## Chapter 7 – Trigonometry

## Curriculum Outcomes 7A & 7B:

Level Criteria	Intervention 1 Spend some extra time with the criteria and ask for help.	Instructional 2 Good start. You are beginning to make sense of this on your own. You are consistent with the basic learning goals for this outcome.	Independence 3 You did it and you did it on your own. You are able to complete the processes for this outcome. Your work is thorough and consistently accurate.	<u>Mastery 4</u> Great work! This is going extra well for you. You have understood the outcome, are able to explain your strategies and apply these to situations. Your work is always accurate.	
Outcome WA 7A Apply understanding of the Pythagorean Theorem to solve problems.	I need more help with becoming consistent with the criteria.	I am able to find the length of the leg or hypotenuse of a right triangle with Pythagorean Theorem given basic information and a diagram. Given the lengths of three sides, I can determine if a triangle is a right triangle.	I can apply an understanding of the Pythagorean Theorem to solve a variety of word problems without being given a diagram.	I can develop, generalize, apply and explain strategies to verify if a corner of a 3-D object is square (90°) or if a parallelogram is a rectangle. Answers must include units of measure.	
Level	Intervention 1 Spend some	Instructional 2 Good start. You are beginning to make sense of	Independence 3 You did it and you did it on your own. You are able to complete the	<u>Mastery 4</u> Great work! This is going extra well for you. You have	

Criteria	extra time with the criteria and ask for help.	beginning to make sense of this on your own. You are consistent with the basic learning goals for this outcome.	own. You are able to complete the processes for this outcome. Your work is thorough and consistently accurate.	well for you. You have understood the outcome, are able to explain your strategies and apply these to situations. Your work is always accurate.
Outcome WA 7B Demonstrate an understanding of primary trigonometric ratios (sine, cosine, and tangent)	I need more help with becoming consistent with the criteria.	I can observe a right triangles and I determine the ratio of the acute angle and the length of the side opposite to the side adjacent, side opposite to the hypotenuse and side adjacent to the hypotenuse. I am able to find the unknown side of a right triangle given the length of one side and an angle measurement given a diagram.	I can apply an understanding of the Sine, Cosine and Tangent ratios to solve a variety of situational questions involving a missing side or a missing angle without a diagram.	I can apply an understanding of the Sine, Cosine and Tangent ratios to solve a variety of multi- step situational questions involving a missing side or a missing angle without a diagram. (eg) find <a to<br="">determine side B in triangles that share an acute angle. Answers must include units of measure.</a>

#### Goals:

- determine the trigonometric ratios
- determine lengths of sides of right triangles using the ratios
- determine the sizes of angles if you know the ratios

#### Key Terms:

- angle of depression
- angle of elevation
- cosine
- sine
- hypotenuse
- tangent
- leg
- Pythagorean Theorem

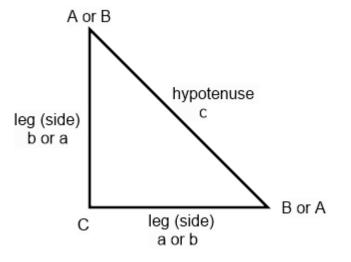
## 7.1 – Right Triangles & The Pythagorean Theorem

- **Trigonometry** the study of relationships between side lengths and angles of triangles.
- **right triangle** a triangle with one 90° angle (right angle)
- hypotenuse the longest side of a right triangle always located across from the 90° angle

#### Labeling triangles:

- angles are labeled with a capital letter
- sides are labeled with a lowercase letter

- the side directly across from an angle receives the same letter label as the angle



The **Pythagorean Theorem** is used to determine the lengths of sides of a right triangle. Carpenters, surveyors, bricklayers, and other professionals use the Pythagorean Theorem on a daily basis.

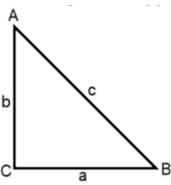
 $a^{2} + b^{2} = c^{2}$ (side)<sup>2</sup> + (side)<sup>2</sup> = (hypotenuse)<sup>2</sup>

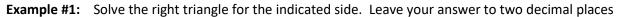


When a ladder is placed against a wall so that a painter can paint the house, the ladder, the wall, and the ground form a right triangle.

