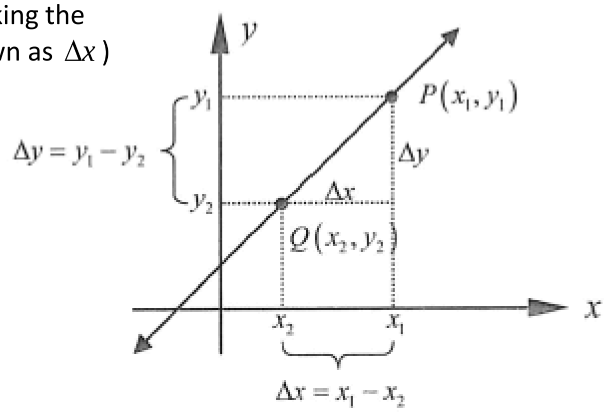
To review the concepts of slope and rate of change.

VIDEO LINKS: a) https://goo.gl/r9fhX3 b)

SLOPE OF A LINE:

- Is a measure of the steepness of a line and is found by taking the ratio of the rise (also known as) Δy to the run (also known as Δx)
- Steep lines have large slopes while lines that are almost horizontal have a small slope
- Horizontal lines have a slope of zero while vertical lines have undefined (infinite) slopes
- The formula for slope is: $m = \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$
- Point Slope Formula for a line: $y - y_1 = m(x - x_1)$
- Slope Intercept Formula for a line: y = mx + b
- Parallel lines have the same slope
- Perpendicular lines have slopes that are negative reciprocals
- When slope is given with units attached it is called a **RATE OF CHANGE**.



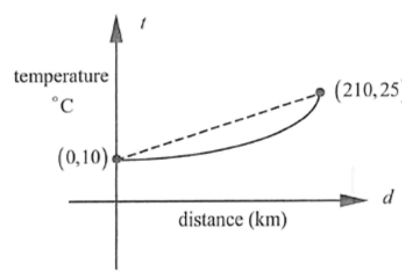
Ex #1:

a) Determine the slope of the line passing through the points (3, -7) and (-24, -28). What is the value of Δy and Δx?

b) Find the equation of the line.

Ex #2:

Anna's car displays the outside temperature. When she leaves her house the temp is 10°C. If the following graph shows the temperature as a function of the distance travelled, find the rate of change.



Ex #3: For a given function f(x), $\dfrac{\Delta y}{\Delta x} = \dfrac{-4}{3}$

a) If x increased by 3, how much does y change?

b) If x decreases by 18, how much does y change by?

Ex #4: A linear function is given by $y = 6 - 5x$. If x increases by 2, how does y change?

$$slope = \frac{\Delta y}{\Delta x} = -5, \ so...$$

Ex #5: Find the equation of the line passing through (1, -2) with a slope of $\frac{2}{3}$.

Ex #6: Find the equation of a secant line that passes through the points (-3, 5) and (-6, 7). Express your answer in slope intercept form.

Unit 2 DAY 1 ASSIGNMENT

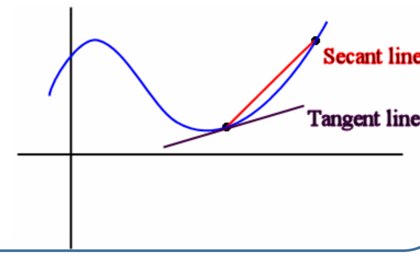
Textbook Page P9 #1 - 6, 12

To introduce the tangent line and to estimate the slope at the tangent line

VIDEO LINKS: a) https://goo.gl/gNkGBX

TANGENT LINE – A line that touches a curve in one place

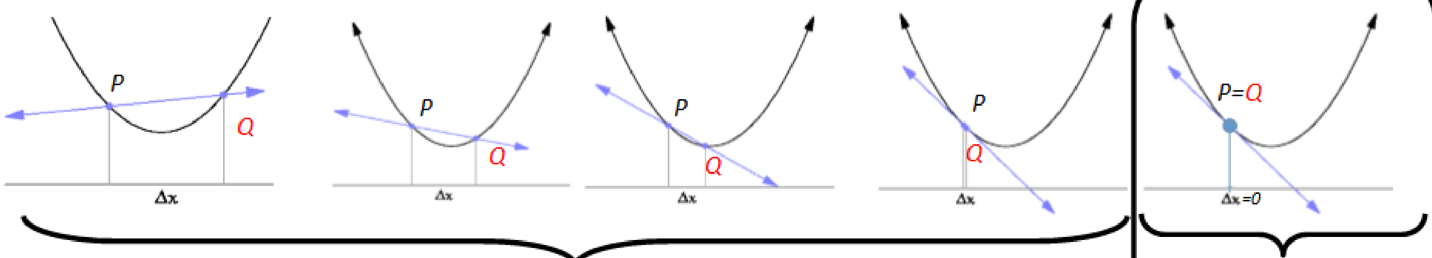
SECANT LINE – A line touches a curve in two or more places



USING THE SLOPE OF A SERIES OF SECANT LINES TO FIND THE SLOPE OF THE TANGENT LINE:

Animation Link:

https://www.google.com/search?q=secant+line+approaching+a+tangent+line&rlz=1C1GCEB_enCA811CA811&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjQi4DG8NrjAhUXZc0KHYPICaUQ_AUIESgB&biw=1366&bih=608&safe=active&ssui=on#imgdii=CwHaxrlkmhbztM:&imgrc=K7TO04Ov_WGaqM:



Secant Lines that are approaching/becoming a Tangent Line as Q moves closer to P and $\Delta x \to 0$

