Chapter 5 – Angles & Parallel Lines

Intervention 1 Spend some extra time with the criteria and ask for help.	Instructional 2 Good start. You are beginning to make sense of this on your own. You are consistent with the basic learning goals for this outcome.	Independence 3 You did it and you did it on your own. You are able to complete the processes for this outcome. Your work is thorough and consistently accurate.	Mastery 4 Great work! This is going extra well for you. You have understood the outcome, are able to explain your strategies and apply these to situations. Your work is always accurate.
I need more help with becoming consistent with the criteria.	I can determine a complimentary and supplementary angle to a given angle. Given a angle measurement, I can determine the size of the bisected angle and name the original angle. I can use referents to estimate angle measurements (eg) 22.5^0, 45^0, 60^0. Given parallel or perpendicular lines, I can determine the size of angles including corresponding, alternate interior, same side interior etc..	Given parallel or perpendicular lines, I can determine and explain the reasons for the size of angles including vertically opposite, corresponding, alternate interior, same side interior etc. I can state the true bearing given a picture or basic description or given the true bearing I can state the direction. I can apply knowledge and skills to situational questions involving angles, parallel, perpendicular, and transversal lines. I can replicate, construct, and bisect angles using compass and/or protractor.	I can do multi step true bearing questions. I can describe and apply strategies for determining if lines or planes are perpendicular or parallel in situational questions. I can do multi step true bearing questions. I can create and solve relevant situational questions that involve angles and/or parallel lines and transversals, including perpendicular transversals, and explain the reasoning.

Goals:

- measure, draw, and describe angles
- estimate the measure of angles
- use certain angles to determine whether two lines are parallel
- solve problems involving angles and pairs of angles, and parallel, non-parallel, perpendicular, and transversal lines.

Key Terms:

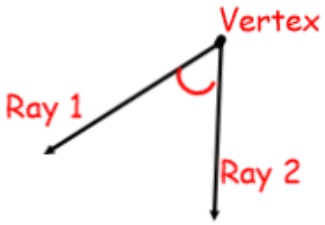
* angle * angle measure * degree* parallel lines * perpendicular lines * transversal

---

5.1 – Measuring, Drawing & Estimating Angles

ANGLES:

An angle is formed when two rays meets at a common endpoint called a vertex. Angles are measured with tools, such as a protractor, that are marked in degrees.

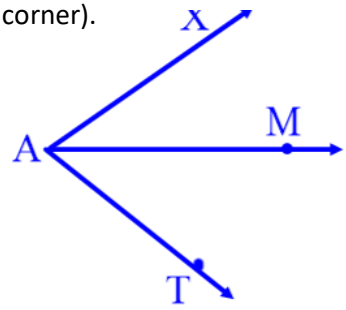


Naming Angles:

Angles are names using 3 letters. The middle letter is always the vertex of the angle(corner).

One angle is ∡MAT (or ∡TAM)

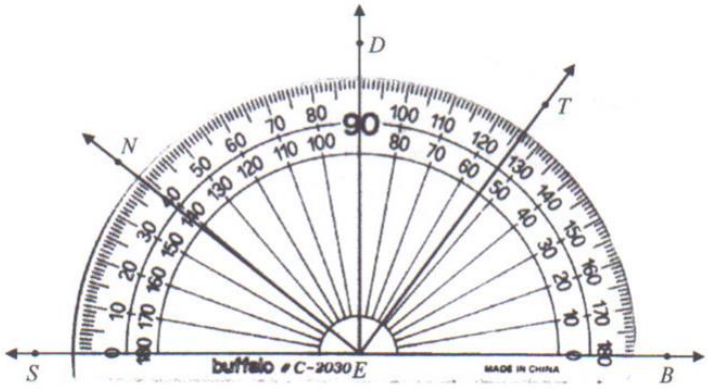
Name the other two angles in the diagram



Angle measure: measure of an expressed angle in degrees.

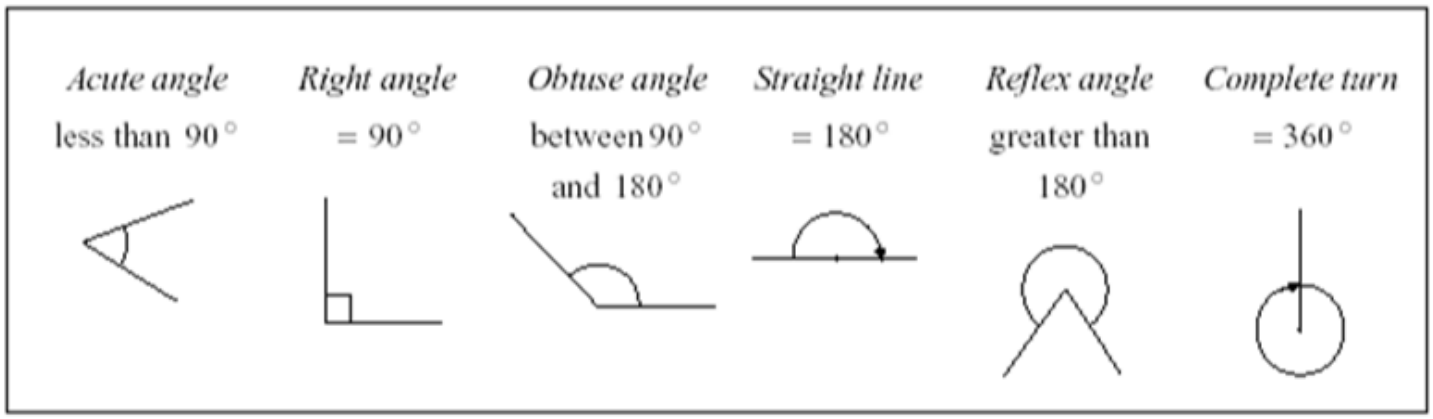
Example 2: Use the angles on the protractor to find the angle measure of the following angles

a) ∠SED =
b) ∠NEB =
c) ∠SET =
d) ∠NET =



Names of Angles

Acute Angles: measure between 0° and 90°
Right Angles: measure 90°
Obtuse Angles: measure between 90° and 180°
Straight Angles: measure 180°
Reflex Angles: measure between 180° and 360°



To estimate an angle you can use **Referent Angles**. These angles are easy to visualize and can help you determine the approximate size of a given angle.

