# BACKGROUND REVIEW (OUTCOME 1)

- The following topics are required review material from PC 20 and PC 30. We will NOT be spending class time reviewing the majority of these skills.
- In order to review this content, you may visit my website at https://carignanmath.weebly.com/, go to the Calculus 30 Topic 1 page.
- In RED you will find review videos for each of these topics. I usually provide several videos for each from different Saskatchewan teachers – pick a teaching style that best matches your learning style. Please use these videos as needed to review skills. Please review appropriately.
- There will be four Summative Delta Math Assignments that you will be assigned to demonstrate understanding of these topics.

  - **Interval Notation:**
  - **Zeros and Dividing**:
  - **Solving Inequalities**:
  - **Absolute Value**
  - **Function Notation**
  - **Classifying Functions**
  - **Piecewise Functions**
  - **Function Characteristics**
  - **Function Transformations:**
  - **Domain & Range (Interval and Set Notation)**
  - **Function Operations**

- Please sign up for both my google classrooms (One is for documents and one is for videos of daily lessons). The Code is on your Course Outline.
- On the Document Google Classroom you will find instructions for signing up for DELTA MATH (I am also giving you this as a handout).
- Please sign up for Delta Math on day 1. I will spend a bit of time going over Delta Math in class – if you still have questions PLEASE ASK! You will have numerous summative assignments to do on Delta Math throughout the semester.

### SHORT REVIEW OF SOME TOPICS:

**1. Absolute value functions defined as a piecewise function:** $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

**EX:** Write the following as a piecewise function: $y = |3x - 4|$

## 2. Piecewise Functions:

Equations that need to be described by more than one equation. Following is the form of a piecewise equation:

$$y = \begin{cases} First\ Equation\ Piece, followed\ by\ its\ domain \\ Second\ Equation\ Piece, followed\ by\ its\ domain \\ Additional\ Pieces\ of\ Equation,\ their\ domain \end{cases}$$

## Interval Notation:

- CLOSED intervals contain their boundary points
- OPEN intervals contain NO boundary points.
- NOTE: IF you learned this in French Immersion you learned it slightly differently. Please take note of this method for University!
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| Open or closed? | Interval notation | Set notation | Graph of all points x |
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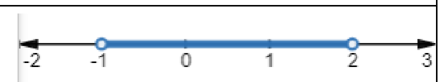
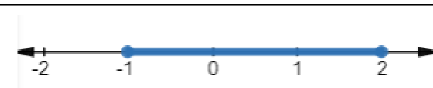
| | | |  |
| | | |  |
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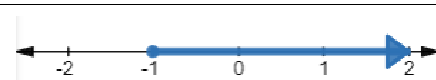
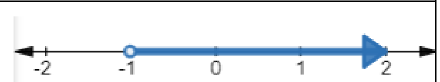
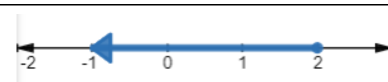
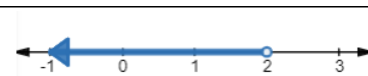
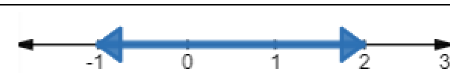
| | | |  |
| | | |  |
| | | |  |
| | | |  |
| | | |  |

- **Union** $(A \cup B)$ consists of all elements that are in A or in B or in both.
- **Intersection** $(A \cap B)$ consists of all elements that are found in both A and B.

- The following are the four Delta Math Assignments for Outcome 1. Please check the due dates on the Delta Math website or on my google calendar https://carignanmath.weebly.com/

**OUTCOME 1 DELTA MATH ASSIGNMENTS**

**Calc #1: Review of Factoring (Outcome 1)**
**Calc #2: Review of Trigonometry (Outcome 1)**
**Calc #3: Graph Analysis (Outcome 1)**
**Calc #4: Review of Special Functions (Outcome 1)**

# CALCULUS 30: UNIT 1 DAY 1 – FACTORING (OUTCOME 2/3)

**To factor using a GCF that has negative and rational exponents. To factor the sum and difference of cubes.**

VIDEO LINKS: a) https://goo.gl/P11Guu b) https://goo.gl/Pqg6DP

## REVIEW: Types of Factoring

1) **GCF:** Always take out a Greatest Common Factor first. To do this see if all numbers can be divided by the same number. If there are the same variable in all of the terms, take out the lowest exponent:

a) $-2x^2 + 12x - 4$

b) $12xyz - 24x^2y^3 + 3xy + 15x^5z^3$

c) $5c^4 + \frac{7}{3}c^2d$

**2) Polynomials of the form x²+bx+c**

- Take out GCF
- Find 2 numbers to multiply to give you the "c" value and add together to give you the "b" value

**Ex: Fully factor the following:**

a) $x^2 - 5x - 14$

b) $-3x^2+15x-18$

## 3) Polynomials of the form $ax^2+bx+c$

- Take out GCF
- Use the window/box method: https://goo.gl/dMqSeB or decomposition https://goo.gl/jg9P7e or guess and check

**Ex: Fully factor the following:** $2x^2 - 4x + 10$

## 4) Difference of Squares

**Ex. Fully factor the following:**

a) $2x^2 - 8$

b) $2x^4 - 18x^2$

c) $x^4 - 16y^4$

## 5) Factoring 4 or more terms:

- Take out GCF
- METHOD 1: Use synthetic division to factor

**Ex. Fully factor the following:**

a) $2x^3 - 5x^2 - 4x + 3$

- Method 2: Factor by Grouping

b) $2x^3 - x^2 + 6x - 3$

## 6) Factoring with Rational or Negative Exponents

To take out the GCF when the exponents are fractions, take out the smallest exponents.

