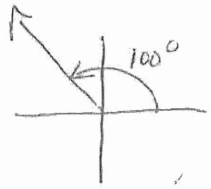
Pre – Calculus 30
Unit 4 Review
Trigonometry and the Unit Circle

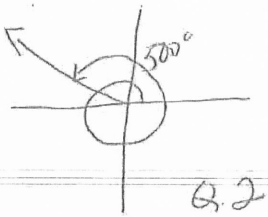
1. If each angle is in standard position, in which quadrant does it terminate? Sketch each angle.

a) 100°



Q.2

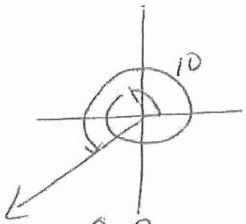
b) 500°



Q.2

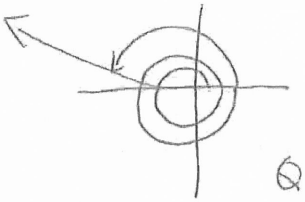
c) 10

$10 \div \pi$
$= 3.2$



Q.3

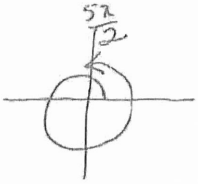
d) $\frac{29\pi}{6}$. $4\frac{5\pi}{6}$



Q.2

2. Draw each angle in standard position. Convert each degree measure to radian measure and each radian measure to degree measure. Give answers as **exact values**

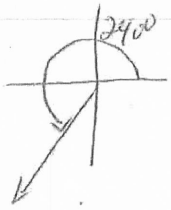
a) $\frac{5\pi}{2} = \frac{5(180°)}{2} = 450°$



b) 240°

$240\left(\frac{\pi}{180}\right) = \frac{240\pi}{180}$

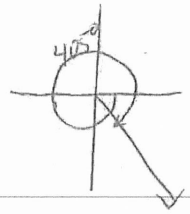
$= \frac{4\pi}{3}$



c) –405°

$-405\left(\frac{\pi}{180}\right)$

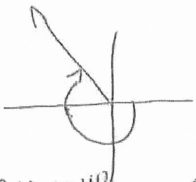
$= \frac{-405\pi}{180} = \frac{-9\pi}{4}$



d) –3.5

$\frac{\pi}{180} = \frac{-3.5}{x}$

$x = \frac{-3.5(180)}{\pi} = -200.54° = \frac{-630}{\pi}$



3. Determine the measure of all angles coterminal with each angle in the domain $-720 \le \theta \le 720$ or $-4\pi \le \theta \le 4\pi$. Draw a diagram showing the quadrant in which each angle terminates.

a) 6.75

$\rightarrow 6.75 - 2\pi(1) = 0.467$

$\rightarrow 6.75 - 2\pi(2) = -5.816$

$\rightarrow 6.75 + 2\pi(1) = 13.033$

$\rightarrow 6.75 + 2\pi(2) = 19.316$

SS: $\{-5.816, 0.467, 13.033, 19.316\}$

b) 400°

$\rightarrow 400° - 360°(1) = 40°$

$\rightarrow 400° - 360(2) = -320°$

$\rightarrow 400 + 360(1) = \cancel{760°}$

$\rightarrow 400 + 360(2) = \cancel{1120°}$

$\rightarrow 400 - 360(3) = -680°$

not in the domain

Answers are $\{-680°, -320°, 40°\}$

c) $-\frac{3\pi}{5}$

$\rightarrow -\frac{3\pi}{5} + 2\pi(1) = -\frac{3\pi}{5} + \frac{10}{5}\pi = \frac{7\pi}{5}$

$\rightarrow -\frac{3\pi}{5} + 2\pi(2) = -\frac{3\pi}{5} + \frac{20\pi}{5} = \frac{17\pi}{5}$

$\rightarrow -\frac{3\pi}{5} - 2\pi(1) = -\frac{3\pi}{5} - \frac{10\pi}{5} = -\frac{13\pi}{5}$

$\rightarrow -\frac{3\pi}{5} - 2\pi(2) = -\frac{3\pi}{5} - \frac{20\pi}{5} = -\frac{23\pi}{5}$ not in the domain