90°

45°

30°

CONSTRUCTIONS

You have used a protractor and ruler to draw angles. You can also draw certain angles with a ruler and compass, and you can replicate any angle with these tools.

Example 1: Use a ruler and compass to create the following angles.

a) Draw a 90^0 angle to a point on a line https://www.mathsisfun.com/geometry/construct-perponline.html

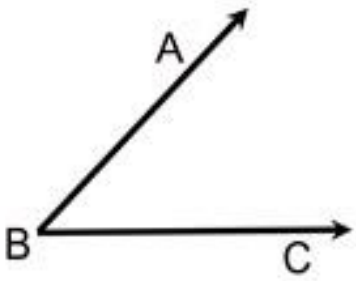
Given: point *P* on a given line
Construct: a line through *P* perpendicular (90^0)to given line.

Bisect an angle

STEPS:
1. Place your compass point on *P* and swing an arc of any size below the line that crosses the line twice. You will be drawing at least a semicircle.
(*Note:* While you can draw this arc above or below the line, below the arc keeps the construction lines from bumping into one another.). The only part of this semicircle that we really need are the two parts that cross ouroriginal line.
2. Stretch the compass LARGER!
3. Place the compass point where the arc crossed the line on one side and make a small arc above the line (the arc could be below the line if you prefer).
4. Without changing the span on the compass, place the compass point where the first arc crossed the line on the OTHER side and make another arc. Your two small arcs should be intersecting.
5. Using a straightedge, connect the intersection of the two small arcs to point *P*.

a) Construct an angle congruent (exactly the same size as) another angle
https://www.mathsisfun.com/geometry/construct-anglesame.html

Given: ∠*ABC*
Construct: an angle congruent to ∠*ABC*.
(*make a copy of the angle*)

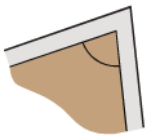


Copy an angle

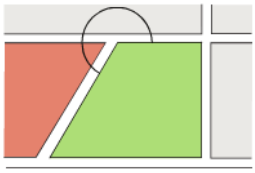
STEPS:
1. Using a straightedge, draw a reference line, if one is not provided.
2. Place a dot (starting point) on the reference line.
3. Place the point of the compass on the vertex of the given angle, ∠*ABC* (vertex at point *B*).
4. Stretch the compass to any length that will stay "on" the angle.
5. Swing an arc so the pencil will cross BOTH sides (rays) of the angle.
6. Without changing the size of the compass, place the compass point on the starting point (dot) on the reference line and swing an arc that will intersect the reference line and go above the reference line.
7. Go back to the given angle ∠*ABC* and measure the span (width) of the arc from where it crosses one side of the angle to where it crosses the other side of the angle. (Place a small arc to show you measured this distance.)
8. Using this width, place the compass point on the reference line where the previous arc crosses the reference line and mark off this new width on your new arc.
9. Connect this new intersection point to the starting point (dot) on your reference line.
10. Label your copy.

Mental Math Estimations are made in many trades that use angles. Imagine that you are working as a tradesperson in these situations and make the following estimations (aim to be within 5^0)

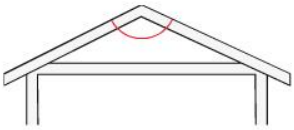
a) a landscaper estimating the angle of the corner of a garden bed



b) a surveyor estimating the angle of a property boundary line on a map



c) a roofer estimating the angle of the peak of a roof



d) a cabinet-maker estimating the angles of two corners of a shelf

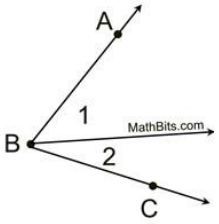
Example 2:

Estimate the measure of these angles without using a measuring device.

Adjacent Angles are two angles that share a common vertex, a common side, and no common interior points. (They share a vertex and side, but do not overlap.)

$\angle 1$ and $\angle 2$ are adjacent angles.

$\angle ABC$ and $\angle 1$ are NOT adjacent angles. ($\angle ABC$ overlaps $\angle 1$.)



Example 3:

Sort the following angles into pairs of <u>complementary angles</u> (two angles that have measures that add up to 90^0) and <u>supplementary angles</u> (two angles that have measures that add up to 180^0).