**Ex. Factor the following**

a) $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

b) $-5x^{-\frac{1}{2}} - 15x^{-\frac{3}{2}} + 10x^{-\frac{5}{2}}$

c) $\frac{1}{6}x^{-\frac{2}{5}} + \frac{7}{6}x^{\frac{3}{5}}$

d) $-\frac{5}{3}x^{\frac{1}{2}}+\frac{2}{9}x^{-\frac{3}{2}}$

## NEW: SUM & DIFFERENCE OF CUBES

**CHARACTERISTICS OF A SUM OR DIFFERENCE OF TERMS**

- Two Terms
- The terms are separated by a + or a – sign
- Each term is a perfect cube

**FORMULA FOR FACTORING A SUM OR DIFFERENCE OF CUBES (Always do GCF first!)**

- $x^3+y^3=(x+y)(x^2-xy+y^2)$
- $x^3-y^3=(x-y)(x^2+xy+y^2)$

**Ex #1:** Factor the following

a) $x^3 + t^3$

b) $x^3 + 125$

c) $8x^3 - 27$

d) $1000x^{12} + 343y^6$

e) $9x^4 - 9x$

f) $(x + 6)^3 - y^3$

g) $27 - (2a - 5)^3$

**Ex #2:** Factor the following.

a) $3(x - 3)^3 - 4(x - 3)^4$

b) $4 - x^{-2}$

## NEW: CRAZY GCF (This will be used a LOT this year

**RULES FOR FACTORING:**

- You need to factor out all coefficients that are fractions (all denominators must be factored out completely, use the GCF of the numerators and take it out – you may leave )
- If the first term is negative, you must factor it out
- Factor out the smallest exponent (the "coldest" exponent") of any variable that appears in each term OR any expression that appears in each term
- Leave all factors simplified (if you can add/subtract or multiply within a bracket you must do so)

**Ex #2:** Factor the following

a) $\left(x^3+2\right)^{1/3}+\left(x^3+2\right)^{-5/3}$

b) $-12x^3(3x+5)^3+3x^2(3x+5)^4$

c) $6x(x^2+1)^2(2-3x)^4-12(2-3x)^3(x^2+1)^3$

d) $2x^3(x-2)^{-1}(x+1)^{\frac{3}{4}}-4x^2(x-2)(x+1)^{-\frac{1}{4}}$

e) $\frac{5}{2}(2x^2+3)^2(5x-1)^{-\frac{1}{2}}+8x(5x-1)^{\frac{1}{2}}(2x^2+3)$

f) $\frac{3}{10}(x-1)^{-2}(2x+1)^{-\frac{3}{4}}-\frac{9}{10}(x-1)^{-2}(2x+1)^{\frac{1}{4}}$

**RULES FOR FACTORING:**

- You need to factor out all coefficients that are fractions
  - note that this means that you must completely factor out all denominators that appear in all coefficients of each term in your original question. Your final answer should not have any denominators (other than one) within the bracketed components in your answer
  - When it comes to the numerators of these coefficients, factor out their GCF (I tend to cover up the denominators with my hand and just factor out GCF's like you did in grade 10). You may end up with coefficients in the numerator (which will be over one) within the bracketed components in your answer
- If the first term is negative within a polynomial raised to the power of one, you must factor it out
- Factor out the smallest exponent (the "coldest" exponent") of any variable that appears in each term OR any expression that appears in each term
- You need to simplify the interior of each "bracketed" portion of your answer. If you end up with polynomials raised to a power of three or two within another set of brackets, you need to expand those interior polynomials and simplify the resulting terms.
  - ie: If you end up with the following as part of your answer $\left[(x-3)^2+(2x+1)^3\right]$, you would need to expand both the $(x - 3)^2$ and $(2x + 1)^3$, and then combine their terms to get $[9x^3 + 6x^2 + 18x - 7]$
- You may leave factors or variables with negative and/or rational exponents
- Each factor is written in descending order of powers

**Duo-Tang: *Unit 1 Assignment 1* P1 #1-6**

**NOTE: Question 6will be handed in for marking**

To rationalize numerators or denominators of a given expression.

VIDEO LINKS:

**RATIONALIZAING A NUMERATOR OR DENOMINATOR**

- Will turn that numerator or denominator into a RATIONAL expression (will remove the roots)
- To rationalize the numerator or the denominator, multiply both the numerator and the denominator by the conjugate of the numerator or denominator that you are rationalizing
    - REMEMBER: The **CONJUGATE** of a binomial is a binomial that is identical to the original binomial but containing the opposite middle sign
    - Hint: Do NOT multiply out or distribute the part of the fraction that you are not rationalizing (ie if you are rationalizing the number, only FOIL the numerator, do not expand the denominator)

**Ex #1:** State the conjugate of each of the following:

a) $\sqrt{a}-\sqrt{b}$

b) $\sqrt{x+4}+2$

**Ex #2:**

a) Rationalize the numerator of

$$\frac{\sqrt{x+4}-2}{x}$$

b) Rationalize the denominator of

$$\frac{5}{\sqrt{x+3}+\sqrt{x}}$$

**UNIT 1: ASSIGNMENT #2**

**Text: P4 #1, 2**

# CALCULUS 30: UNIT 1: DAY 3 – THE LIMIT OF A FUNCTION (VISUAL PERSPECTIVE) (OUTCOME 2/3)

**To be able to understand graphically what a limit is, to find the limit graphically, to learn graphically when a limit doesn't exisit and to learn the proper notation to writing limits (Textbook Section 1.2).**

VIDEO LINKS: a) https://goo.gl/3Rchor (a bit long – start at about the 15 min mark)
b) https://goo.gl/9h3Upx c) https://goo.gl/NgjAtc

**Example 1:** The following table indicates the number of homework questions that Ms. C gave over the course of a week. What does the limit of the number of questions appear to be? Partial questions indicate questions were only some of parts a, b, c, etc. that were assigned.

| Day (d) | Number of Questions (n) |
|---|---|
| 1 | 75.5 |
| 15 | 74.75 |
| 21 | 75.5 |
| 32 | 74.375 |
| 45 | 74.3 |
| 60 | 74.25 |
| 72 | 74.214 |
| 81 | 74.188 |
| 90 | 74.167 |

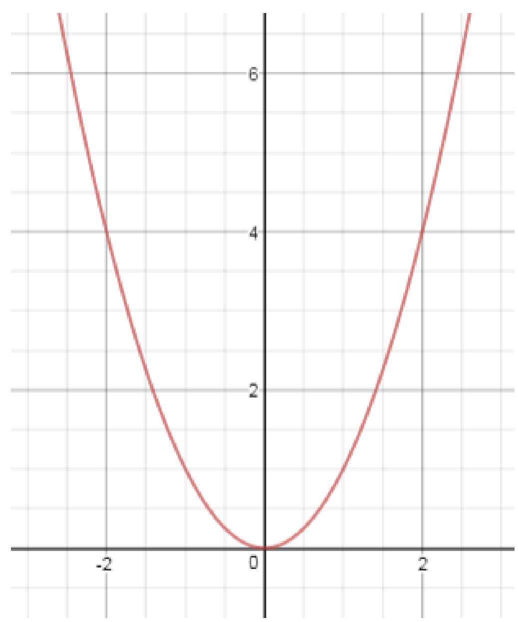
- What is the limit to the number of questions?
- Fill in the following blanks: The limit to the number of questions assigned is n = ____________ as d goes towards _____________

