $f(x)=\dfrac{\sqrt{x}-\sqrt{3}}{x-3}$

4. Consider the function $f(x)=\begin{cases} x^2+5x+2, & x<1 \\ cx-3, & x\ge 1 \end{cases}$. Find the value of c if $f(x)$ is to be continuous.

5. Consider the function $g(x)=\begin{cases} 2x^2-dx+3, & x<-3 \\ x^2+x+9, & x\ge -3 \end{cases}$. Find the value of d if $g(x)$ is to be continuous.

$\{4x+5,\ x\le -1$

6. Consider the function $h(x)=\begin{cases}\dfrac{ax+b}{x-1}, & -1<x<2 \\ x^2-1, & x\geq 2\end{cases}$. Find the value of a and b if $h(x)$ is to be continuous.

7. Consider the function $i(x)=\begin{cases}\dfrac{x^4-x}{x-1}, & x\neq 1 \\ e, & x=1\end{cases}$. Find the value of e if $i(x)$ is to be continuous.

8. Each of the following functions has a vertical asymptote and a hole. Determine the equation of the vertical asymptote and the coordinates of the hole.

   (a) $f(x)=\dfrac{3x-x^2}{x^2+x}$ (b) $f(x)=\dfrac{x^3+3x^2-10x}{x^2+x-6}$

9. Using the continuity principles, explain why the function $f(x)=\left(2^{x+5}\right)\sqrt[3]{x^2-x-12}$ is continuous

10. Determine the points, if any, at which each function is discontinuous. For those that are discontinuous, state the location of the discontinuities and classify them.

   (a) $f(x)=\sin x+\cos x$
   (b) $f(x)=\dfrac{x-3}{x^2+4}$
   (c) $f(x)=\dfrac{x+2}{x^2-2x-3}$
   (d) $f(x)=\left|x^2-6x\right|$
   (e) $f(x)=\dfrac{|x-1|}{x^2-x}$
   (f) $f(x)=\dfrac{|x+3|}{|x+3|}$
   (g) $f(x)=\dfrac{x-4}{\sqrt{x}-2}$
   (h) $f(x)=\dfrac{1}{2}x^2-\dfrac{3}{4}x+\dfrac{2}{5}$
   (i) $f(x)=\dfrac{x+5}{\log_2\left(x^2+1\right)}$
   (j) $f(x)=\dfrac{x+6}{|x-6|}$
   (k) $f(x)=\dfrac{x^2+x}{x^4-x}$
   (l) $f(x)=\begin{cases}2^x+1, & x\geq 2 \\ 2^{x+1}-3, & x<2\end{cases}$
   (m) $f(x)=\begin{cases}-4, & x<-3 \\ x-1, & -3\leq x\leq 2 \\ x^2, & x>2\end{cases}$
   (n) $f(x)=\tan x$
   (o) $f(x)=\sqrt[3]{9-x^2}$

11. Use the three-part definition of continuity to determine if the given functions are continuous at the indicated values of *x*.

$$g(x)=\begin{cases}\dfrac{x^2-9}{x+3} & x\neq -3 \\ 5 & x=-3\end{cases} \quad \text{at } x=-3$$

DELTA MATH (Summative) : Calc #7. Continuity

CALCULUS 30: UNIT 1 REVIEW

1.

If $f(x) = \begin{cases} 2x - 1 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

a. Find $\lim_{x \to 2} f(x)$, and show $\lim_{x \to 2} f(x) \neq f(2)$.

b. Draw the graph of f.

2.

. Given a function f defined by $f(x) = \begin{cases} x^2 - 9 \text{ if } x \neq -3 \\ 4 \text{ if } x = -3 \end{cases}$

find $\lim_{x \to -3} f(x)$ and show that $\lim_{x \to -3} f(x) \neq f(-3)$.

3.