SS $= \{-\frac{13\pi}{5}, \frac{7\pi}{5}, \frac{17\pi}{5}\}$

d) −105°

$-105° - 360°(1) = -465°$

$-105° + 360°(1) = 255°$

$-105° - 360(2) = \cancel{-825}$

$-105 + 360(2) = 615°$

SS $= \{-465°, 255°, 615°\}$

4. Write an expression for all angles coterminal with each angle. Indicate what your variable represents.

a) 250° $250° \pm 360°(n), n \in N$

b) $\frac{5\pi}{2}$ $\frac{5\pi}{2} \pm 2\pi n, n \in N$

c) −300° $-300° \pm (360°)n, n \in N$

d) 6 $6 \pm 2\pi n, n \in N$

5. Use the information in each diagram to determine the value of the variable. Give your answers to the nearest hundredth of a unit.

a)

$\theta = \frac{a}{r}$

$= \frac{14}{6}$

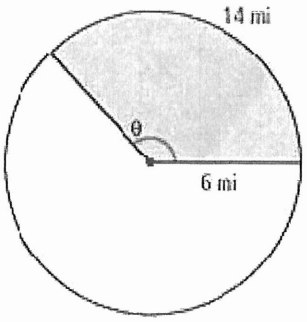
$= 2.33$

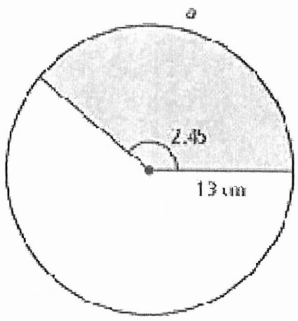
b)

$a = \theta r$

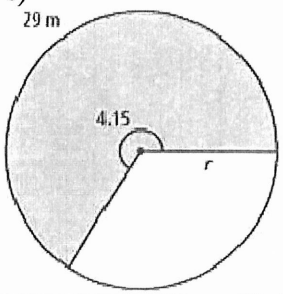
$= 2.45(13)$

$= 31.85\text{cm}$





c)

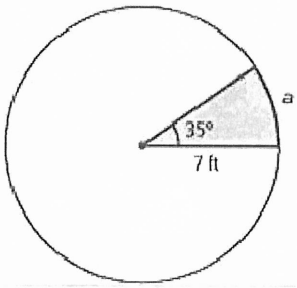


$\theta = \frac{a}{r}$

$4.15 = \frac{29m}{r}$

$r = 6.99$

d)

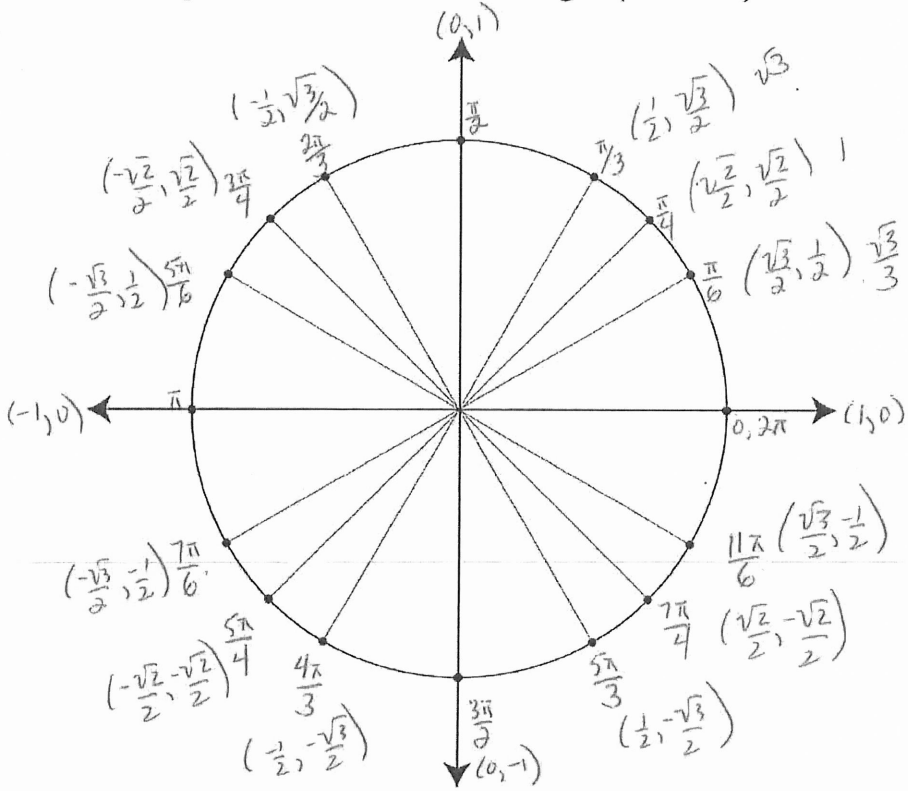


$a = \theta r$

$a = \frac{35\pi}{180}(7)$

$= 4.28 ft.$

6. Complete the unit circle with the angles (in radians) and their coordinates.



7. $P(\theta) = (x, y)$ is the point where the terminal arm of an angle θ intersects the unit circle. What are the coordinates for each point?