THESE ARE ALL INDIVIDUAL AVERAGE RATES OF CHANGE

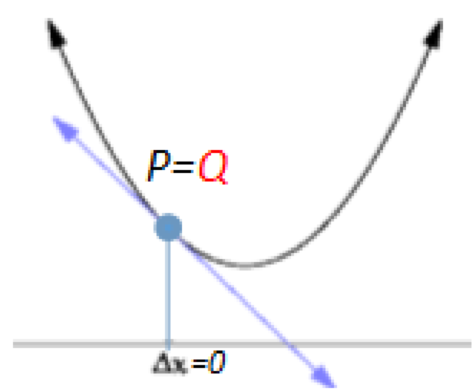
This is the Tangent Line where P=Q and $\Delta x = 0$

THE TANGENT IS THE INSTANTANEOUS RATE OF CHANGE

INTUITIVE DEFINITION OF A TANGENT LINE:

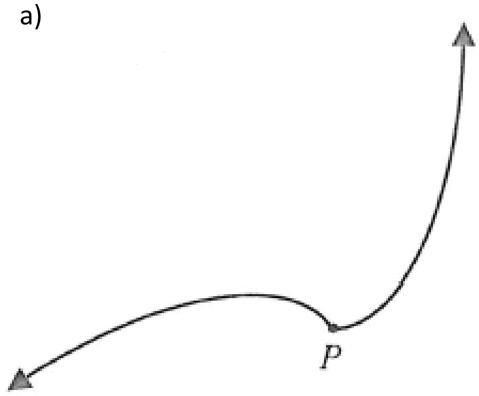
If, as we change the sequence of points Q_n, the secant lines $\overleftrightarrow{PQ_n}$ all approach the same unique line regardless as to whether the points Q_n are on the left or the right side of P, the unique line that is formed is called the TANGENT line to the curve at point P, and we say that the tangent line exists.

- In our above example, we only approached from the right. In the following diagram of the tangent, please draw three secants from the right in one colour and three secants from the left in another colour. Do your two colours merge at point P?

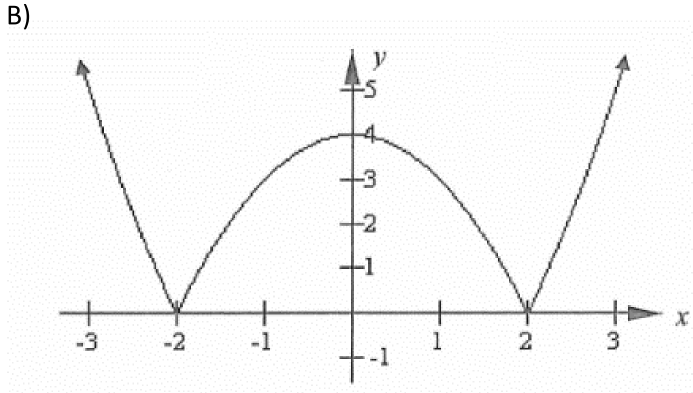


Ex #1: Explain where the following graphs do not have tangent lines and why.

a)



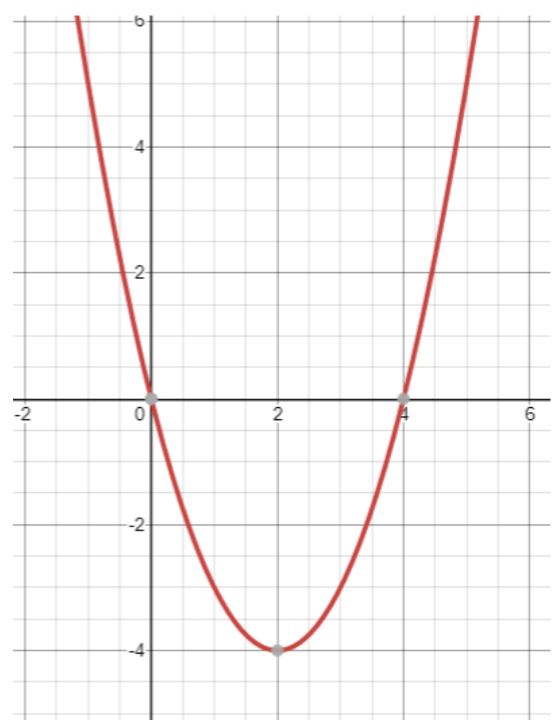
B)



PLACES WHERE TANGENT LINES DO NOT EXIST:
Tangent lines do not exist as places where the slope approached from the left does not equal the slope approached from the right. This occurs in places where the graph has:

A ________________ or a __________________

Ex #2: The following diagram is a graph of $f(x) = x^2 - 4x$. Sketch a tangent line at x = 3 and x = 0 then estimate the slope of each tangent line.

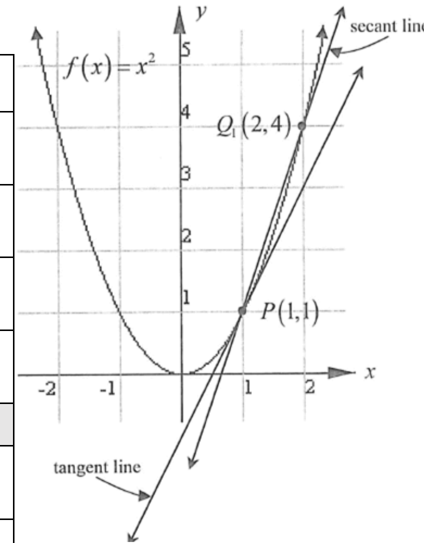


b) What is the slope when x = 2?

Ex #3:

a) Find the slope of the secant line PQ.

- **Estimate** the slope of the tangent line drawn to the function f(x) = x^2 at the point P(1, f(1)) by filling in the following table..