4x

For the function defined by $f(x) = \begin{cases} \dfrac{x + \quad}{x} \text{ if } x > 0 \\ -x^2 \text{ if } x \leq 0 \end{cases}$

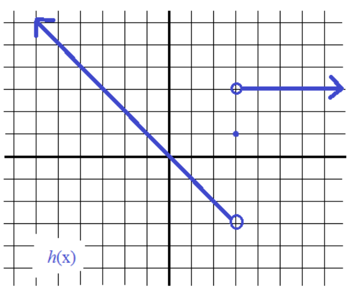
a. sketch the graph of f.

b. Find the left-hand limit $\lim_{x \to 0^-} f(x)$.

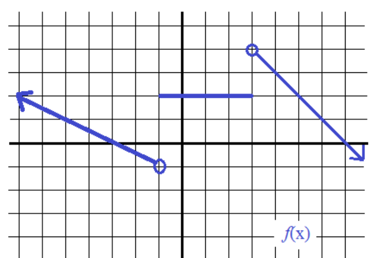
c. Find the right-hand limit $\lim_{x \to 0^+} f(x)$.

4. Write the equation of each of the following

a)



b)



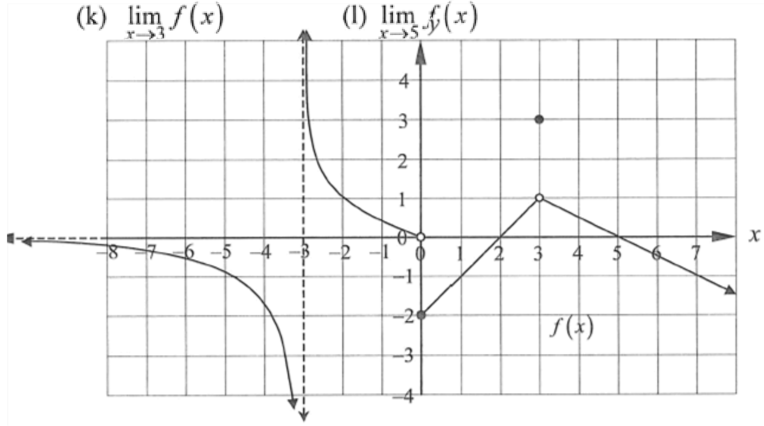
5. By examining the graph of the function $f(x)$ below, determine each of the following. Just record your answer. There is no need to show any work.

(a) $f(0)$ (b) $f(3)$ (c) $f(-3)$ (d) $\lim_{x \to -3^+} f(x)$

(e) $\lim_{x \to -3^-} f(x)$ (f) $\lim_{x \to -3} f(x)$ (g) $\lim_{x \to -2} f(x)$ (h) $\lim_{x \to 0^+} f(x)$

(i) $\lim_{x \to 0^-} f(x)$ (j) $\lim_{x \to 0} f(x)$ (k) $\lim_{x \to 3} f(x)$ (l) $\lim_{x \to 5} f(x)$

(m) $\lim_{x \to -\infty} f(x)$ (n) $\lim_{x \to \infty} f(x)$



6. There are three conditions that a function must satisfy in order to be continuous at a point. In light of these conditions, explain why function $f(x)$, above, is not continuous at:

(a) $x=-3$ (b) $x=0$ (c) $x=3$

7. Evaluate each of the following limits. Do not merely record your answer, but show your work, step-by-step.

(a) $\lim_{x\to 5}\frac{x^2-x-12}{x+5}$ (b) $\lim_{x\to -4}\frac{2x^2-7x-4}{4-x}$ (c) $\lim_{x\to -3}\frac{x^2-x-12}{x+3}$

(d) $\lim_{x\to 1}\frac{x^3-1}{x^4-1}$ (e) $\lim_{x\to 0}\frac{(5+x)^2-3(5+x)-10}{x}$ (f) $\lim_{x\to 0}\frac{(x-1)^2-1}{3x}$

(g) $\lim_{x\to 0}\frac{1-\sqrt{x+1}}{x}$ (h) $\lim_{x\to 3}\frac{3-x}{\sqrt{4-x}-\sqrt{x-2}}$ (i) $\lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1}$