a) $P\left(\frac{5\pi}{6}\right)$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $P(-150°)$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) $P\left(-\frac{11\pi}{2}\right)$ $(0,1)$

d) $P(45°)$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

e) $P(120°)$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

f) $P\left(\frac{11\pi}{3}\right)$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

8. Identify all measures for θ in the interval $-2\pi \le \theta \le 2\pi$ such that $P(\theta)$ is the given point.

a) $(0,1)$ $\theta = \frac{\pi}{2}, -\frac{3\pi}{2},$

b) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\theta = \frac{11\pi}{6}, -\frac{\pi}{6}$

c) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\theta = \frac{3\pi}{4}, -\frac{5\pi}{4}$

d) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\theta = \frac{5\pi}{3}, -\frac{\pi}{3}$

9. If $P(\theta) = \left(\frac{\sqrt{5}}{3}, -\frac{2}{3}\right)$ what is the measure of θ?

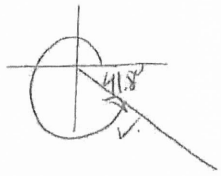
$\cos\theta = \frac{\sqrt{5}}{3}$ $\theta = 41.8°$

$\sin\theta = \frac{-2}{3}$

Q4

$\therefore 360° - 41.8°$

$= 318.2°$



10. Without using a calculator, determine the exact value of each trigonometric ratio.

a) $\sin\left(-\frac{3\pi}{2}\right) = 1$

b) $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

c) $\cot\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2} \div \frac{-1}{2}$

$= -\frac{\sqrt{3}}{2} \cdot \frac{2}{-1}$

$= \sqrt{3}$

d) $\sec(-210°)$

$= \frac{1}{\cos(-210°)}$

$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

e) $\tan(720°)$

$= 0$

f) $\csc(300°)$

$= \frac{1}{\sin 300°}$

$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

11. If $\cos\theta = \frac{1}{3}, 0° \le \theta \le 270°$, what is the value of each of the other 5 trigonometric ratios of θ? (/5)

$x = 1$

$r = 3$

$y = ?$

$x^2 + y^2 = r^2$

$1 + y^2 = 3^2$

$y^2 = 8$

$y = \pm 2\sqrt{2}$

$y = 2\sqrt{2}$ (Q_1)

S	A ✓
T	C

$x^2 + y^2 = r^2$

$(\frac{1}{3})^2 + y^2 = 1^2$

$\sin\theta = \frac{y}{r} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

$\csc\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\sec\theta = 3$

$\cot\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

12. Determine the approximate measure of all angles that satisfy the following. Give answers to the nearest hundredth of a unit. Draw a sketch to show the quadrant(s) involved.