Approach x = 1 from the left

Approach x = 1 from the right

POINT	x	f(x)	Slope of $\overleftrightarrow{PQ_n}$
Q_1	0		
Q_2	0.5		
Q_3	0.6		
Q_4	0.9		
P	**1**	**1**	
Q_5	1.01		
Q_6	1.1		
Q_7	1.5		
Q_8	2		

b) Find the slope of the tangent line by taking the limit of the slope of the secant line. This can be expressed as:

$$\lim_{Q \to P} m_{PQ} = m \quad \text{or} \quad m = \lim_{x \to p} \frac{f(x) - f(1)}{x - p} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$$

c) Now that we know that the tangent line passes through P(2, 4) and has slope 2, what is the equation of the tangent line?

SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 1):

- Given a function f(x), the slope of a line tangent to a curve at a specific point (a, f(a)) is defined as follows:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Ex #3: **Find** the slope and the equation of the tangent line to the curve $y = 4x^2 - 3x - 1$ at the point (2, 9) by using the above slope formula.

b) What is the EQUATION of the tangent line at the point (2, 9)?

Unit 2 DAY 2 ASSIGNMENT

Duo Tang P13 # 1 – 6 Textbook Page 35 #2 & 7 (Formula 1)

CALCULUS 30: UNIT 2 DAY 3– FINDING SLOPES OF TANGENT LINES USING LIMITS

Using Limits to find slopes of tangent lines at Specific Points

VIDEO LINKS: a) https://goo.gl/wCNKSW b) https://goo.gl/NSrJFS c) https://goo.gl/KpZWGz

Today we are going to get a little more specific in how we can use limits to find the slope of a tangent line. Last day we used a table to estimate the slope of the tangent line, and then we used a new limit formula to find the slope without using a table. Today we are going to adapt that formula in such a way that it will help us transition to the next concept in Calculus 30. When we used the table to estimate the limit, we kept moving the position of Q closer and closer to the value of P. Today, instead of actually giving different values for Q, we are instead going to focus in on the horizontal distance that exists between Q and P, and see what happens when we take the limit of that horizontal distance (which we are going to call h) as it approaches zero.

Demonstration Ex #1: Determine the slope of the tangent line to the function $f(x) = x^2$ at the point $P(1, f(1))$

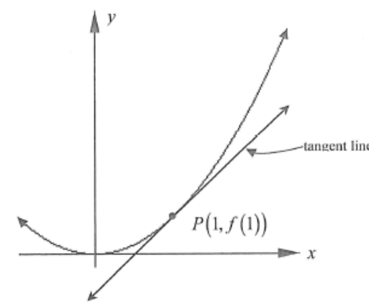


Figure 1

Step 1 Begin by drawing a sketch of the function near point P. Label point P, with its coordinates, and draw what you consider to be the tangent line at P as in Figure 1. Place point Q on the graph of the function to the right of point P. How far to the right? However far you care to go. Suppose that Q is h units horizontally to the right of P. Since the x-coordinate of P is 1, then the coordinates of point Q will be $(1+h, f(1+h))$. Draw the secant $\overleftrightarrow{PQ}$ as shown in Figure 2.

Step 2 Determine the slope of the secant line $\overleftrightarrow{PQ}$. Since the coordinates of both points P and Q can be determined, the slope can be found using $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{f(1+h) - f(1)}{(1+h) - 1}$

$$= \frac{(1+h)^2 - 1^2}{h} = \frac{1 + 2h + h^2 - 1}{h} = \frac{2h + h^2}{h} = \frac{h(2+h)}{h} = \mathbf{2} + \boldsymbol{h}.$$

This result is very significant. It says that if point Q is h units to the right of P, the slope of the secant line $\overleftrightarrow{PQ}$ will be h units larger than 2. Recall that the x-coordinate of P is 1. Thus if point Q has an x-coordinate of 1.0001, h would be 0.0001 since Q is 0.0001 units to the right of P. The slope of the secant line would thus be $2 + h = 2 + 0.0001 = 2.0001$. Check the bottom row of the table in example 2 of section 4.2. That is precisely the result we obtained.

Step 3 We know that the tangent line is the limiting position of the secant line as Q approaches P. Of course as Q approaches P, the horizontal separation between them, namely h, gets closer and closer to 0. Thus by finding $\lim_{h \to 0}(\text{secant slope})$ we can obtain the slope of the tangent line. Since the secant slope was $2 + h$, we have $\lim_{h \to 0}(2 + h) = 2 + 0 = 2$. Thus the slope of the tangent line is 2.

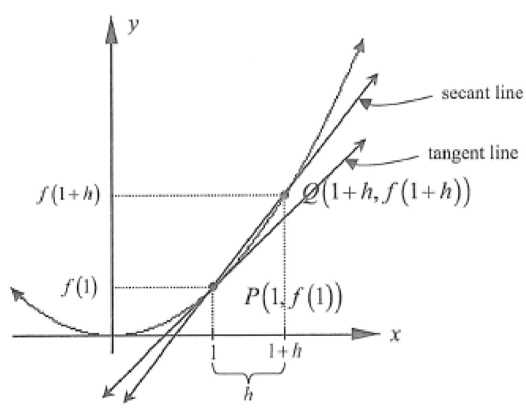


Figure 2

Note that h is always considered to be positive.

SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 2):

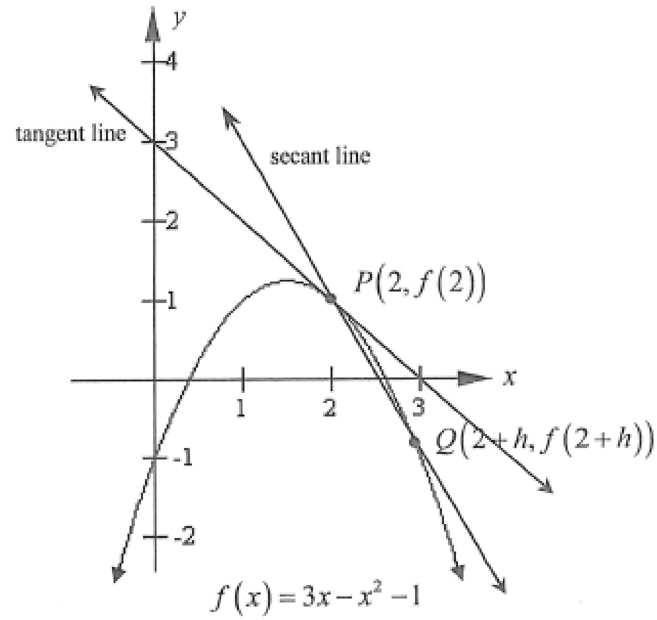
- The slope of a line tangent to a curve at a point (a, f(a+h)) is defined as follows:

$$m == \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

Ex #2:

a) Find the slope of the tangent line drawn to the function $f(x) = 3x - x^2 - 1$ at the point P(2, f(2)).
Begin by identifying the following:

- a=
- (a + h)=
- f(a)=
- f(a+h)=



$$m == \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \boxed{\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$$

b) Find the equation of the tangent line at (2, f(2))

Ex #3:

Find the equation to the tangent line drawn to the function $g(x) = x^3$ at the point (1, g(1))

Unit 2 DAY 3 ASSIGNMENT

Textbook Page 35 (ONLY USE FORMULA 2 TODAY) #1, 6ab (for i), 7abc for i, ii, v, 8ac, 9, 10

VIDEO LINKS:

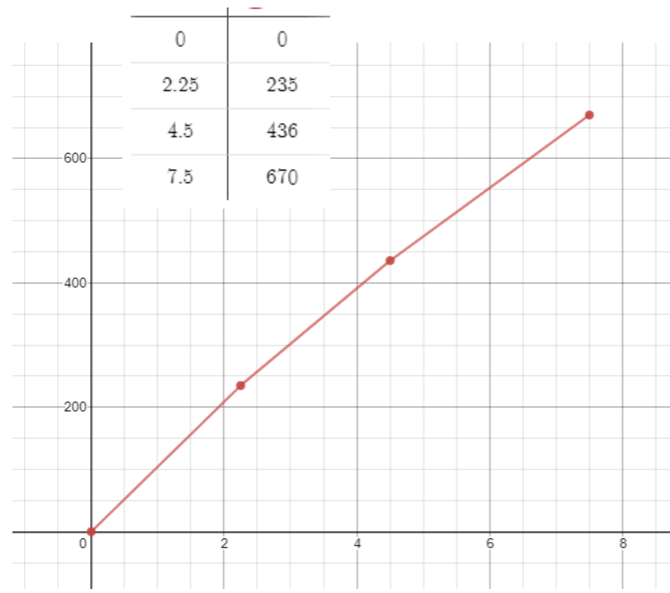
a) https://goo.gl/dWdxV6 b) https://goo.gl/9QtnkR c) https://goo.gl/alYXOd

AVERAGE VELOCITY:

$$\text{average velocity}=\frac{\text{distance travelled}}{\text{time elapsed}}=\frac{\Delta s}{\Delta t}$$

Ex #1: You drive from Regina to Calgary. It takes 2.25 hours to reach Swift Current, which is 235km from Regina. Then you stop briefly in Medicine Hat to get gas. You notice that you have now travelled a total distance of 436 km and it has taken you 4.5 hours in all. By the time you reach Calgary, you have travelled 670 km in 7.5 hours.

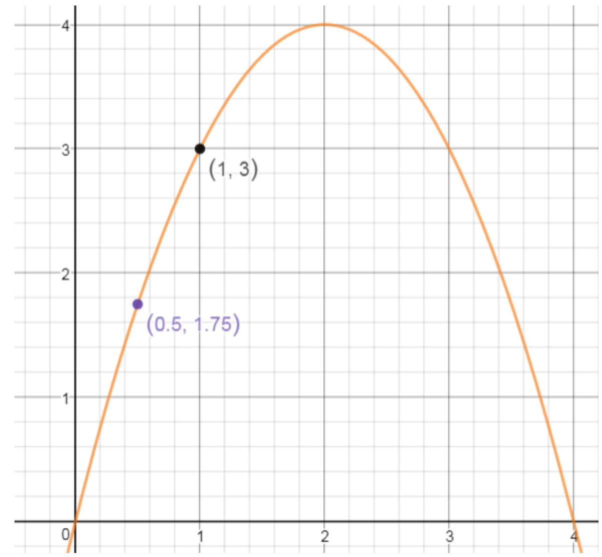
a) What is your average velocity between Regina and Swift Current?
https://www.desmos.com/calculator/yuyqkt0fhz



b) What is your average velocity between Swift Current and Medicine Hat?

c) What is your average velocity between Regina and Calgary?

Ex #2: At the end of grade 12 an unnamed student in Calculus 30 is so distraught (due to the fact that the class is over) that she tosses her binder of calculus notes into the air. Even with her grief she is able to quickly note that the path that the binder follows is the equation $y = -(x-2)^2 + 4$, where y is the height of the binder in m and x is the number of seconds that the binder is in the air.



a) Use the following graph and the equation to determine the average velocity of the binder between the 0.5s mark and the 2 second mark.

b) Can you use the equation of the function f(x) to approximate the average velocity between 0.9 seconds and 1.1 seconds?

c) How would you be able to find the instantaneous velocity at 1 second?