(j) $\lim_{x\to\infty}\frac{5x^2+1}{x^3+x^2}$ (k) $\lim_{x\to 2^+}\frac{1+x^2}{x^2+3x-10}$ (l) $\lim_{x\to 0}\frac{x-1}{x^2(x+3)}$

(m) $\lim_{x\to 10^-}\frac{|x-10|}{x-10}$ (n) $\lim_{x\to 3}\frac{|x^2-9|}{x-3}$ (o) $\lim_{x\to -\infty}\frac{\sqrt{x^2+6x}}{2x-4}$

8. Find the limits.

a. $\lim_{x\to\infty}\frac{3x+2}{5x+3}$ b. $\lim_{x\to\infty}\frac{3x^2-2}{2x^2+3}$ c. $\lim_{x\to\infty}\frac{x+3}{3x^2+1}$

d. $\lim_{x\to\infty}\frac{\sqrt{x^2+4}}{x+4}$ e. $\lim_{x\to-\infty}\frac{\sqrt{x^2+4}}{x+4}$ f. $\lim_{x\to+\infty}\frac{2x^2+x-3}{x+5}$

9. Identify the type of discontinuity each function has at $x=1$. Explain.

(a) $f(x)=\frac{\sin\left(\frac{\pi x^2}{2}\right)}{x^2-1}$ (b) $f(x)=\frac{x-1}{x^2-1}$ (c) $f(x)=\frac{|x-1|}{x-1}$

10 Find the value(s) of x, if any, at which the function $f(x)=\frac{x|x-4|}{x^3-2x^2-8x}$ is discontinuous. Explain your reasoning. Classify each discontinuity.

11. Find the value of c if $f(x)=\begin{cases} cx+9, & x<4 \\ x^2+3x+1, & x\geq 4\end{cases}$ is to be continuous.

12. Evaluate each of the following limits if $f(x)=\begin{cases} x^2-4, & x<-2 \\ 2x+4, & -2\leq x<3 \\ 12-x, & x\geq 3\end{cases}$.

(a) $\lim_{x\to -2^-}f(x)$ (b) $\lim_{x\to -2^+}f(x)$ (c) $\lim_{x\to -2}f(x)$

(d) $\lim_{x\to 3^-}f(x)$ (e) $\lim_{x\to 3^+}f(x)$ (f) $\lim_{x\to 3}f(x)$

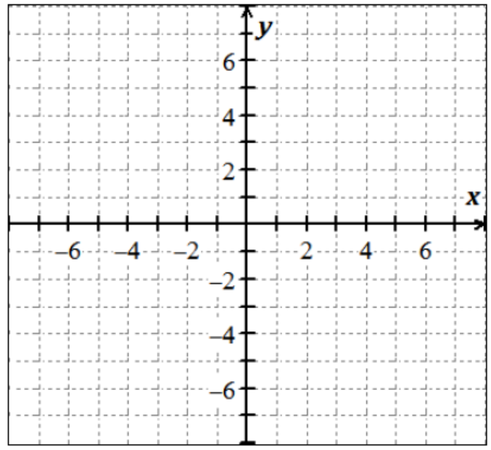
13. Sketch the graph of an example of a function f that satisfies all of the following conditions:

$\lim_{x\to 0^+} f(x) = -2$ $\quad$ $\lim_{x\to 0^-} f(x) = 1$

$f(0) = -1$ $\quad$ $\lim_{x\to\infty} f(x) = 3$

$\lim_{x\to 2^-} f(x) = \infty$ $\quad$ $\lim_{x\to 2^+} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = 4$



IF YOU NEED EXTRA PRACTICE WITH FACTORING, DO THE FOLLOWING AS WELL:

Factor each expression over the set of real numbers.