$\angle 1 = 42^0$ $\angle 5 = 121^0$

$\angle 2 = 107^0$ $\angle 6 = 31^0$

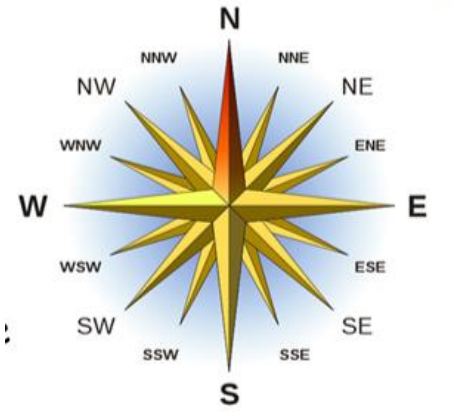
$\angle 3 = 59^0$ $\angle 7 = 19^0$

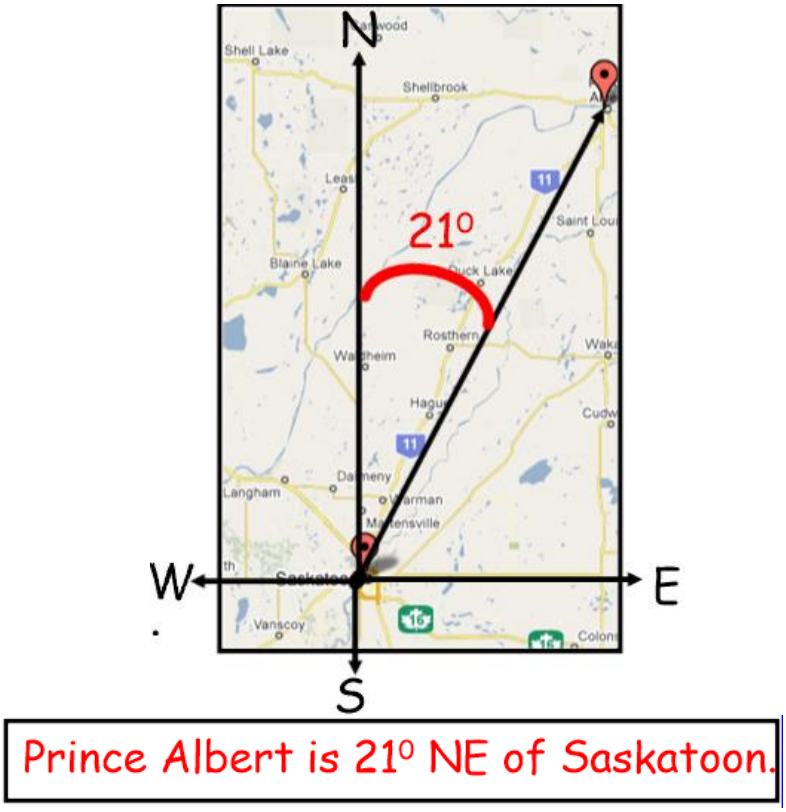
$\angle 4 = 48^0$ $\angle 8 = 73^0$

<u>complementary angles</u>

<u>supplementary angles</u>

In navigation, the angle is measured relative to true north, which is 0^0 and may be expressed as a bearing.
A true bearing describes the number of degrees, measured clockwise between an imaginary line pointing towards true north and another imaginary line pointing towards an intended direction. For example:
East is at a 90^0 angle from true north.

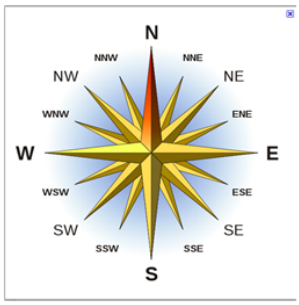


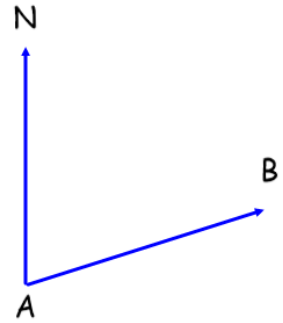


Compass roses (above) were originally created to navigate at sea. Although marine navigators now use other technologies, compass roses are still used extensively today to describe weather patterns and wind directions relative to true north.

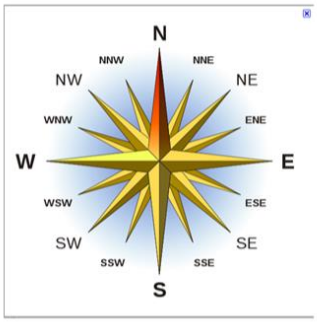
Example 4:

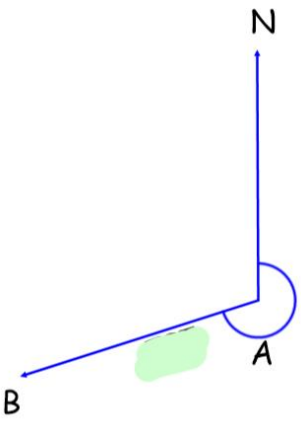
a) Determine the true bearing between A and B





b) Determine the true bearing between A and B





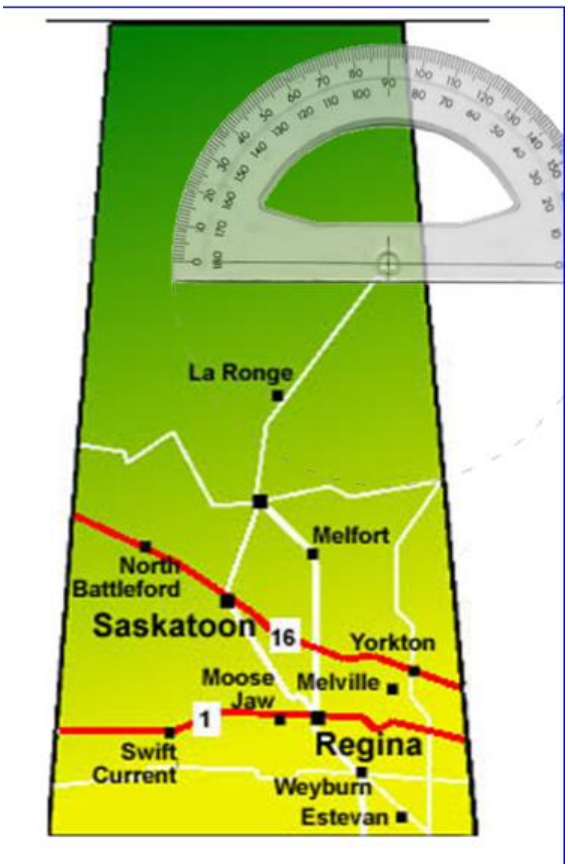
Example 5:

True Bearing with Multiple Points.

You are travelling from Saskatoon to Swift Current. What is your true bearing?

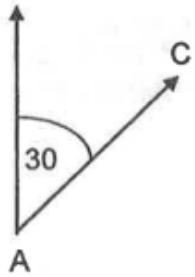
From Swift Current, you need to go to Regina. What is your true bearing?

From Regina you will travel back to Saskatoon. What is your true bearing?



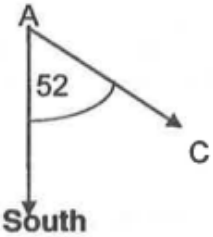
There are two ways to express a navigational heading or a "bearing".

1. When a single angle is given, it is understood that the bearing is measured in a clockwise direction from due north. The bearing from A to C is 30 .



2. The other system starts with a north or south line and uses an acute angle to show direction.

The bearing from A to C is S 52^0 E (52 East of South)



Example 6:

Give a diagram that represents each bearing.