**Most of us understand what the word limit means outside of the world of mathematics**

- the speed that you are allowed to drive on a highway
- the amount of weight you can bench press in the gym
- how far you can argue with your parents
- how high a ball will bounce if you let it drop from your hand

**Mathematically, the limit of a function is essentially the height (the y value) that a function approaches as you approach a certain value for x.**

**Ex #1:** Given the function f(x) = $x^2$ , determine the limit as x approaches 2.




- Mathematically this would be written as follows: If f(x) = $x^2$, find $\lim\limits_{x \to 2} f(x)$
- To find this answer, you must try approaching the indicated x value of two from BOTH the left and the right side of the graph. This means you must complete the following two calculations:
  - $\lim\limits_{x \to 2^-} f(x) = \boxed{\phantom{xx}}$
    (this means to "drive" from the **left** towards x = 2 ON the graph and see how high you are on the y axis)

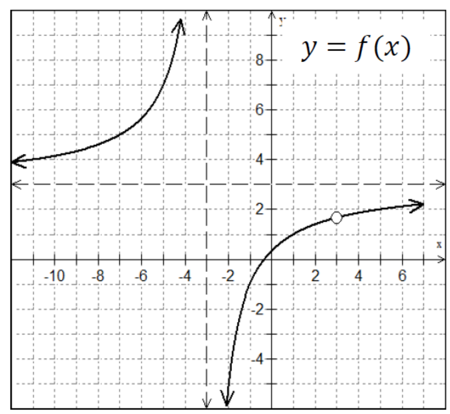
    And
  - $\lim\limits_{x \to 2^+} f(x) = \boxed{\phantom{xx}}$
    (this means to "drive" from the **right** towards a value of x = 2 ON the graph and see how high you are on the y axis
- If both of your answers in the above step were the same, you have found the limit! Therefore $\lim\limits_{x \to 2} f(x) = \boxed{\phantom{xx}}$
- **NOTE: the function does not actually have to exist at the value of the limit – there can be a hole or an asymptote at the actual location!**

Limits are the "backbone" of understanding that connect algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific *x*-value.

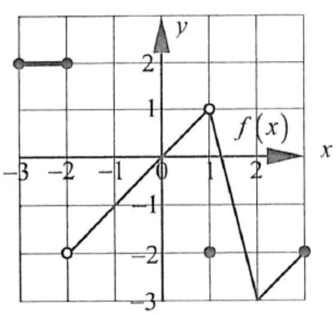
| PROPER LIMIT NOTATIONS | | |
|---|---|---|
| **TYPE OF LIMIT** | **PROPER NOTATION** | **VERBALLY:** |
| **Right-hand limit** | | |
| **Left-hand limit** | | |
| **General limit** | | |

**Ex #2:** : Use the graph to complete the table below.

| Pre-Calculus Notation vs. Calculus Notation | |
|---|---|
| As $x \to -\infty$, the graph of $f(x) \to$ ______ | As $x \to \infty$, the graph of $f(x) \to$ ______ |
| As $x \to -3$ from the right, the graph of $f(x) \to$ _____ | As $x \to 3$ from the left, the graph of $f(x) \to$ ______ |
| As $x \to -3$ from the left, the graph of $f(x) \to$_____ | As $x \to 3$ from the right, the graph of $f(x) \to$_____ |




**Ex #2:** Using the given graph, calculate each limit:




(a) $\lim_{x \to 2} f(x)$

(b) $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to -2.5} f(x)$

(d) $\lim_{x \to 1} f(x)$

(e) $f(1)$

(f) $f(-2)$

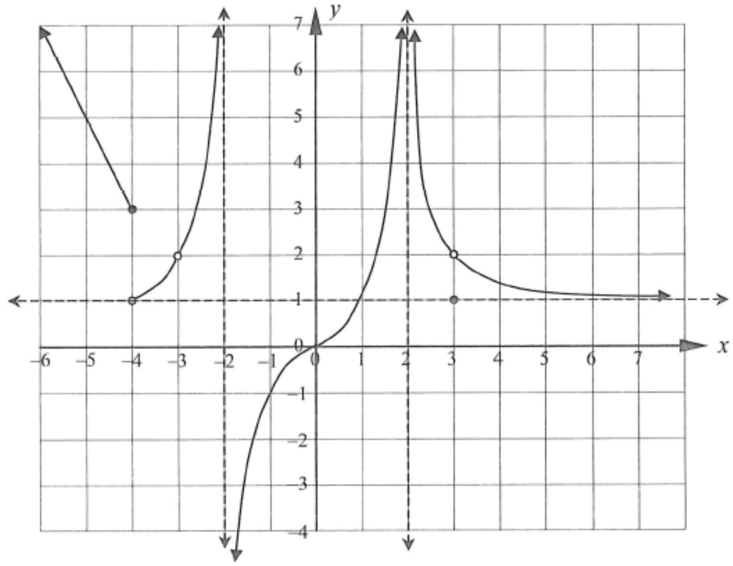
(g) $\lim_{x \to -2^+} f(x)$

(h) $\lim_{x \to -2^-} f(x)$

(i) $\lim_{x \to -2} f(x)$

**Ex #3:** Using the given graph, calculate each limit:

(a) $f(-6)$ (b) $f(0)$ (c) $f(3)$ (d) $f(-4)$ (e) $f(-3)$




(f) $\lim\limits_{x\to 1} f(x)$ (g) $\lim\limits_{x\to -5} f(x)$ (h) $\lim\limits_{x\to 0} f(x)$ (i) $\lim\limits_{x\to 3} f(x)$

(j) $\lim\limits_{x\to -4^+} f(x)$ (k) $\lim\limits_{x\to -4^-} f(x)$ (l) $\lim\limits_{x\to -4} f(x)$ (m) $\lim\limits_{x\to 2^+} f(x)$

(n) $\lim\limits_{x\to 2^-} f(x)$ (o) $\lim\limits_{x\to 2} f(x)$ (p) $\lim\limits_{x\to -2^+} f(x)$ (q) $\lim\limits_{x\to -2^-} f(x)$

(r) $\lim\limits_{x\to -2} f(x)$ (s) $\lim\limits_{x\to \infty} f(x)$ (t) $\lim\limits_{x\to -\infty} f(x)$

- NOTE: If a limit goes to either $\pm\infty$ , the BEST answer (and the one I expect) will be to first show that it goes to either $\pm\infty$ and THEN conclude that the limit DNE for that reason. IF you just say $\pm\infty$ or just say DNE you will not get full points. If the limit DNE because the limits on either side of an asymptote change between $\pm\infty$, for full marks you need to show that the limit from the left does not equal the limit from the right and then conclude the limit DNE.