(a) d^2-8d-9 (b) $18v^2+45v-50$ (c) $3x^2-15$ (d) j^3+125

(e) $8b^3-7$ (f) $250s^3-10s$ (g) x^2-8x+9 (h) $w^2-6w+10$

(i) $x^{7/3}-4x^{1/3}$ (j) $t^{-1}+4t^{-2}+4t^{-3}$ (k) $2am^{5/3}-6am^{2/3}+4am^{-1/3}$ (l) $x^{7/2}y^{-3/2}+x^{1/2}y^{3/2}$

(m) $6-\frac{1}{6}y^2$ (n) $\frac{1}{5}t^2+\frac{1}{30}t-\frac{1}{2}$ (o) p^4-4 (p) h^7-1

(q) $x^3-3x^2-10x+24$ (r) $x^4+x^3-10x^2-4x+24$ (s) $2x^3+6x^2-18x-54$

CALCULUS 30: SOLUTIONS TO UNIT 1 ASSIGNMENTS

SOLUTIONS TO: UNIT 1 ASSIGNMENT #1

1. a) $(a+b)(a^2-ab+b^2)$ b) $(m-2b)(m^2+2mn+4b^2)$ c) $(3t-1)(9t^2+3t+1)$
   d) $2(3c+2d)(9c^2-6cd+4d^2)$ e) $x(x-1)(x^2+x+1)$ f) $2xy(2y-x)(4y^2+2yx+x^2)$
   g) $(x+2)(x^2+x+1)$ h) $x(x^2+6x+12)$ i) $(4x^2+3y^3)(16x^4-12x^2y^3+9y^6)$
   j) $18(a^2+3)$ k) $(x-2)(x+2)(x^4+x^2+7)$ l) $(x-2)(x+2)(x^4-14x^2+52)$
2. (x-4)(x+4)(x-1)
5. a) $x^{1/2}(x-1)(x+1)$ b) $x^{-1}(x+2)(x+3)$ c) $x^{-1/2}(x+4)(x-2)$
   d) $2x^{1/2}(x-1)(x^2+x+1)$ e) $x^{-2}(x+1)^2$ f) $(x^2+1)^{-1/2}(x^2+4)$
6. To be handed in for marking

SOLUTIONS TO: UNIT 1 ASSIGNMENT #3

3.

x	2.1	2.01	2.001	2.0001	2.00001	2.000001
$\frac{x^2+x-6}{x^2-2x}$	2.428571429	2.492537313	2.499250375	2.499925004	2.4999925	2.49999925

x	1.9	1.99	1.999	1.9999	1.99999	1.999999
$\frac{x^2+x-6}{x^2-2x}$	2.578947368	2.507537688	2.500750375	2.500075004	2.5000075	2.50000075

A reasonable estimate for $\lim_{x\to 2}\frac{x^2+x-6}{x^2-2x}$ would be 2.5.

4.

x	0.1	0.01	0.001	0.0001	0.00001	0.000001
$\frac{3^x-1}{x}$	1.16123174	1.104669194	1.099215984	1.098672638	1.09861832	1.0986129

x	−0.1	−0.01	−0.001	−0.0001	−0.00001	−0.000001
$\frac{3^x-1}{x}$	1.040415402	1.092599583	1.098009035	1.098551943	1.098606254	1.09861169

A reasonable estimate for $\lim_{x\to 0}\frac{3^x-1}{x}$ would be 1.09861.

5.

x	25.1	25.01	25.001	25.0001	25.00001	25.000001
$\frac{x-25}{\sqrt{x}-5}$	10.00999002	10.0009999	10.0001	10.00001	10.000001	10

x	24.9	24.99	24.999	24.9999	24.99999	24.999999
$\frac{x-25}{\sqrt{x}-5}$	9.98998998	9.9989999	9.999900001	9.99999	9.999999	10

A reasonable estimate for $\lim_{x\to 25}\frac{x-25}{\sqrt{x}-5}$ would be 10.

6.

x	0.1	0.01	0.001	0.0001	0.00001
$\frac{\sin x}{x}$	0.9983341665	0.9999833334	0.9999998333	0.9999999983	1

x	−0.1	−0.01	−0.001	−0.0001	−0.00001
$\frac{\sin x}{x}$	0.9983341665	0.9999833334	0.9999998333	0.9999999983	1

A reasonable estimate for $\lim_{x\to 0}\frac{\sin x}{x}$ would be 1.