1. bearing of 32^0

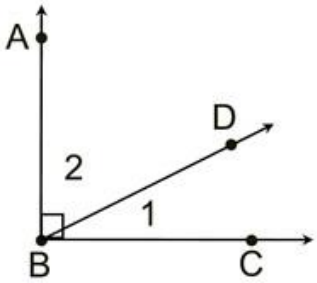
2. bearing of 304^0

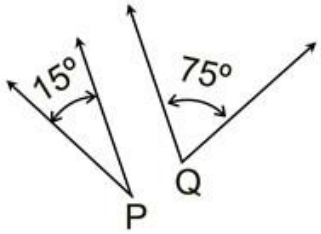
3. N 42^0 E

4. S 31^0 E

5. N 52^0 W

Complementary Angles are **two** angles the sum of whose measures is 90º.

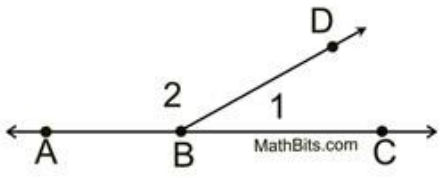


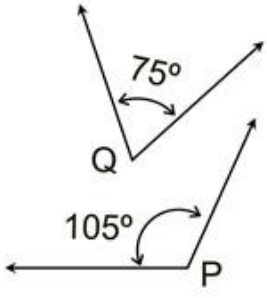


Complementary angles can be placed so they form perpendicular lines, or they may be two separate angles.
∠1 and ∠2 are complementary.
∠*P* and ∠*Q* are complementary.

$\overline{AB} \perp \overline{BC}$

Supplementary Angles are **two** angles the sum of whose measures is 180º.





Supplementary angles can be placed so they form a linear pair (straight line), or they may be two separate angles.
∠1 and ∠2 are supplementary.
∠*P* and ∠*Q* are supplementary.

The line through points *A*, *B* and *C* is a straight line.

Example 7:

Given each of the following angles, determine the size of the **complement** and/or the size of the **supplement** (if they exist).

a) 75°

b) 43°

c) 103°

d) 87°

e) 300°

5.1 Assignment : WORKBOOK
Build Your Skills – Page 215 #1, Page 217 #2-4, P 219-220 #6-9, P 222 #9
Practice Your Skills Page P 222-224 #1-5

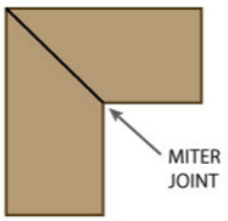
5.2 – Angle Bisectors & Perpendicular Lines

Bisecting an object involves dividing it into two congruent (equal) parts. When you bisect an angle, such as a 76^0 angle, you divided into two 38^0 angles. The line, line segment, or ray that separates the two halves of a bisected angle is called the angle bisector.

A right (90^0) angle can be thought of as a bisected straight (180^0) angle. Perpendicular lines, and line segments form right angles. Can you identify any perpendicular lines or line segments in your classroom? You can probably identify several rectangular or square objects that contain them.

Perpendicular lines and line segments are drawn using the same techniques that are used to bisect angles. Because perpendicular lines and line segments are made so often, specialized tools such as a framing square (also known as a carpenters' square) were created.

Mitre joints are common in woodworking. The ends of two pieces of wood are cut with angles having the same measure. When they are joined, they create a 90^0 (right) angle. The mitre joint acts as a bisector. What would happen if the angles of the cuts were even slightly off?



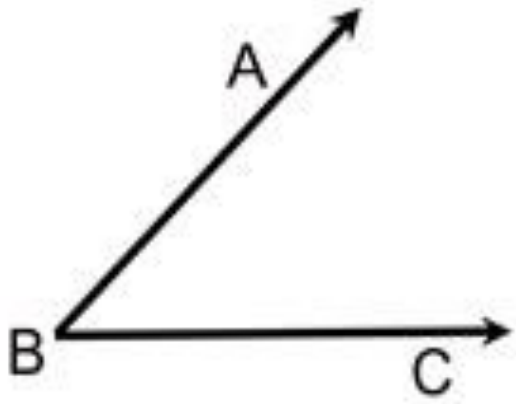
CONSTRUCTIONS

Example 1: Use a ruler and compass to bisect the following angle.

a) Bisect the given angle https://www.mathsisfun.com/geometry/construct-anglebisect.html

Given: $\angle ABC$

Construction: bisect $\angle ABC$.



Bisect an angle

STEPS:

1. Place compass point on the vertex of the angle (point *B*).

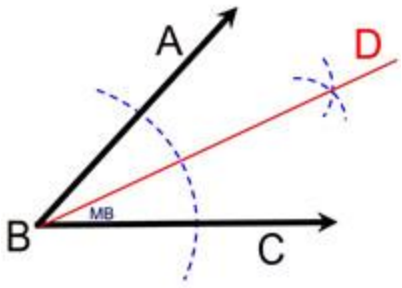
2. Stretch the compass to any length that will stay ON the angle.

3. Swing an arc so the pencil crosses both sides (rays) of the given angle. You should now have two intersection points with the sides (rays) of the angle.

4. Place the compass point on one of these new intersection points on the sides of the angle.
If needed, stretch the compass to a sufficient length to place your pencil well into the interior of the angle. Stay between the sides (rays) of the angle. Place an arc in this interior (it is not necessary to cross the sides of the angle).

5. Without changing the span on the compass, place the point of the compass on the other intersection point on the side of the angle and make a similar arc. The two small arcs in the interior of the angle should be intersecting.

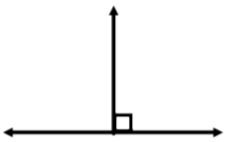
6. Connect the vertex of the angle (point *B*) to this intersection of the two small arcs.
You now have two new angles of equal measure, with each being half of the original given angle.