AVERAGE VELOCITY:

- We can expand upon our earlier definition of average velocity by saying that it is slope of a secant line between two points (x, y) and (x_1, y_1) on a distance time graph where y is distance and x is time.

$$\text{Average Velocity}=\frac{\text{distance travelled}}{\text{time elapsed}}=\frac{\text{change in distance}}{\text{change in time}}=\frac{\Delta y}{\Delta x}$$

Often the variables used are not x and y. Instead of y they often use "s". Instead of x they often use "t". In that case we would say that

$$\text{Average Velocity}=\frac{\text{change in distance}}{\text{change in time}}=\frac{\Delta s}{\Delta t}$$

- Instantaneous velocity is slope of the TANGENT line at a certain point in time. This will be the limit of an average velocity as Δt approaches zero

$$\text{Instantaneous Velocity}=v(a)=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

To actually use this limit algebraically to find an answer to the instantaneous velocity of a function at time "a", we need to apply the formula we learned in the last section:

$$\boxed{\text{Instantaneous Velocity}=v(a)=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$$

Ex #3: A ball is dropped from the top of a 400 foot tall building and falls such that its distance from the ground at *t* seconds is $s = -16t^2 + 400$ feet.

a) What is the average velocity of the ball for the first 4 seconds?

b) What is the instantaneous velocity at 4 seconds?

c) Oops – made a mistake in b. I actually wanted you to find the instantanteous velocity at 6 seconds. Nope – make that 12 seconds.

- If that actually happened and you had to redo part b twice more, would you steps looks very similar?
- Instead of redoing part b twice more, is there a more generic formula that you could find for any value of "a" that would calculate v(a)?

Ex #4: The displacement, "s"in metres, of a particle is given by $s = t^2 - 4t + 3$, where t is measures in seconds. Find the velocity of the particle at time t = a. Use this expression to find the velocity of the particle after 5 seconds.

Unit 2 DAY 4 ASSIGNMENT

Textbook Page 43 # 1a (i & ii), 1b, 2a(i & iv), 2b, 3, 5a(i only), 5b (sketch the graph), 9

Definition of a Derivative.

VIDEO LINKS:

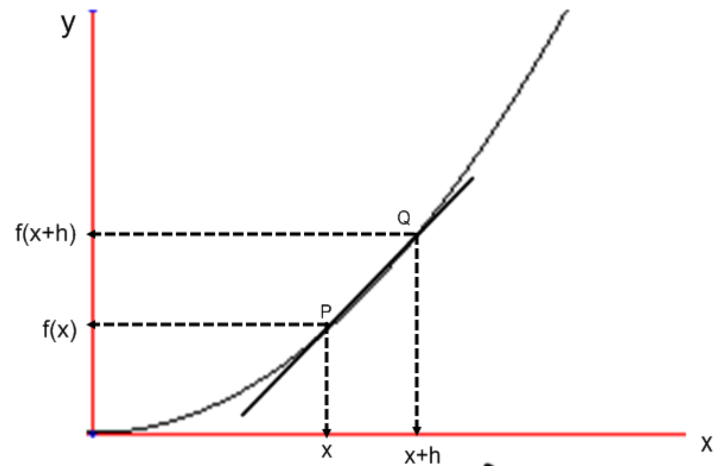
a) https://goo.gl/E3dkvY b) https://goo.gl/xYh5Li c) https://goo.gl/DM3kio

- Earlier we learned to find the slope of a tangent line to a point on a graph by using a secant line where the distance between the two points of the secant line, h, approaches zero. This slope is called **THE DERIVATIVE.**

- Slopes of Tangents at a general point (x, f(x)) = Finding the derivative

- The slope of the SECANT line PQ is a value: $m = \frac{f(x+h)-f(x)}{h}$

- In order to turn secant PQ into a tangent line (going through just P), we continually move point Q closer to P until the distance between them, h, approaches zero. To find the numerical value of the slope of the tangent line we need to use limits in the above formula.



DERIVATIVE: The derivative of a function f(x) represents the slope of a line tangent to a curve at any point (x, f(x+h)) and is defined as follows:

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

- When the above formula is used for any value of x, we leave the value of x in the formula. The answer we get will not be a numerical value for a specific slope, rather it will be a general formula that can be used to find the value of the slope at a specific value of x, x = a.

Sometimes the formula is modified to look like $f'(a) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

- This formula does not give us a general slope equation, rather it gives us the specific numerical value of the slope at a value of x = a on the graph.

- The original equation is very useful if we will be determining multiple slopes on a single equation as it gives us a simple formula that can be used easily with different values of x = a. The second equation is useful if we know for sure that we will only be computing one slope for the given function f(x)

Example 1: Use the definition of the derivative to find the slope of the tangent line to the function

a) $f(x) = x^2 \ at \ (2, f(2))$

b) $f(x) = x^2 \ at \ (-3, f(-3))$

c) Find the function f'(x) that will allow us to easily find the derivative function at any point f'(a)

Example 2:

a) use the definition of the derivative to find $f'(x)$ if $f(x) = 2x^2 - x^3$

- ask yourself – is this asking for f'(a) or f'(x)? What is the difference between the two?

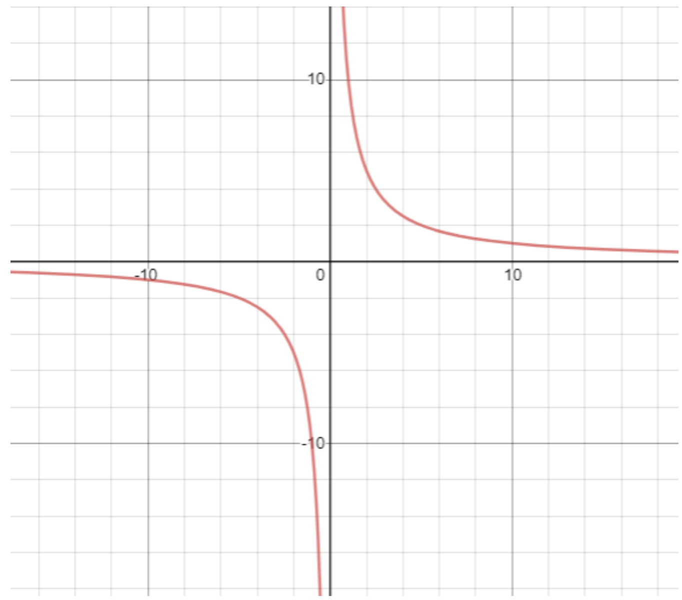
b) Use your answer from part a to find f'(3)

c) Find the coordinates of the two points on the curve $f(x) = 2x^2 - x^3$ at which the slope of the tangent line is 1.