**Ex #4:** The following is a table of values for the function $y=\dfrac{x^3-8}{x-2}$. Use the table to predict $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$

| **x** | $y=\dfrac{x^3-8}{x-2}$ |
|---|---|
| 1.9 | 11.41 |
| 1.999 | 11.994001 |
| 1.9999 | 11.9994001 |
| 2 | Undefined |
| 2.0001 | 12.000600001 |
| 2.001 | 12.006001 |
| 2.01 | 12.0601 |

https://www.desmos.com/calculator/smojrmbqu5

**Ex #5:** Explain the difference between the limit of a function and the value of a function.

**DEFINITION OF A LIMIT**

- If, as x approaches b from both the right and the left, f(x) approaches the single real number *L*, then *L* is called the limit of the function f(x) as x approaches b and we write

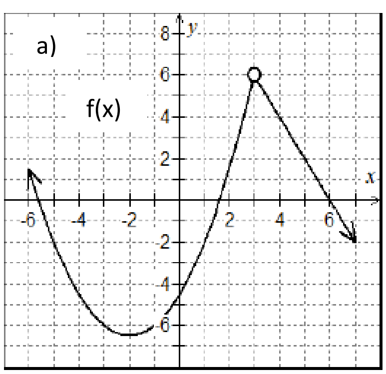
$$\lim_{x\to b} f(x) = L$$

- In order for $\lim_{x\to b} f(x)$ to exist, the limit as you approach b from either side must be the same number. That is that

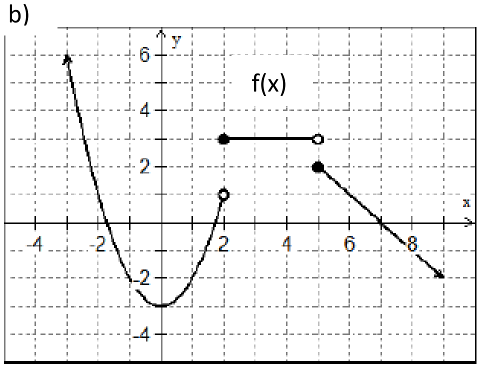
$$\lim_{x\to b^+} f(x) = \lim_{x\to b^-} f(x) = L$$ where *L* is a real number

- If $\lim_{x\to b^+} f(x) \neq \lim_{x\to b-} f(x)$ then we say that "THE LIMIT DOES NOT EXIST" or DNE
- If the limit of the graph at a value b seems to approach infinity, we can say that $\lim_{x\to b} f(x) = \infty, \therefore DNE$. Some texts will just answer $\infty$, some texts will just answer DNE but the best answer (and the answer I would like is $\infty, \therefore DNE$. Just answering $\infty$ is a bit problematic in that $\infty$ is not a number as required. Just answering DNE is somewhat ambiguous.

**Ex #5:** Use the graph to find the limit, if it exists. If the limit does not exist, explain why.




| | |
|---|---|
| **A.** $\lim_{x\to -3} f(x)$ | **B.** $\lim_{x\to -\infty} f(x)$ |
| **C.** $\lim_{x\to 6} f(x)$ | **D.** $\lim_{x\to 1} f(x)$ |
| **E.** Does $\lim_{x\to 3} f(x)$ exist? Why or why not? | |




| | |
|---|---|
| A. $\lim_{x\to 0} f(x)$ | B. $\lim_{x\to 2^-} f(x)$ |
| C. $\lim_{x\to 4} f(x)$ | D. $\lim_{x\to 5} f(x)$ |
| E. $\lim_{x\to -\infty} f(x)$ | F. $\lim_{x\to 7} f(x)$ |

**UNIT 1: ASSIGNMENT #3**

**Duo-Tang: P 2&3 # questions 3 – 14**

**DELTA MATH (Summative) : Calc 30 #5: Limits with Graphs**

To be use and choose from different methods to solve limits as x approaches a specific value (Textbook 1.2).

VIDEO LINKS:

a) https://goo.gl/MSqw2e b) https://goo.gl/e3nNb3 c) https://goo.gl/vwy4AF

- When we don't have the graph of the function that we are finding the limit of, we need to use algebraic techniques in order to find the limit
- Today you will learn 5 different techniques. Sometimes only one of the five methods will work and sometimes more than one will work (in that case you want to work to try and find the most efficient method).

## METHOD 1: SUBSITUTION

- This method involves directly substituting the value that the variable is approaching into the expression
- This method should always be the first thing you try (but you can't use it if the value that the variable is approaching is itself a non-permissible value of the expression)

**Ex #1:** Find the following limits: https://www.desmos.com/calculator/7abk3gyxrn https://www.desmos.com/calculator/6lzuwjbqm3

a) $\lim_{x \to 3} \left(x^2 - 5x + 4\right)$

b) $\lim_{x \to -3} \frac{x+9}{x-3}$

NO CALCULATOR FOR C & D

c) $\lim_{x \to 9} \frac{\log_3 x}{\sin\left(\frac{\pi x}{18}\right)}$

d) $\lim_{\theta \to \frac{2\pi}{3}} \frac{\cos\theta}{\theta}$

https://www.desmos.com/calculator/wpkamhkmoc https://www.desmos.com/calculator/c3y6y8qrwu

## METHOD 2: FACTOR AND REDUCE

- If you have a rational function, you may be able to factor the numerator and denominator and reduce the function by cancelling. At that point you may be able to use the first method of SUBSTITUTION to find the limit.

