## - Know all of your antiderivative formulas for the CH 6 FORMULA TEST

Riemann Sums - You might be given a function's equation, data in graphs, or data in charts.

1. If $f(x)=-x^{2}+2$, estimate the value of $\int_{0}^{2} f(x) d x$ using 4 subintervals of equal length and the approximation method indicated. (Use a calculator and as always, include 3 decimal places if necessary.)
(a) A left Riemann sum
(b) A right Riemann sum
(c) A midpoint approximation
(d) A trapezoidal approximation
2. Use your calculator to evaluate the integral from question 1. Then for each approximation draw a sketch of the strips used and justify why the value is an overestimate or underestimate.
3. For each graph of the function $y=f(x)$, draw in the area strips and estimate $\int_{0}^{16} f(x) d x$. (a) A left Riemann sum with 8 equal subintervals.

(b) A right Riemann sum with 8 equal subintervals.

(c) A midpoint approximation with 4 equal subintervals.

(d) A trapezoidal approximation with 4 equal subintervals.


| $x$ | 0 | 0.8 | 2 | 3.6 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 9.58 | 3.5 | -0.82 | 2.38 | 4.78 |

4. Selected values for the function $f$ are given above. Approximate the value of $\int_{0}^{4} f(x) d x$ using: (a) A left Riemann sum using 4 subintervals.
(b) A right Riemann sum using 4 subintervals.

| $x$ | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 | 2.4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 1 | 1.16 | 1.64 | 2.44 | 3.56 | 5 | 6.76 |

5. Selected values for the function $f$ are given above. Approximate the value of $\int_{0}^{2.4} f(x) d x$ using:
(a) A midpoint approximation using 3 subintervals.
(b) A trapezoidal approximation using 3 equal subintervals.

Be able to describe what you are finding.
6. The function $v(t)$ gives a particle's velocity at a time $t$ in meters per second.
(a) Using correct units, explain what the integral $\int_{1}^{3} v(t) d t$ would find.
(b) Using correct units, explain what $v^{\prime}(4)$ would find.

## Integrals can be expressed using summation notation.

7. Write an integral that the following Riemann sum estimates:

$$
\left[3\left(\frac{1}{40}\right)^{3}+3\left(\frac{2}{40}\right)^{3}+3\left(\frac{3}{40}\right)^{3}+\ldots \ldots .+3\left(\frac{40}{40}\right)^{3}\right] \frac{1}{40}
$$

8. Write an integral that is equal to the limit given below: (Can you write a second one?)

$$
\lim _{n \rightarrow \infty} \sum_{k=1}^{n} 4\left(1+\frac{2 k}{n}\right)^{2} \frac{2}{n}
$$

9. Write the following definite integrals as limits.
(a) $\int_{1}^{2} x^{2} d x$
(b) $\int_{1}^{2} 4 x^{2} d x$
(c) $\int_{0}^{1}(1+x)^{2} d x$
(d) $\int_{0}^{1} 4(1+x)^{2} d x$

Know how to manipulate integrals to solve problems:
7. Evaluate $\int_{1}^{5} f(x) d x$ if $\int_{1}^{7} 2 f(x) d x=8$ and $\int_{5}^{7} f(x) d x=3$.

Know how to apply the Fundamental Theorem of Calculus, parts I and II.
8. Find $\frac{d y}{d x}$ when $y=\int_{0}^{x^{2}} \tan 5 t d t$
9. If the antiderivative of $f(x)$ is $\sin \left(x^{2}-3\right)$, evaluate $\int_{1}^{2} f(x) d x$
10. If $g(x)=\int_{1}^{x} f(x) d x$, where the function $f$ is continuous and $f(3)=5, f(1)=2$, and $f(0)=9$. Evaluate $g^{\prime}(3)$.