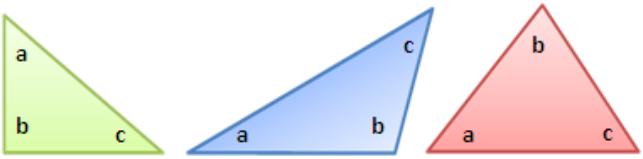
$\angle ABD \cong \angle CBD$

Review of Prior Knowledge

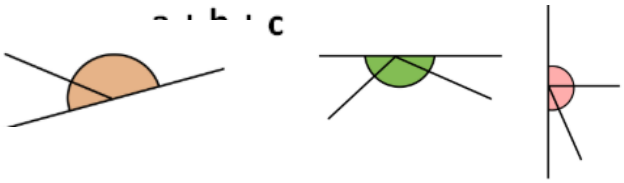
- Perpendicular Lines form a 90^0 angle



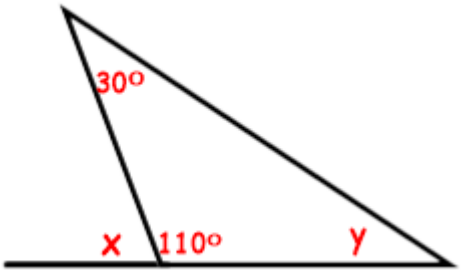
- The sum of angles in a triangle are always 180^0



- Angles that create a straight line total 180^0

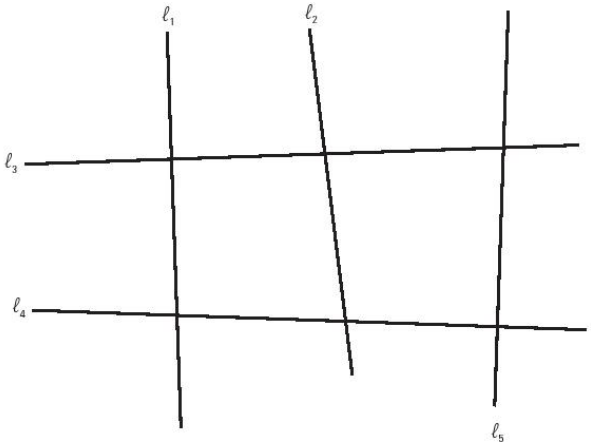


Example 2: Find x and y in the following diagram.



Example 3:

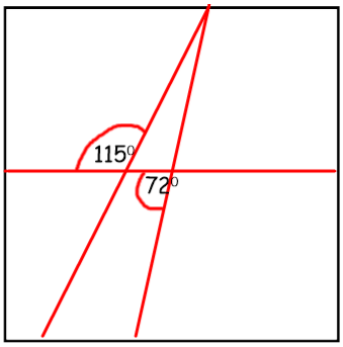
Using a protractor, determine which of the following lines are perpendicular.



5.2 Assignment : WORKBOOK
Build Your Skills – Page 226-227 #1-4 Page 228 #5-7
Practice Your Skills Page P 229 – 230 #1-5

5.3 – Non-Parallel Lines & Transversals

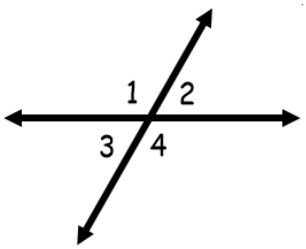
Michelle Diaz is an interior decorator from Winnipeg, MB. She makes multicolored cushion covers by marking off the fabric with chalk. The lines she drew are shown to the right.What are the measures of the six unmarked angles?



A variety of objects and materials such a trusses, railroad tracks, and fabrics contain intersecting lines. The measures of certain angles created by intersecting lines and the ability to identify types of angles can indicate whether these lines are parallel or non-parallel.

VERTIALLY OPPOSITE ANGLES

When two lines intersect each other, four distinct angles are created. The angles that share a side are adjacent angles. Angles that share only a vertex are vertically opposite angles.

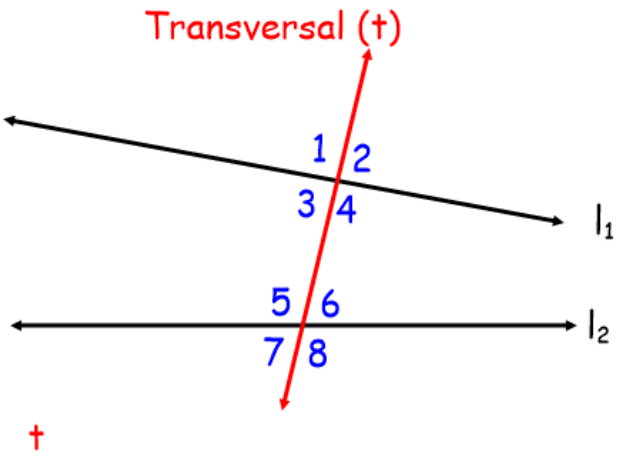


Which angle is vertically opposite to <2?

Which angles are adjacent to <3?

TRANSVERSAL

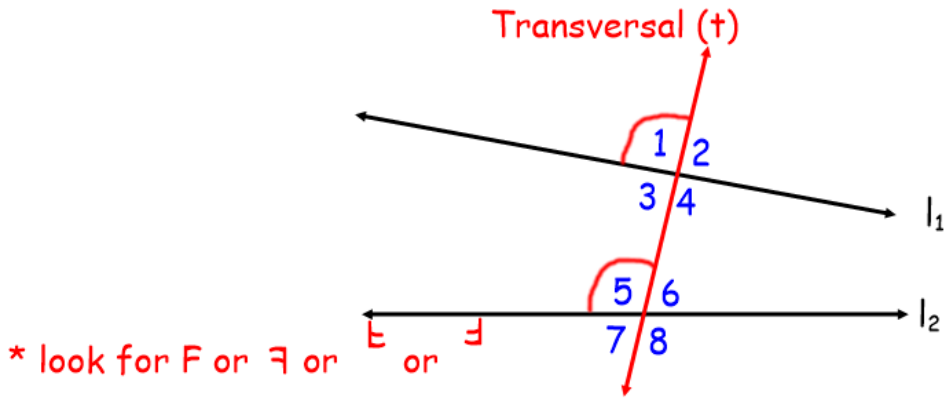
- a line that intersects 2 or more lines in different points.
- a transversal that intersects 2 lines creates 8 different angles that fit into categories based on their relative positions to each other.



CORRESPONDING ANGLES

- corresponding angles are pairs of angles that have the same corresponding positions at the two intersections of lines.

<1 and <5 have the same corresponding positions at the two intersections so they are called corresponding angles. Identify the three other pairs.