**Ex #2:** Find the following limits:

a) $\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$

https://www.desmos.com/calculator/9chsbjbgvb

b) $\lim\limits_{x\to 1}\dfrac{10x^2-10x}{x^3-1}$

https://www.desmos.com/calculator/ulpykmbicw

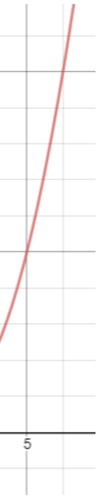
**METHOD 3: SIMPLIFYING**

- If direct substitution leaves you with a zero in the denominator and you can't factor, apply your knowledge of adding/subtracting/multiplying/dividing fractions until you simplify the expression into one that substitution will work to find the limit

**Ex #3:** Find the following limits: https://www.desmos.com/calculator/8kytudylan

a) $\lim\limits_{x\to 1}\dfrac{\dfrac{1}{x+1}-\dfrac{1}{2}}{x-1}$

b) $\lim_{h \to 0} \frac{(-2+h)^3 - 2(-2+h) + 4}{h}$




https://www.desmos.com/calculator/zwn3noae95

**METHOD 4: RATIONALIZING**

- If your function has radicals in either its numerator or its denominator, rationalize to remove the radical by multiplying the numerator and denominator by the CONJUGATE

**Ex #4:** Find the following limits:

a) $\lim_{r \to 6} \frac{\sqrt{3+r} - 3}{r - 6}$


b) $\lim_{h \to 6} \frac{6 - h}{\sqrt{10-h} - \sqrt{h-2}}$

https://www.desmos.com/calculator/xn8w8um444

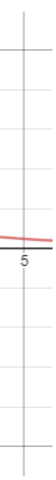
https://www.desmos.com/calculator/ldy20eorah

## METHOD 5: SIGN ANALYSIS

- This method will work with rational functions **IF** the number that the variable is approaching is **ALSO THE LOCATION OF AN ASYMPTOTE** of the function – ie we are approaching some number *a* and there is a vertical asymptote at *x* = *a*.
- When you find *f(a)* and get $\frac{k}{0}$ **OR** you end up with $\frac{k}{0}$ after factoring/canceling, then $\lim_{x \to a} f(x)$ does not exist. (This means that we wouldn't be able to use substitution in this situation because the value that the variable is approaching is also a non-permissible value and will produce a zero in the denominator )
- 
- In order to find the limit, we need to find out how the graph is behaving on either side of the asymptote – we don't actually need any specific value to find the behavior, just the sign. We perform a sign analysis of the function f(x) to see if the graph is approaching $+\infty$ or $-\infty$ on either side of the asymptote
  - If the sign analysis shows that the function is approaching $+\infty$ on both sides of *a*, we can say that $\lim_{x \to a} f(x) = +\infty, \therefore$ Does Not Exist . **NOTE: We use this definition because it gives us a good image of how the graph looks, but technically $+\infty$ is not a defined limit because $+\infty$ is a concept, not a NUMBER (limits are defined to be a REAL NUMBER L)**
  - If the sign analysis shows that the function is approaching $-\infty$ on both sides of *a*, we can say that $\lim_{x \to a} f(x) = -\infty, \therefore$ Does Not Exist
  - If the sign analysis shows that one side is approaching $+\infty$ and one side approaching $-\infty$, the limit doesn't exist because $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

**Ex #5:** Find the following limits: https://www.desmos.com/calculator/r8jd3qws4q

a) $\lim_{x \to 2} \frac{x}{x^2 - 4}$



b) $\lim\limits_{x \to 0} \dfrac{-2x-3}{x^4-4x^3-21x^2}$

https://www.desmos.com/calculator/h4f47vwocq


## LIMIT PROPERTIES

If $c$ and $k$ are real numbers, $n$ is an integer, $\lim\limits_{x\to c} f(x)$ is a real number, and $\lim\limits_{x\to c} g(x)$ is a real number, then:

1. If $f(x)$ is the constant function $f(x)=k$, then $\lim\limits_{x\to c} k = k$.
2. If $f(x)$ is the identity function $f(x)=x$, then $\lim\limits_{x\to c} x = c$.
3. *Sum*: $\lim\limits_{x\to c}\left[f(x)+g(x)\right]=\lim\limits_{x\to c} f(x)+\lim\limits_{x\to c} g(x)$.
4. *Difference*: $\lim\limits_{x\to c}\left[f(x)-g(x)\right]=\lim\limits_{x\to c} f(x)-\lim\limits_{x\to c} g(x)$.
5. *Product*: $\lim\limits_{x\to c}\left[f(x)\cdot g(x)\right]=\lim\limits_{x\to c} f(x)\cdot\lim\limits_{x\to c} g(x)$.
6. *Quotient*: $\lim\limits_{x\to c}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$, provided $\lim\limits_{x\to c} g(x)\neq 0$.
7. *Constant Multiple*: $\lim\limits_{x\to c}\left[k\cdot f(x)\right]=k\cdot\lim\limits_{x\to c} f(x)$ ($k$ is a real number).
8. *Power*: $\lim\limits_{x\to c}\left[f(x)\right]^n=\left[\lim\limits_{x\to c} f(x)\right]^n$, ($n$ is a positive integer).
9. *Root*: $\lim\limits_{x\to c}\sqrt[n]{f(x)}=\sqrt[n]{\lim\limits_{x\to c} f(x)}$, provided the root exists.

Put into words, 3 to 9 say:

3. The limit of a sum is the sum of the limits.
4. The limit of a difference is the difference of the limits.
5. The limit of a product is the product of the limits.
6. The limit of a quotient is the quotient of the limits, provided the denominator is not 0.
7. The limit of a constant multiple of a function is the constant times the limit of the function.
8. The limit of a positive integral power of a function is the power of the limit of the function.
9. The limit of a root of a function is the root of the limit, provided the root exists.

**UNIT 1 ASSIGNMENT #4:**

**Text: P19 #2, 3, 4, 5c, 6, 7, 9, 8, 10, 11, 12, 13**

**To be able to use and choose from different methods to solve limits as x approaches infinity.**

VIDEO LINKS: a) https://goo.gl/kY6D4t b) https://goo.gl/5RLxzV

The Symbol for infinity ∞ does not represent a real number. We use ∞ to describe the behavior of a function when the values in its domain or range outgrow all finite bounds.

- For example, when we say **"the limit of *f* as x approaches infinity"** we mean the limit of *f* (or the height of the y value) as x moves increasingly far to the right on the number line.
- When we say **"the limit of *f* as x approaches negative infinity (-∞)"** we mean the limit of *f* (or the height of the y value) as x moves increasingly far to the left on the number line.

**Ex #1:** Intuitively find the following limits. Imagine placing numbers close to negative or positive infinity into x and evaluating. What limits do you get?