https://www.desmos.com/calculator/0qqoxypp9j

Ex #3: Given the function $f(x) = \frac{10}{x}$, determine:

https://www.desmos.com/calculator/e5prgzf0cy



a) f'(x)

b) f'(2)

c) The equation of the tangent line at the point (2, f(2))

d) The value of x at which the slope of the tangent line to the curve is –5.

e) Where or if the slope of the tangent line will ever be positive?

There are many ways to denote the derivative of a function $y = f(x)$.

Derivative Notation	Words describing derivative notation	Important points about the derivative notation
$f'(x)$		
$f'(a)$		
y'		
$y'(a)$		
$\frac{d}{dx}y = \frac{dy}{dx}$		
$\left.\frac{dy}{dx}\right\vert_{x=a}$		
$\frac{df}{dx} = \frac{d}{dx}f(x)$		

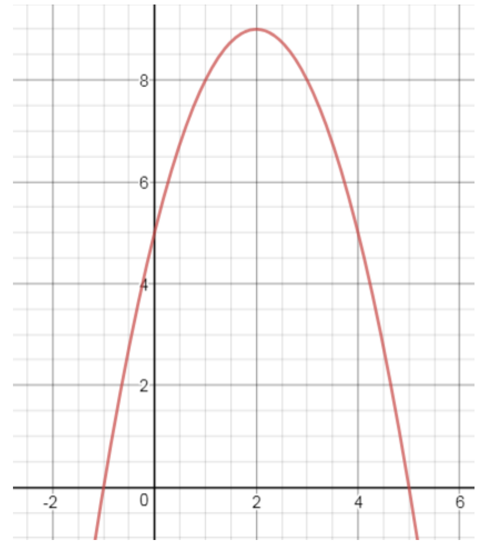
$$\therefore f'(x) = y' = y'(x) = \frac{d}{dx}y = \frac{dy}{dx} = \frac{d}{dx}f(x) = \frac{df}{dx}$$

<u>**NOTE:**</u> $\frac{d}{dx}$ should NOT be thought of as a fraction or as itself a derivative; it should be thought of as an operator that instructs you to take the derivative and treat x as the variable.

Example 4: Complete the table below, stating what each of the indicated limits finds in terms of the derivative of a function, *f*(*x*).

Definition of the Derivative (slope function)	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
Definition of the Derivative (slope value)	$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$	
Alternate Form of the Definition of the Derivative	$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$	

Ex #4: Answer the following questions using the given graph of the quadratic $y = -(x-4)^2 + 9$



a) What is f(0)?

Estimate f'(0).

What is the difference between that which each of the above values represents?

b) Where on the graph is f'(x) = 0? How do you know?

c) Find the equation of the tangent line to the curve of f(x) at x = 2.

Unit 2 DAY 5 ASSIGNMENT

DUO TANG P 14/15: FA #1a-f & h, 2, 3, 4, 5, 7, 9

CALCULUS 30: UNIT 2 DAY 6– DEFINITION OF A DERIVATIVE CONT

Definition of a Derivative.

VIDEO LINKS:

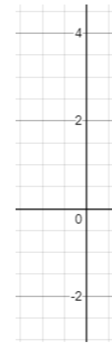
a) https://goo.gl/JN6BMH b) https://goo.gl/UneUEj (this is part one of a six part series)

THE DERIVATIVE:

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NOTE: Sometimes instead of asking for f'(x) they merely say f'

Ex #1: If $f(x)=\sqrt{x-2}$, find $f'(x)$ and state the domains of f and $f'(x)$.



https://www.desmos.com/calculator/rf4xj1l27b

Ex #2: If given the following limit of a function *f* at some value of *a*, state the value of *a* and the equation of f(x)

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{(4+h)^2-16}{h}$$

Ex #3: Find $f\,'$ if $f(x)=\dfrac{x+1}{3x-2}$.

https://www.desmos.com/calculator/7brln65nth

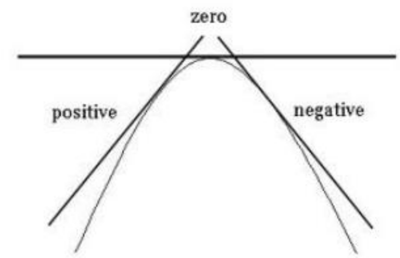
Ex #4: If $f(x) = -5x^2 + 4x$, find $f'(2)$