7.

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$\frac{x^2-1}{\lvert x-1\rvert}$	2.1	2.01	2.001	2.0001	2.00001	2.000001

x	0.9	0.99	0.999	0.9999	0.99999	0.999999
$\frac{x^2-1}{\lvert x-1\rvert}$	−1.9	−1.99	−1.999	−1.9999	−1.99999	−1.999999

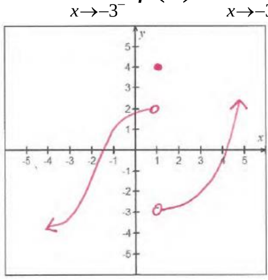
Based on the tables it appears that $\lim_{x\to 1}\frac{x^2-1}{|x-1|}$ does not exist since $\lim_{x\to 1^+}\frac{x^2-1}{|x-1|}$ appears to be 2 while $\lim_{x\to 1^-}\frac{x^2-1}{|x-1|}$ appears to be -2. Since $\lim_{x\to 1^+}\frac{x^2-1}{|x-1|}\neq\lim_{x\to 1^-}\frac{x^2-1}{|x-1|}$, $\lim_{x\to 1}\frac{x^2-1}{|x-1|}$ does not exist.

8. (a) -1 **(b)** 0 **(c)** -2 **(d)** 2 **(e)** 2 **(f)** -1 **(g)** -1 **(h)** -1 **(i)** -2 **(j)** -2 **(k)** -2 **(l)** 1 **(m)** 3 **(n)** does not exist since $\lim_{x\to -1^+} g(x)\neq\lim_{x\to -1^-} g(x)$ **9. (a)** 1 **(b)** 1 **(c)** 1.5 **(d)** 2 **(e)** 2 **(f)** 2 **(g)** -2 **(h)** 1 **(i)** -1 **(j)** does not exist since $\lim_{x\to -1^+} g(x)\neq\lim_{x\to -1^-} g(x)$ **10. (a)** 0 **(b)** ∞ **(c)** $-\infty$ **(d)** does not exist **(e)** 1 **(f)** 1 **11. (a)** ∞ **(b)** ∞ **(c)** ∞ **(d)** ∞ **(e)** $-\infty$ **(f)** does not exist **(g)** -1 **(h)** -1 **12. (a)** ∞ **(b)** ∞

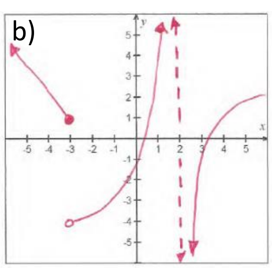
13. a) ∞ b) $-\infty$ c) 1 d) 8 e) ∞ f) 2

g) DNE $\lim_{x\to -3^-} f(x)\neq\lim_{x\to -3^+} f(x)$ h) DNE $\lim_{x\to 2^-} f(x)\neq\lim_{x\to 2^+} f(x)$

14. a)



b)



SOLUTIONS TO: UNIT 1 ASSIGNMENTS 5 & 6

27. 7 **28.** 2 **29.** 4 **30.** 1 **31.** $-\frac{1}{4}$ **32.** $-\frac{2}{9}$ **33.** $-\frac{1}{4}$ **34.** 6 **35.** $\frac{\sqrt{2}}{4}$ **36.** $-\frac{1}{4}$ **37.** 6 **38.** $-\frac{1}{16}$ **39.** 24 **40.** 0
41. 2 **42.** 3 **43.** -4 **44.** 2 **45.** ∞ **46.** $-\infty$ **47.** -2 **48.** 2 **49.** 1 **50.** 0 **51.** 2 **52.** 1 **53.** -1
54. does not exist **55.** $-\frac{1}{4}$ **56.** -3 **57.** -5 **58.** -27 **59.** 27 **60.** does not exist **61.** (a) 16 (b) 16 (c) $\frac{1}{2}$ (d) 2 (e) does not exist **62.** (a) 1 (b) -4 **63.** (a) 2 (b) 2 (c) 2 (d) 2 (e) 5 (f) 6 (g) 5 (h) does not exist (i) 14 **64.** -1 **65.** 0 **66.** since $\sin x$ oscillates between -1 and 1 forever, $\sin x$ will never approach a fixed real number.