a) $\sin\theta = 0.54, -2\pi \le \theta \le 2\pi$

$\theta_R = 0.57$

$Q_1 = 0.57, 5.71$

$Q_2 = \pi - 0.57 = 2.57, -3.71$

✓S	A ✓
T	C

$2\pi - 0.57$

$\pi + 0.57$

b) $\tan\theta = 9.3, -180° \le \theta \le 360°$

$\theta_R = 83.86°$

$Q_1 = 83.86°$, too big

$Q_3 = 180° + 83.86° = 263.86°$

$-180° + 83.86° = -96.14°$

S	A ✓
T ✓	C

c) $\cos\theta = -0.77, -\pi \le \theta \le \pi$

$\theta_R = 0.69$

$Q_2 = \pi - 0.69 = 2.45$

$Q_3 = -\pi + 0.69 = -2.45$

✓S	A
✓T	C

d) $\csc\theta = 9.5, -270° \le \theta \le 90°$

$\csc\theta = 9.5$

$\sin\theta = \frac{1}{9.5}$

$\theta_R = 6.04°$

$Q_1 = 6.04°$, too big

$Q_2 = -$

$-186.04°$

✓S	A ✓
T	C

$-$ -6.04

13. Determine each trigonometric ratio, to three decimal places.

a) $\sin 285° = -0.966$

b) $\cot 130° = \frac{1}{\tan 130} = -0.839$

c) $\cos 4.5 = -0.211$

d) $\sec 7.38 = \frac{1}{\cos 7.38} = 2.191$

14. Factor each trigonometric expression.

a) $\cos^2\theta + \cos\theta$

$\cos\theta(\cos\theta + 1)$

b) $\sin^2\theta - 3\sin\theta - 4$

$(\sin\theta - 4)(\sin\theta + 1)$

c) $\cot^2\theta - 9$

$(\cot\theta - 3)(\cot\theta + 3)$

d) $2\tan^2\theta - 9\tan\theta + 10$

20, -9 $\rangle$ $-4, -5$

$(2\tan\theta - 5)(\tan\theta - 2)$

15. Determine the exact roots for each trigonometric equation.

a) $\csc\theta = \sqrt{2}, 0° \le \theta \le 360°$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = 45°, 135°$

$\{45°, 135°\}$

S | A
T | C

b) $2\cos\theta + 1 = 0, 0 \le \theta \le 2\pi$

$2\cos\theta = -1$

$\cos\theta = \frac{-1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

S | A
T | C

c) $3\tan\theta - \sqrt{3} = 0, -180° \le \theta \le 360°$

$3\tan\theta = \sqrt{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = 30°, 210°, -150°$

S | A
T | C

d) $\cot\theta + 1 = 0, -\pi \le \theta \le \pi$

$\cot\theta = -1$

$\tan\theta = -1$

$\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\{-\frac{\pi}{4}, \frac{3\pi}{4}\}$

S | A
T | C

16. Solve for θ. Give solutions as exact values where possible. Otherwise, give approximate measures, to the nearest thousandth.

a) $2\sin x + 1 = 0, 0 \le x \le 2\pi$

$2\sin x = -1$

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

S | A
T | C

b) $\cos^2 x = \cos x, 0 \le \theta \le 360°$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ $\quad$ $\cos x = 1$

$x = 90°, 270°$ $\quad$ $x = 0°, 360°$

$\tan x(\tan^2 x - 1) = 0$

$\tan x = 0$ $\quad$ $\tan^2 x = 1$

$x = 0°, 180°$ $\quad$ $\tan x = \pm 1$

$x = 45°, 135°, 225°, 315°$

c) $\tan^3 x - \tan x = 0, 0 \le x \le 360°$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

d) $4\cos^2 x - 3 = 0, 0 \le x \le 2\pi$

e) $2\sin^2 x = -3\sin x - 1, 0 \le x \le 2\pi$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = \frac{-1}{2}$ $\quad$ $\sin x = -1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad$ $x = \frac{3\pi}{2}$

f) $6\cos^2\theta + \cos\theta = 1, 0 \le \theta \le 360°$ $\quad$ $+3, -2$

$6\cos^2\theta + \cos\theta - 1 = 0$

$(3\cos\theta - 1)(2\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{3}$ $\quad$ $\cos\theta = \frac{-1}{2}$

~~$\theta = 70.539°$~~ $\quad$ ~~$\theta_R = 60°$~~

$\theta = 289.471°$ $\quad$ $\theta = 120°, 240°$

17. Determine the ***general solution*** for the following equations.

Radians

a) $12\sin^3 x - 3\sin x = 0$

$3\sin x(4\sin^2 x - 1) = 0$

$3\sin x(2\sin x - 1)(2\sin x + 1) = 0$

$3\sin x = 0$ $\quad$ $\sin x = \frac{1}{2}$ $\quad$ $\sin x = \frac{-1}{2}$

$\sin x = 0$

$x = 0, \pi$ $\quad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\{0 + 2\pi n, \pi + 2n\pi, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi\}$

b) $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = \frac{-1}{2}$ $\quad$ $\cos x = -1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $x = \pi$

$\{\frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \pi + 2\pi n$

c) $4\sin^2 x - 3 = 0$

$4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \frac{\pm\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\{\frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

d) $\tan^3 x = 3\tan x$

$\tan^3 x - 3\tan x = 0$

$\tan x(\tan^2 x - 3) = 0$

$\tan x = 0$ $\quad$ $\tan^2 x = 3$

$x = 0, \pi,$ $\quad$ $\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\{0 + 2\pi n, \pi + 2\pi n, \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n,$

$\frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$