(Note: Draw two round tables at the intersections of the transversal and both lines. Corresponding angles will be the angles in the same chair position

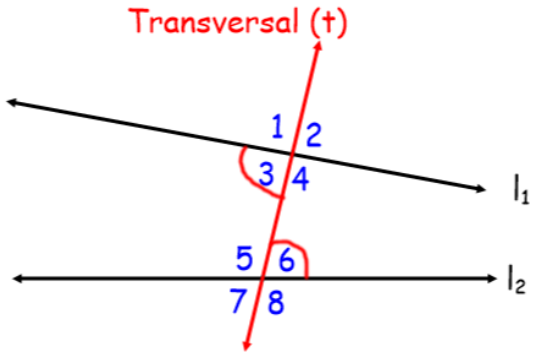
ALTERNATE INTERIOR ANGLES

- Angles between the two main lines are interior angles. Two interior angles that are on alternate sides of the transversal are called alternate interior angles.

<3 & <6 are one pair of alternate interior angles.

Name the 2nd pair.

* Look for a Z or Σ shape



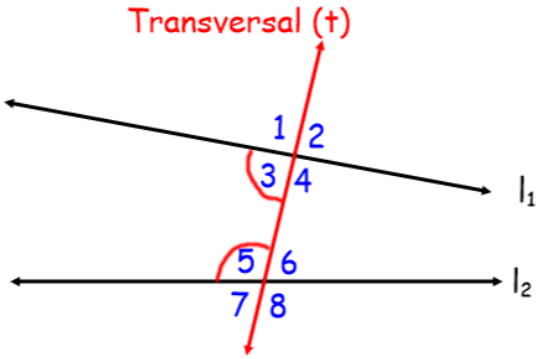
SAME SIDE INTERIOR ANGLES

- Same side interior angles are two interior angles that areon the same side of the transversal.

<3 & <5 are same side interior angles

Name the 2nd pair

* look for [or]

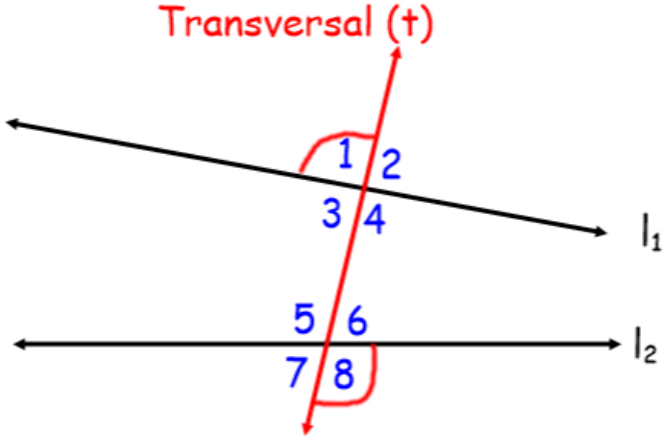


ALTERNATE EXTERIOR ANGLES

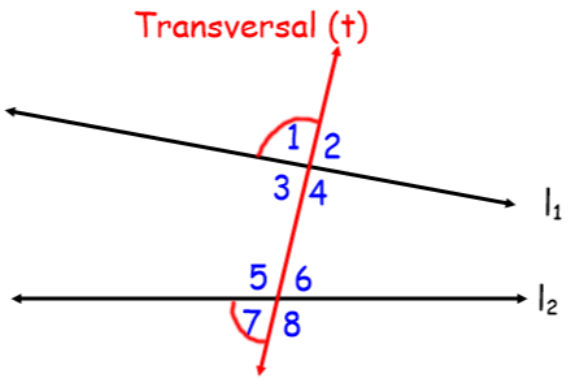
- Angles outside the two main lines are exterior angles. When they are outside the lines and are on alternate sides of the transversal, they are called Alternate Exterior Angles.

<1 & <8 are Alternate Exterior Angles

Name the 2nd pair.



EXAMPLE 1: Two exterior angles on the same side of the Transversal are highlighted in the diagram below. Identify the other pair of exterior angles on the same side of the transversal.

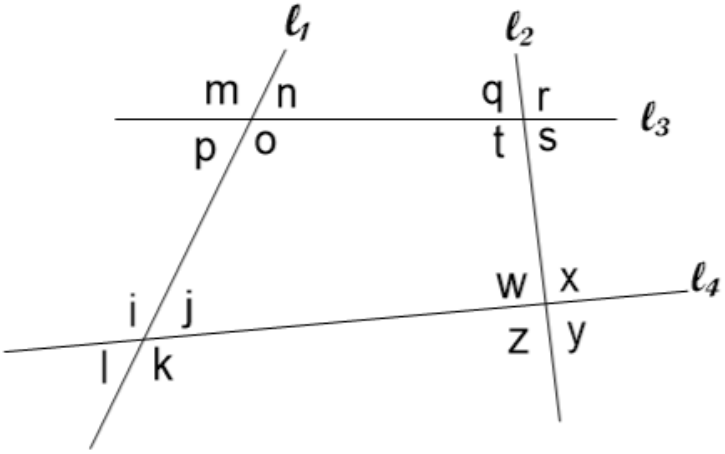


EXAMPLE 2:

a) Name a pair of lines and the transversal.

b) Name two pairs of:

- vertically opposite angles
- corresponding angles
- alternate interior angles
- alternate exterior angles



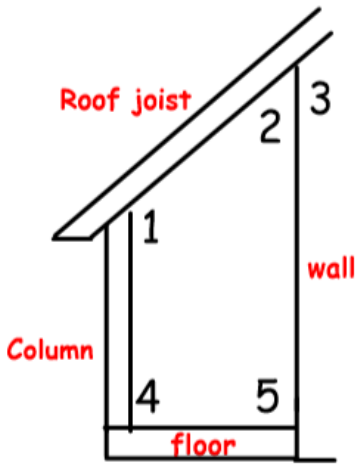
EXAMPLE 3:

To the right is a side diagram of a verandah that is attached to a house. For each pair of angles listed below, identify the kind of angle pair as well las the parts of the verandah that make up the angle pair's lines and transversals.

a) <1 & <4

b) <3 & <5

c) <1 & <3



EXAMPLE 4: Lattice towers are free-standing structures that have cross-bracing to give the structure the strength and rigidity needed to stand by themselves without additional support. The lattice that exists on each side of a lattice tower is essentially a series of pairs of line segments and transversals.