67. The numerator will oscillate between -1 and 1 as x increases, but the denominator grows ever larger, resulting in the fraction approaching a value of 0. **68.** $\lim_{x\to\infty}\frac{x^2}{2^x}=0$. Judging from the calculator values, the denominator outgrows the numerator, sending the value of the fraction towards 0.

69. (a) 1 (b) $\frac{1}{2}$ (c) does not exist **70.** 12 **71.** (a) $f(x)=\frac{x^2+2x}{4x+4}$ (b) $\lim_{x\to 0^+} f(x)=0$. As the width

SOLUTIONS TO: UNIT 1 ASSIGNMENT 7

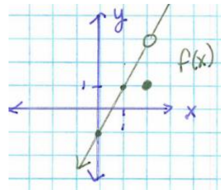
1. There is a **jump discontinuity** at $x=-6$. The second condition is not satisfied since $\lim_{x\to -6} g(x)$ does not exist. There is an **infinite discontinuity** at $x=-4$. The first condition is not satisfied since $g(-4)$ does not exist. There is a **removable discontinuity** at $x=-2$. The first condition is not satisfied since $g(-2)$ does not exist. By defining $g(-2)$ as 0, the discontinuity could be removed. There is a **removable discontinuity** at $x=1$. The third condition is not satisfied since $g(1)\neq \lim_{x\to 1} g(x)$. By defining $g(1)$ as 3, the discontinuity could be removed. There is a **jump discontinuity** at $x=4$. The first condition is not satisfied since $g(4)$ does not exist **2.** (a) The first condition is not satisfied since $f(-3)$ does not exist. $f(-3)$ results in $\frac{12}{0}$. There is an infinite discontinuity at $x=-3$. (b) The first condition is not satisfied since $f(\pi)$ results in $\frac{0}{0}$ which is indeterminate. Note that $\lim_{x\to\pi}\frac{x-\pi}{x^2-\pi^2}=\lim_{x\to\pi}\frac{\cancel{x-\pi}}{(\cancel{x-\pi})(x+\pi)}=\frac{1}{2\pi}$.

Thus there is a removable discontinuity at $x=\pi$. By defining $f(\pi)$ as $\frac{1}{2\pi}$, the discontinuity can be removed. (c) The second condition is not satisfied since $\lim_{x\to 3} h(x)$ does not exist because $\lim_{x\to 3^+} h(x)=17$ but $\lim_{x\to 3^-} h(x)=15$. There is a jump discontinuity at $x=3$. (d) The third condition is not satisfied since $\lim_{x\to -1} i(x)=1$ but $i(-1)=2$. Thus there is a removable discontinuity at $x=-1$. By redefining $i(-1)$ to be 1, the discontinuity can be removed. (e) The first condition is not satisfied since $j(-4)$ yields $\frac{0}{0}$.

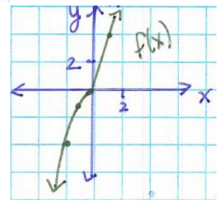
SOLUTIONS TO: UNIT 1 REVIEW

REMEMBER TO GIVE CALCULUS APPROPRIATE ANSWERS (NOT PRECALCULUS JUSTIFICATIONS!)