Ex #5: The first set of graphs represent a set of original functions f(x) and the second set of graphs represent the graphs of the derivatives f'(x) of the original functions. Match the original graph f(x) with its derivative f'(x)

https://www.intmath.com/differentiation/derivative-graphs.php

https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

Graphical Interpretation of Derivatives given *f(x)*



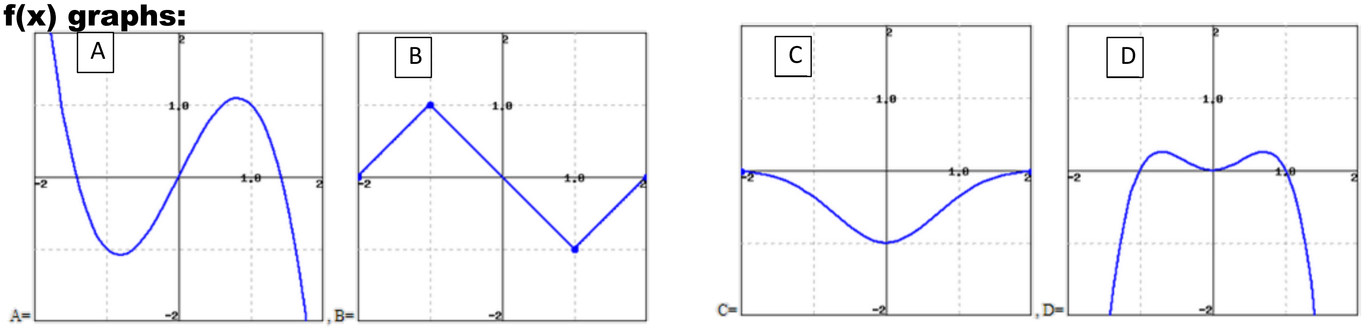
At a point $x = a$ on $f(x)$,

- If the tangent line has a positive slope, then the derivative $f'(a)$ is a positive value and is above the x-axis of $f'(x)$.
- If the tangent line has a negative slope, then the derivative $f'(a)$ is a negative value and is below the x-axis of $f'(x)$
- If the tangent line has slope zero (is horizontal), then the derivative $f'(a)$is zero and is on the x-axis of $f'(x)$
  https://www.intmath.com/differentiation/derivative-graphs.php

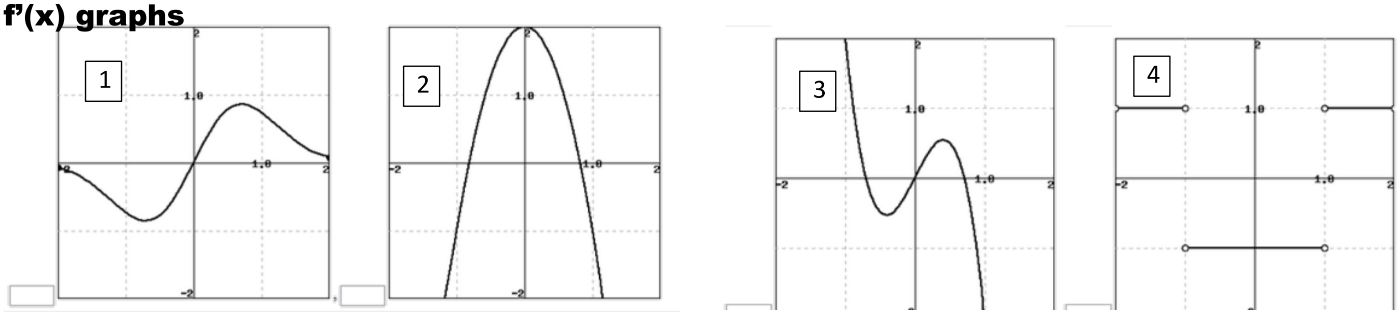
  https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

Example 6: Match the graph of f(x) with its derivative, f'(x)

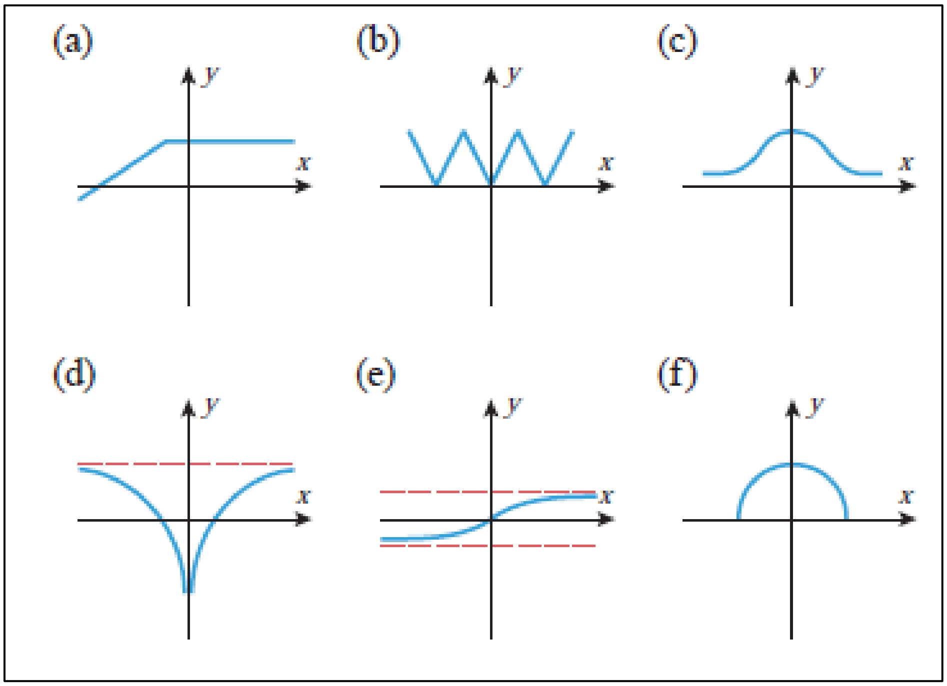
f(x) graphs:

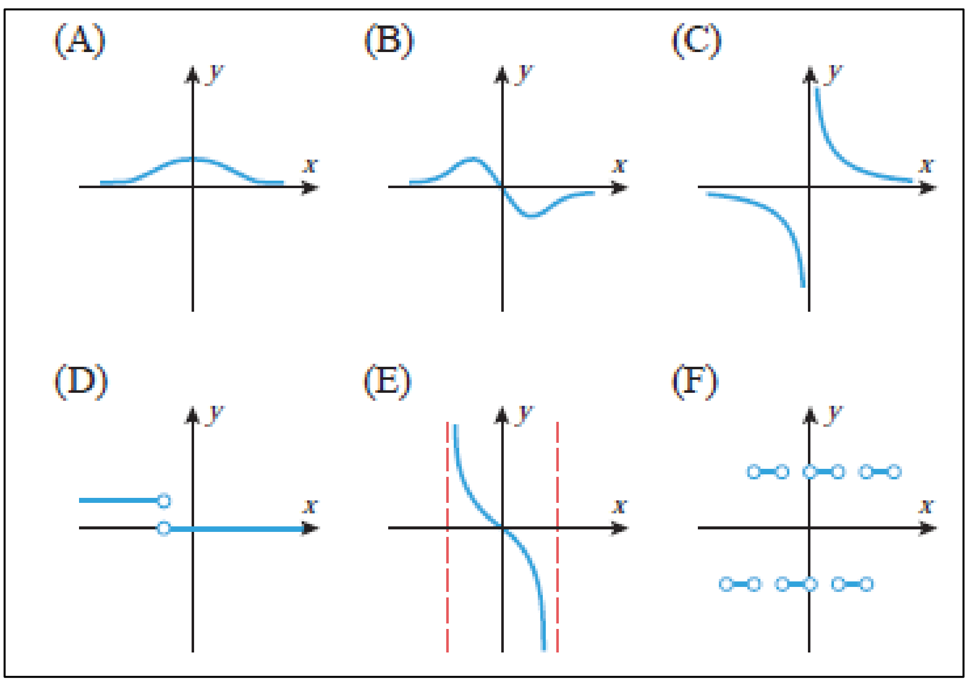


f'(x) graphs

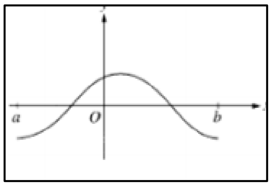


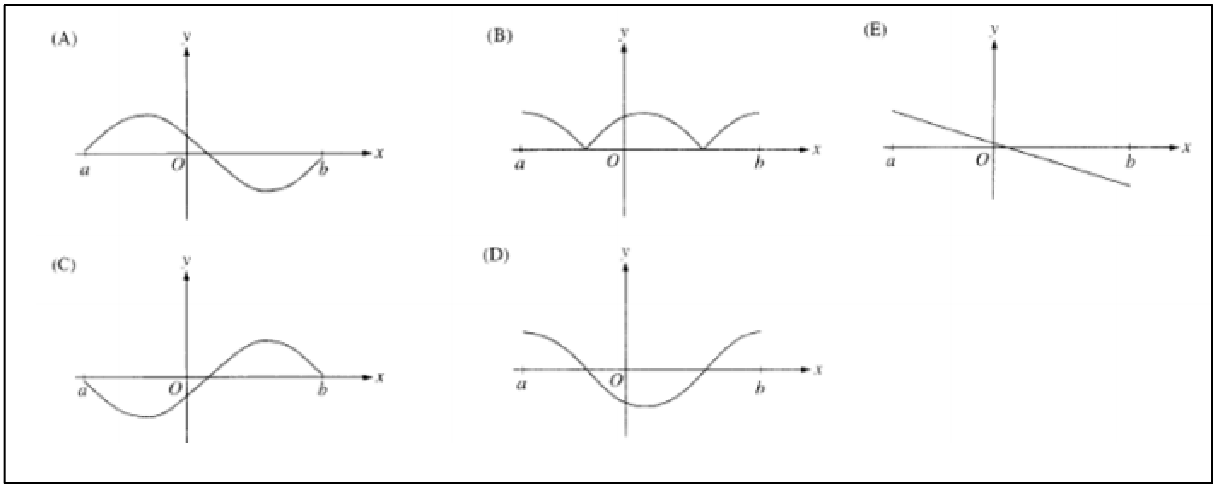
Example 7: Match the graph of the functions shown in a-f with the graphs of their derivatives in A-F



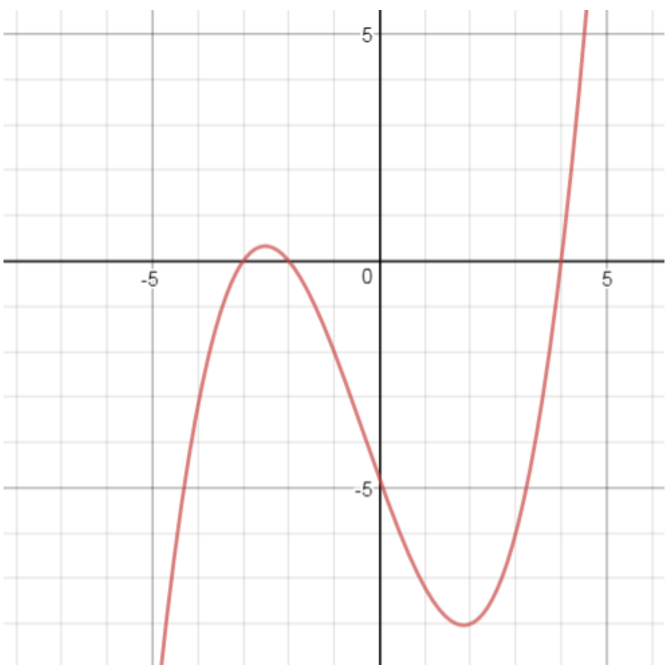


Example 8: The graph of *f* is shown in the figure below. Which of the graphs below could be the graph of the derivative of *f*?





Example 9: Given the following graph of f(x), sketch the graph of f′(x)



https://www.desmos.com/calculator/rmzuqwiyho

Unit 2 DAY 6 ASSIGNMENT

Textbook Page 75 #1b-d, 2, 3, 5, 6, 8bc, 10b, 11a, 12cd, 13, 14