(a) $\lim\limits_{x\to\infty} x$ (b) $\lim\limits_{x\to\infty} x^2$ (c) $\lim\limits_{x\to-\infty} x$ (d) $\lim\limits_{x\to-\infty} x^2$

**QUESTION:**

- What happens if I take a number such as 6, and continually try dividing it by larger and larger numbers? What will the answer eventually approach? $\frac{6}{2},\frac{6}{4},\frac{6}{6},\frac{6}{8},\ldots\ldots,\frac{6}{1000},\frac{6}{100\ 000\ 000},\ldots,\frac{6}{\infty}$
- What happens if I take the same question but square all of the numbers on the bottom?

$$\frac{6}{2^2},\frac{6}{4^2},\frac{6}{6^2},\frac{6}{8^2},\ldots\ldots,\frac{6}{1000^2},\frac{6}{100\ 000\ 000^2},\ldots,\frac{6}{\infty^2}$$

**DIVIDING BY INFINITY**

- Any number that is divided by a very large number (like $\infty$ ) will get so close to zero that we may as well say it is equal to zero
- $\lim\limits_{x\to\infty}\frac{1}{x}=0$ (additionally this is true for higher powers of x: $\lim\limits_{x\to\infty}\frac{1}{x^2}=0, \lim\limits_{x\to\infty}\frac{1}{x^3}=0,\ldots$ )

**Ex #2:** Given that $f(x)=\frac{x+1}{x}$, use a graph and tables to find the following: https://www.desmos.com/calculator/qyhomijc9c

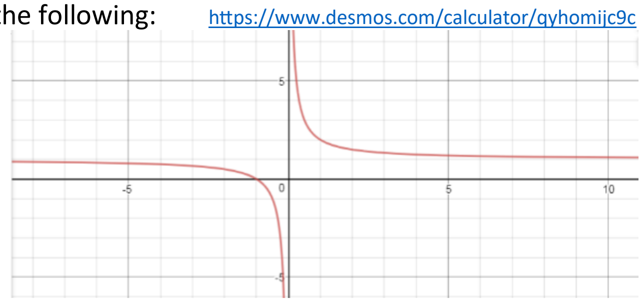
a) $\lim\limits_{x\to\infty} f(x)$

b) $\lim\limits_{x\to-\infty} f(x)$

c) Identify all horizontal asymptotes

| X | Y1 |
|---|---|
| -150 | .99333 |
| -100 | .99 |
| -50 | .98 |
| 0 | ERROR |
| 50 | 1.02 |
| 100 | 1.01 |
| 150 | 1.0067 |

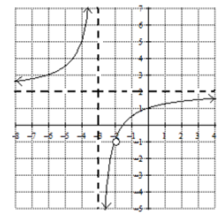
Y1=(X+1)/X



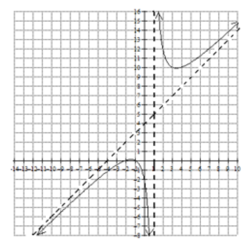
In pre-calculus, we learned three rules for determining the existence of horizontal asymptotes of rational functions. When a rational function had a horizontal asymptote, the end behavior was always such that as $x \to -\infty$ or $\infty$, then the graph of $f(x) \to$ the horizontal asymptote. We learned three rules for determining the horizontal asymptote, if one existed, for rational functions. We are about to use the idea of a limit and calculus to find out why those rules are such as they are. For each function below, divide every term in both the numerator and the denominator by the highest power of $x$ that appears in the denominator. Then, evaluate the indicated limit. Does the result of each limit make sense based on the graph that is pictured?

**Ex #3:**

a) $\lim\limits_{x\to\infty} \dfrac{2x^2+7x+6}{x^2+5x+6}$



b) $\lim\limits_{x\to\infty} \dfrac{x^2+3x+2}{x-1}$



The line $y=b$ is a **horizontal asymptote** of the graph of a function $y=f(x)$ if either

$$\lim_{x\to\infty} f(x)=b \quad \text{or} \quad \lim_{x\to-\infty} f(x)=b$$

**FINDING THE LIMIT OF A RATIONAL EXPRESSION AS $x \to \infty$ or $x \to -\infty$**

- Try and divide each term in the function by the variable with the highest degree that exists in the denominator

**Ex #4:** Find the following limits. Explain what the result of the limit means about the graph of each rational function.

a) $\lim\limits_{x\to\infty} \dfrac{2x-3}{x^2-1}$

b) $\lim_{x \to -\infty} \frac{2x^2+5}{3x^2-4x+1}$

(c) $\lim_{x \to \infty} \frac{4x^3-2x^2}{6-5x}$

https://www.desmos.com/calculator/7vwykqg69u

**FINDING THE LIMIT OF A RATIONAL EXPRESSION AS $x \to \infty$ or $x \to -\infty$**

- Multiply the numerator and denominator by the reciprocal variable with the highest degree in the denominator, simplify and apply the limit.
- If the degree of the numerator is the same as the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to ______________________________________________
- If the degree of the numerator is less than the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to ______________________________________________
- If the degree of the numerator is greater than the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to ______________________________________________

**FINDING THE LIMIT OF A RADICAL EXPRESSION AS $x \to \infty$ or $x \to -\infty$**

- NOTE: Because $\sqrt{x^2} = \pm x$, we can really say that $\sqrt{x^2} = |x|$
- With a radical expression we need to factor out a GCF from under the root sign that you will be able to take the exact root of. If there is a non radical numerator or denominator, you need to take out a GCF of the highest power of its variable
- You need to consider whether the question is asking for $x \to \infty$ or $x \to -\infty$ as this will direct you as to whether you are using $\pm x$ when the question has a $\sqrt{x^2}$ or a $|x|$

**Ex #5:** Find the following limits:

a) $\lim_{x\to-\infty}\dfrac{\sqrt{x^2-5x}}{3x+2}$

b) $\lim_{x\to\infty}\dfrac{x^2+4}{\sqrt{4x^4+x^2+1}}$

**DEFINITION Vertical Asymptote**

The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if either

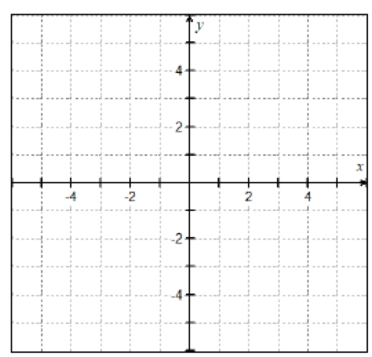
$$\lim_{x\to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x\to a^-} f(x) = \pm\infty$$

**Ex #6:** Sketch a function that satisfies the stated conditions. Include asymptotes.

$\lim_{x\to3} f(x) = -2$  $\lim_{x\to-2^-} f(x) = -\infty$

$\lim_{x\to-2^+} f(x) = +\infty$  $\lim_{x\to-\infty} f(x) = 3$

$\lim_{x\to+\infty} f(x) = \infty$




**UNIT 1: ASSIGNMENT #5**
**Duo- Tang: P4 #40-51**

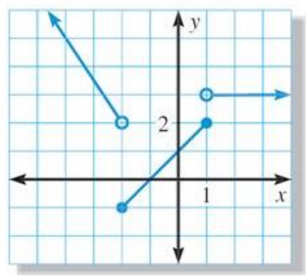
**To find the limits of absolute value functions and piecewise functions.**

VIDEO LINKS:

a) https://goo.gl/XZSHrE  b) https://goo.gl/mJiqcL  c) https://goo.gl/iZRGK3

A **piecewise function** is defined by more than one equation. Each equation corresponds to a different part of the domain of the function.