1. a) $\lim\limits_{x\to 2} f(x) = 3$; since f(2)=1 then $\lim\limits_{x\to 2} f(x) \neq f(2)$ b)



2. $\lim\limits_{x\to -3} f(x) = 0$; since f(-3)=4 then $\lim\limits_{x\to -3} f(x) \neq f(-3)$
3. b) $\lim\limits_{x\to 0^-} f(x) = 0$ c) $\lim\limits_{x\to 0^+} f(x) = 0$ 3 a)



4. a) $f(x) = \begin{cases} -x, & x < 3 \\ 1, & x = 3 \\ 3, & x > 3 \end{cases}$ b) $f(x) = \begin{cases} -\frac{1}{2}x - \frac{3}{2}, & x < -1 \\ 2, & -1 \geq x \geq 3 \\ -x+7, & x > 3 \end{cases}$
5. a) -2 b) 3 c) undefined d) $\infty \therefore$ Does not Exist e) -$\infty \therefore$ Does not Exist f) Does Not Exist g) 1
h) -2 i) 0 j) Does Not Exist k) 1 l) 0 m) 0 n) -$\infty \therefore$ Does not Exist

6. a) $\lim\limits_{x\to -3^-} f(x) = -\infty, \lim\limits_{x\to -3^+} f(x) = +\infty$, therefore there is a vertical asymptote at x=-3, which means there is an infinite discontinuity at x=-3 and f(-3) does not exist
b) $\lim\limits_{x\to 0^+} f(x) = -2, \lim\limits_{x\to 0^-} f(x) = 0, f(0) = 0$. Therefore $\lim\limits_{x\to 0^+} f(x) \neq \lim\limits_{x\to 0^-} f(x)$ which means there is a jump discontinuity at x=0
c) Since $\lim\limits_{x\to 3} f(x) = 1$ and f(3)=3, $\lim\limits_{x\to 3} f(x) \neq f(3)$ and there is a removable discontinuity at x=3

7. a) 4/5 b) 7 c) -7 d) ¾ e) 7 f) -2/3 -1/2 h) 1 i) 3/5 j) 0 k) $\infty \therefore$ Does not Exist
l) -$\infty \therefore$ Does not Exist m) -1 n) Does Not Exist
8. a) 3/5 b) 3/2 c) 0 d) 1 e) -1 f) $\infty \therefore$ Does not Exist

9. a) $\lim\limits_{x\to 1^-} f(x) = -\infty, \lim\limits_{x\to 1^+} f(x) = +\infty$, therefore there is a vertical asymptote at x=1, which means there is an infinite discontinuity at x=1 and f(1) does not exist b) $\lim\limits_{x\to 1^-} f(x) = \frac{1}{2}, \lim\limits_{x\to 1^+} f(x) = \frac{1}{2}, f(1)$ Does Not Exist. Since $\lim\limits_{x\to 1} f(x) \neq f(1)$, there is a hole at x=1 and an removable discontinuity at x=1 c) $\lim\limits_{x\to 1^-} f(x) = -1, \lim\limits_{x\to 1^+} f(x) = 1$, therefore $\lim\limits_{x\to 1^-} f(x) \neq, \lim\limits_{x\to 1^+} f(x)$ and there is a jump discontinuity at x=1
10. There are three discontinuities:
a) $\lim\limits_{x\to -2^-} f(x) = \infty, \lim\limits_{x\to -2^+} f(x) = -\infty$, therefore there is a vertical asymptote at x=-2, which means there is an infinite discontinuity at x=-2 and f(-2) does not exist.
b) $\lim\limits_{x\to 0^-} f(x) = -\frac{1}{4}, \lim\limits_{x\to 0^+} f(x) = \frac{1}{4}, f(0)$ Does Not Exist. Since $\lim\limits_{x\to 0} f(x) \neq f(0)$, there is a hole at x=0 and a removable discontinuity at x=1.
C) $\lim\limits_{x\to 4^-} f(x) = -\frac{1}{6}, \lim\limits_{x\to 4^+} f(x) = \frac{1}{6}$, therefore $\lim\limits_{x\to 4^-} f(x) \neq, \lim\limits_{x\to 4^+} f(x)$ and there is a jump discontinuity at x=4
11. c=5
12. a) 0 b) 0 c) 0 d) 10 e) 9 f) Does Not Exist
13. Answers May Vary