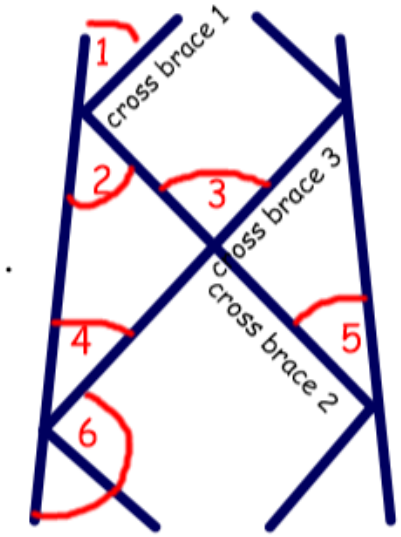
Using the diagram below, answer the questions.

Determine which two parts of the tower make up the main line segments, and which part makes up the transversal that forms each of these pairs of angles.

a) Angles 3 and 4 are corresponding angles.

b) Angles 2 and 5 are alternate interior angles.

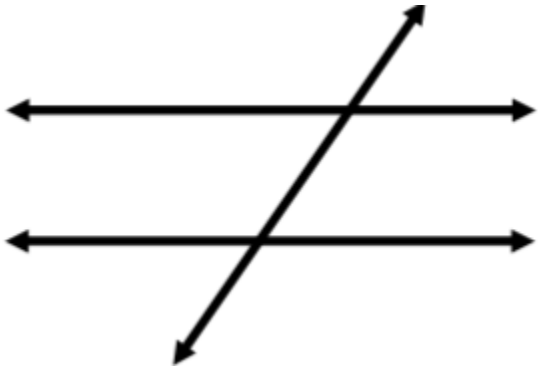
c) Angles 1 and 6 are exterior angles on the same side of the transversal.



5.3 Assignment: WORKBOOK
Build Your Skills – Page 233–234 #1-3 P 235-236 #4-6
Practice Your Skills Page P 236 – 238 #1-5 Plus questions textbook activity 5.6 on page 201

5.4 – Parallel Lines & Transversals

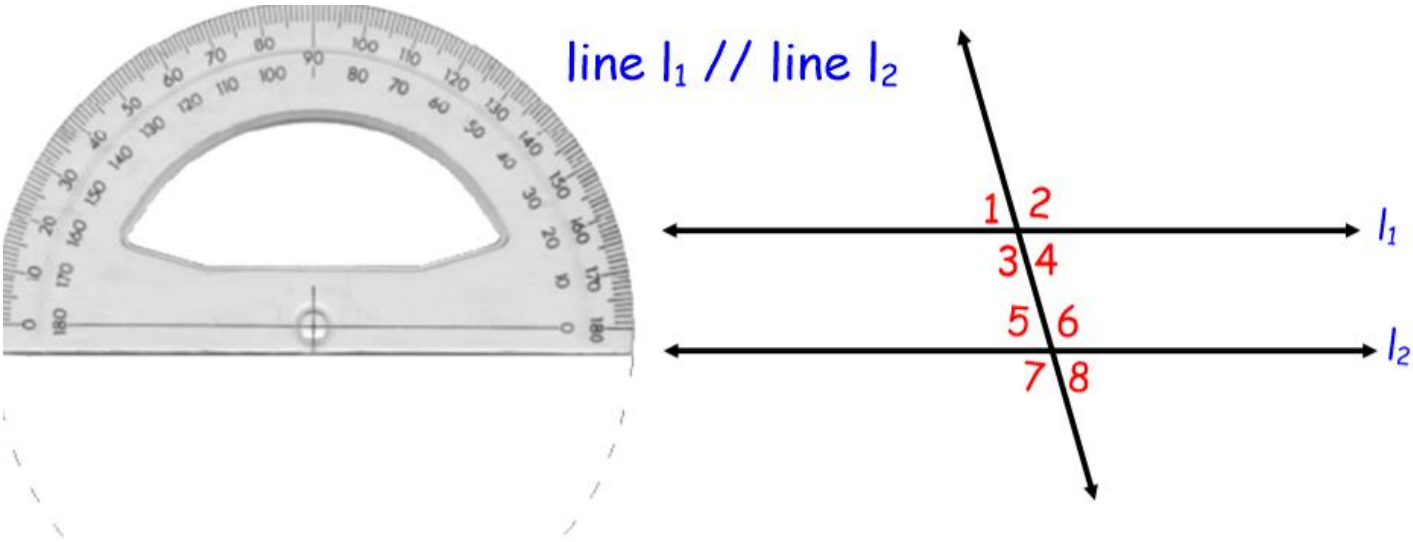
Two lines are parallel if they never intersect each other. This only happens when the lines are a constant distance from each other. What would happen if lines that were supposed to be parallel, like lines on roads to define lanes, become closer and closer together?



If two lines are parallel and are intersected by a transversal, the angles we discussed in the previous section (eg) corresponding angles, will have certain properties.

Use your protractor and the diagram below to measure the following pairs of angles:

- Corresponding angles
- Alternate interior angles
- Same Side Interior angles
    - Find the sum of these angle measurements
- Alternate Exterior angles
- Same Side Exterior angles
    - Find the sum of these angle measurements



ANGLES IN PARALLEL LINES

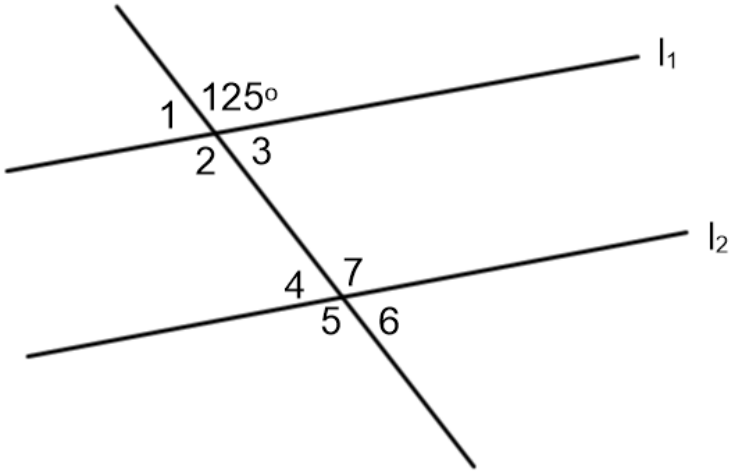
When two lines are parallel and intersected by a transversal:

- the measures of pairs of corresponding angles will be equal
- the measures of pairs of corresponding angles alternate interior angles will be equal
- the measures of pairs of alternate exterior angles will be equal
- pairs of same side interior angles will be supplementary (add to 180°)
- pairs of same side exterior angles will be supplementary (add to 180°)

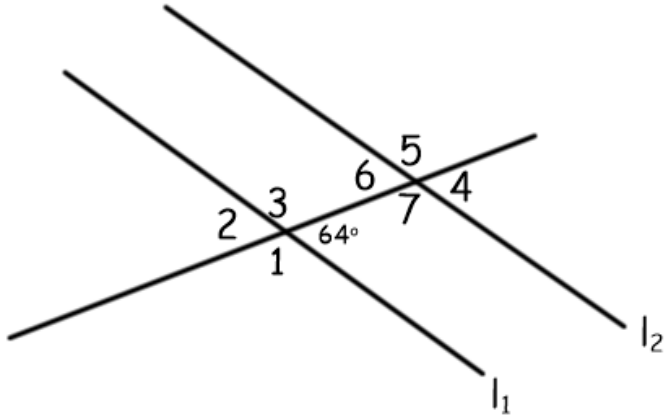
CONVERSE OF THE ABOVE STATEMENTS:

In a diagram of two lines and a transversal, if the corresponding angles are equal, the lines are parallel.

EXAMPLE #1: If the two lines are parallel, Find the measures of the missing angles. Put the parallel markings on the diagram as well!



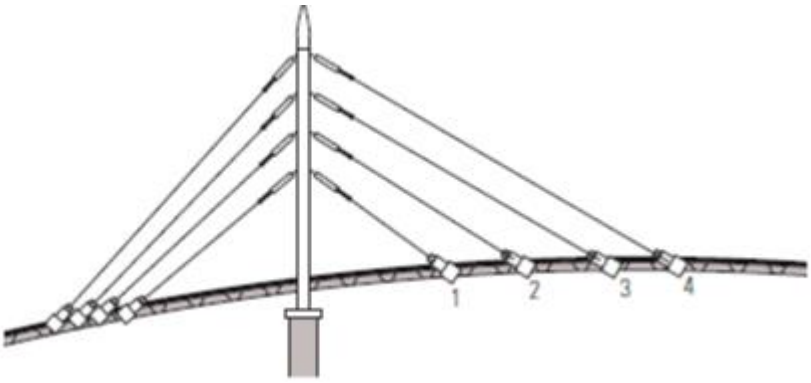
EXAMPLE #2: Consider the diagram below where l_1 is parallel to l_2. What are the measures of the indicated angles?



EXAMPLE #3:

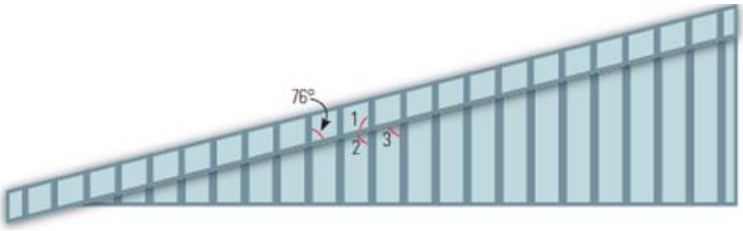
A cable stay bridge is made of a support tower and cables that reach down to the bridge deck. The cables, which can be parallel or angles, suspend the bridge above water. One of Canada's most well-know cable stay bridges is the Esplanade Riel in Winnipeg.

Use the diagram determine which of the four indicated cables are parallel to each other and which ones are not. How do you know?



EXAMPLE #4: Danielle is a sheet metal worker. She specializes in sheet metal roofing that can be purchased in segments, snapped into place, and secured with screws or nails.

The diagram below shows two segments of sheet metal roofing. The horizontal line, or transversal, represents where the two segments meet. The vertical lines are ridges. One angle is given. State the measures that angle 1, 2, and 3 must have if the ridges are parallel to each other. Explain why they must have those measures.

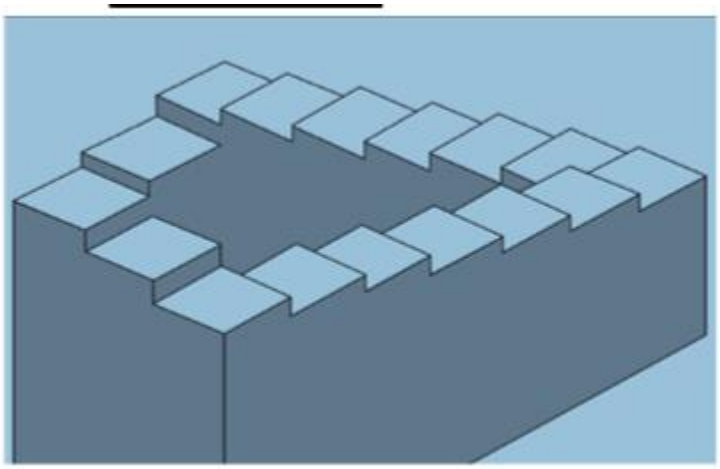


5.3 Assignment: WORKBOOK
Build Your Skills – Page 240 – 245 #1 - 9
Practice Your Skills Page P 246 – 247 #1-4 Plus following question

Puzzle It Out!

The Impossible Staircase

Follow the staircase around. Can you explain how the illusion works???



Unit Review
Chapter Test

Page 248 - 252

Questions
1 - 10

Remember to Practice!

Replicating Angles and Bisecting Angles with a Compass and a Ruler!