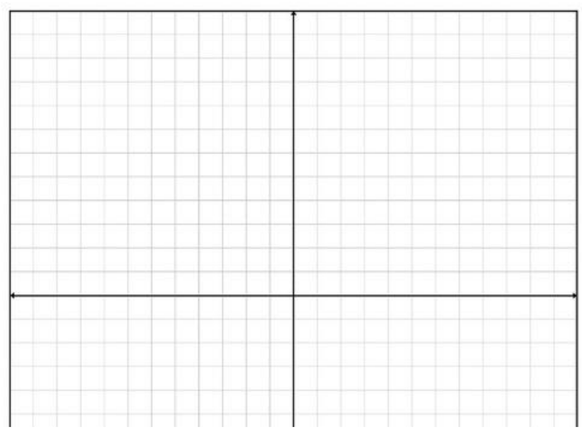
$$f(x)=\begin{cases}-\frac{3}{2}x-1, & \text{if } x<-2\\ x+1, & \text{if } -2\le x\le 1\\ 3, & \text{if } x>1\end{cases}$$



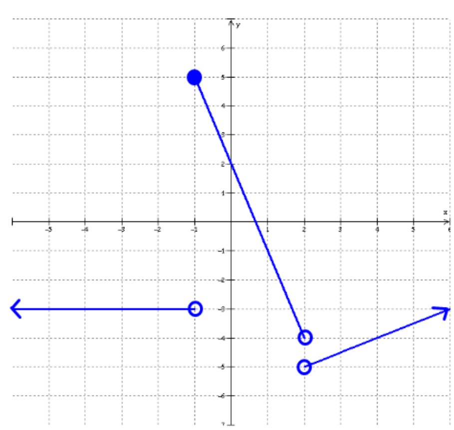

**Ex #1:** Sketch the following piecewise function:

a)

$$f(x)=\begin{cases}-1, & x\in(-\infty,-3)\\ 2x+4, & x\in[-3,1)\\ -(x-1)^2+7, & x\in[1,\infty)\end{cases}$$



**Ex #2:** Find the equation of the following piecewise function.



**Ex #3:** Given the function $f(x)=\begin{cases}(x+3)^3, x\geq -2\\(x+1)^3, x<-2\end{cases}$, determine the following limits:

a) $\lim\limits_{x\to 1} f(x)$

b) $\lim\limits_{x\to -3} f(x)$

c) $\lim\limits_{x\to -2} f(x)$

https://www.desmos.com/calculator/1dctnn6xvf

**REVIEW: ABSOLUTE VALUE AS A PIECEWISE FUNCTION**

- Recall that absolute value graphs can also be written as piecewise functions. In general,

  $y=|x|$ can also be written as $y=\begin{cases}x, x\geq 0\\-x, x<0\end{cases}$

**Ex #4:** Evaluate the following limits:

a) $\lim\limits_{x\to -1^+} \dfrac{|x^2-2x-3|}{x+1}$

b) $\lim_{x \to -1^-} \frac{|x^2 - 2x - 3|}{x+1}$

c) $\lim_{x \to -1} \frac{|x^2 + x - 20|}{x-4}$

- Remember when taking the limit of absolute value and piecewise functions you need to approach the limit from both the positive and negative sides of the x value. If the limit from the left does not equal the limit from the right, the limit at that value does not exist.

**UNIT 1: ASSIGNMENT #6**

**Duo- Tang: P4 #52-63**

**DELTA MATH (Summative) : Calc #6. Limits Analytically**

To learn what is meant by a continuous function and to learn about the three different types of discontinuities..

VIDEO LINKS:

a) https://goo.gl/xkXnq3 b) https://goo.gl/ryNScH c) https://goo.gl/F6x416

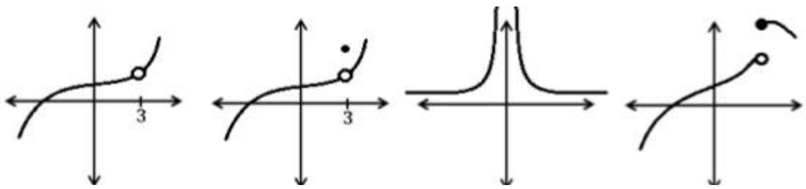
## CONTINUOUS FUNCTIONS: Formal Definition

- A function is considered to be continuous at a specific x value of "a" if the following is true:

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$$

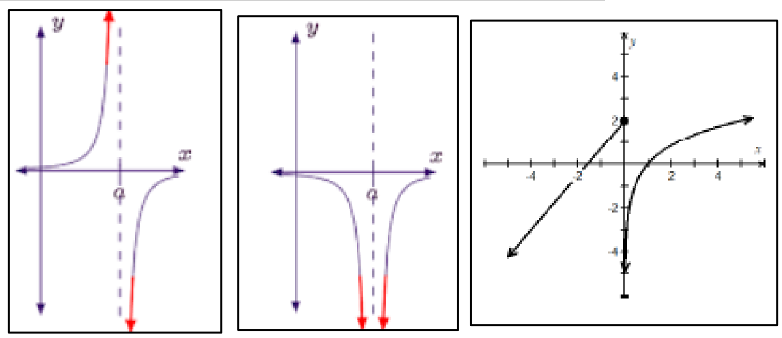
The right limit at a = The left limit at a = The actual height of the function at a

- Possible Problems:



## THERE ARE FOUR DIFFERENT WAYS THAT A FUNCTION CAN BE DISCONTINUOUS:

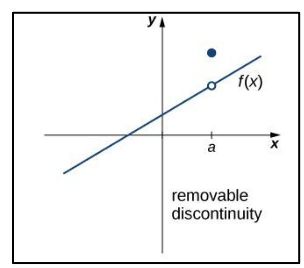
### 1. INFINITE DISCONTINUITY



- This is where the graph has a vertical asymptote and where the limit of a graph approaches $\infty$ or $-\infty$
- They can be found algebraically by finding where the values of x where the denominator of a rational function will be zero
- ***At least one of the one sided limits does not exist***

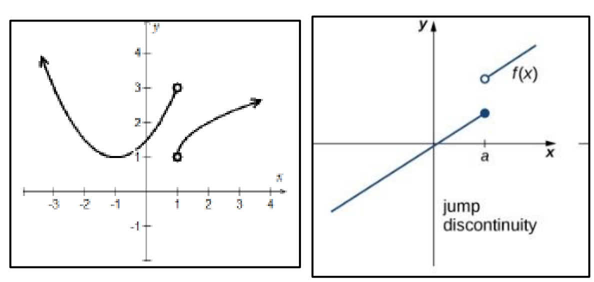
### 2. REMOVABLE DISCONTINUITY

- On the graph there will be a hole
- This happens in equations where a factor in the numerator cancels with a factor in the denominator
- The ordered pair of the hole can be found algebraically by cancelling the common factor and then substituting the x value of that factor into the resulting equation
- ***The two sided limit exists at the hole***



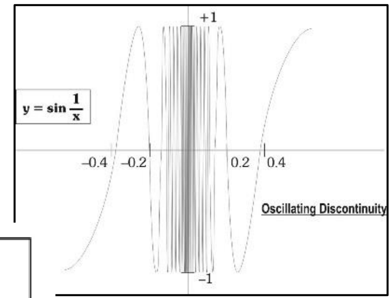

### 3. JUMP DISCONTINUITY

- On the graph this will look like the graph changes or jumps from one part of a graph to another at a specific x value
- These will usually be piecewise or absolute value functions



## 4. OSCILLATING DISCONTINUITY (NOT PART OF THE CURRICULUM)

- An **oscillating discontinuity** exists when the values of the function appear to be approaching two or more values simultaneously.




### Definition of Continuity - More Facts and Theorems

**One-Sided Continuity**

A function $f(x)$ is called

- ❑ Left-continuous at $x = c$ if $\lim_{x \to c^-} f(x) = f(c)$
- ❑ Right-continuous at $x = c$ if $\lim_{x \to c^+} f(x) = f(c)$

**Continuity at a Point**

Suppose $f(x)$ is defined on an open interval containing $x = c$.
Then $f$ is continuous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c)$$

| Continuity on an Open Interval | Continuity on a Closed Interval |
| --- | --- |
| A function is continuous on an open interval $(a, b)$ if it is continuous at each point in the interval. | A function is continuous on a closed interval $[a, b]$ if it is continuous on the open interval $(a, b)$ and the function is continuous from the right at $a$ and continuous from the left at $b$. |

**Continuity Laws of Some Basic Functions**

- ❑ Polynomial functions $P(x)$ are continuous over reals
- ❑ Rational Functions $P(x)/Q(x)$ is continuous on its domain such that $Q(c) \neq 0$.
- ❑ In addition, the composites of continuous functions are continuous. If f(x) and g(x) are continuous, f(g(x)) and g(f(x)) are continuous.

**Ex #1:** Use CALCULUS to determine whether or not the following functions are continuous (using PC 30 methods will not earn you any credit). If it is not continuous, identify the x value or the point where the discontinuity occurs and classify the discontinuity.

a) $f(x) = \dfrac{x^2 - 4}{x - 2}$

b) $f(x) = \begin{cases} x^2, x \neq 2 \\ 1, x = 2 \end{cases}$

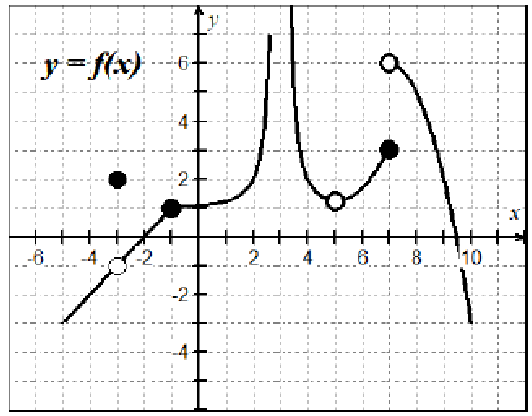
c) $f(x)=\begin{cases} x+1, x\leq 2 \\ \dfrac{x-5}{x-3}, x>2 \end{cases}$ https://www.desmos.com/calculator/1iauu8svdp

d) $f(x)=\dfrac{\left|x^2-3x+2\right|}{x-2}$ (Ignore the Vertical Portion) https://www.desmos.com/calculator/a96viw3nb5

e) $f(x) = \sqrt{x - 25}$

f) $f(x) = \sqrt[3]{x - 5}$

**Ex #3:** **For $c = -3, c = 3,$ and $c = 7$, find $f(c)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c^+} f(x)$, and $\lim_{x \to c} f(x)$. Justify your findings using the three-part definition of continuity.**



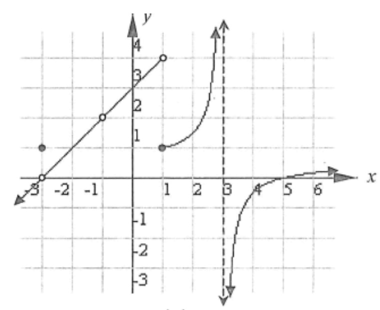

**Ex #4:** Use the three-part definition of continuity to determine if the given functions are continuous at the indicated values of *x*. State the type of discontinuity.

a. $f(x) = \begin{cases} -5, & x < -4 \\ 2x + 5, & -4 \le x \le -1 \\ 7, & x > -1 \end{cases}$ at $x = -1$

b) $g(x)=\begin{cases} x^2+3, & x<2 \\ x+5, & x>2 \end{cases}$ at $x = 2$

**Ex #4:** For example 3b above, make one change in each function that would create continuity at the indicated point.

**Ex #5:** Identify and classify the discontinuities in m(x). Use Calculus to explain why the discontinuity exists.




$m(x)$

**UNIT 1: ASSIGNMENT #7**

**Duo- Tang: P5 & 6 #1-11**

**DELTA MATH (Summative) : Calc #7. Continuity**