In this example, the wall and the ground form the right angle and the ladder forms the hypotenuse. The ground and the wall are the legs or sides.

Triangle ABC represents the house and ladder. Leg BC of the triangle is said to be adjacent to angle B and opposite to angle A. In general, a leg of a right triangle is adjacent to the angle if it, along with the hypotenuse, forms the acute angle. The other leg is said to be opposite that acute angle. In triangle ABC, leg AC is opposite angle B.



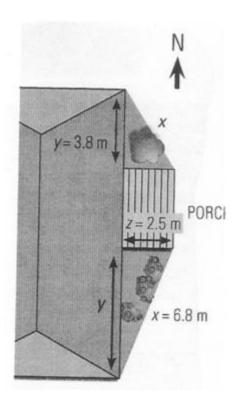




#### Example 2:

Mary has submitted a plan to plant a herb garden in front of the proposed Centre Communitaire Beaumont Community Centre in Beaumont, Alberta. The garden will be made of two triangular pieces of earth on either side of the centre's porch. It will be used to grow traditional French herbs.

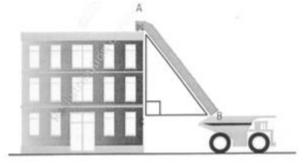
a) Given the dimensions of the legs of the triangle as shown in the diagram, what will be the length of the hypotenuse of the plot to the north of the porch?



b) How far along the front of the house will the garden in the plot to the south of the porch reach?

### Mental Math and Estimation Examples:

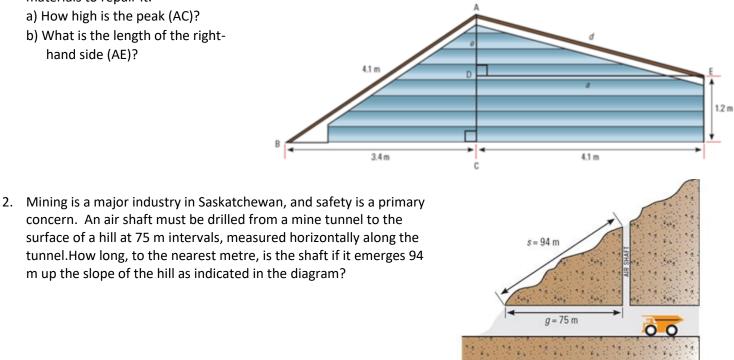
a) In the diagram shown, if AB is 30 ft and AC is 25 ft, approximately how far from the building is the back of the truck?



b) In a right triangle, if the hypotenuse is 20 in and one leg is 12 in, how long is the other leg?

## 7.1 Assignment : WORKBOOK Build Your Skills – Page 290 #1 & 2, Page 294 #6-8 Practice Your Skills Page P 295 – 296 #1-5 Plus The Following Two Questions #1 & 2

1. The roof of a shed is offset as in the diagram shown. Ben must determine its measurements so that he can order materials to repair it.



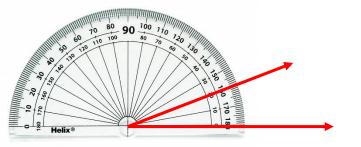
Extra Notes Handout Page 4

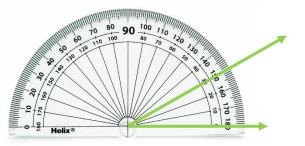
## 7.2 – The SINE Ratio

## ACTIVITY

1. Using the following protractors, draw two right triangles which have the following angle measures.Triangle #1: 20°, 90°, 70°Triangle #2: 30°, 90°, 60°

Use the protractor on the page to draw the first angle in each triangle (I've started the sides of the angle for you). Use a straight edge and an actual protractor to draw the other angles and sides. Your sides may be any length you wish.





- 2. Label your triangle as follows: Call the angle in the left corner "A", call the side opposite that angle "OPPOSITE", call the hypotenuse "HYPOTENUSE" and name the bottom side beside the original angle "ADJACENT"
- 3. Carefully measure, to the nearest millimetre, the length of the OPPOSITE and HYPOTENUSE of each triangle. Record your findings in the table provided.

Angle Size	Opposite	Hypotenuse	MAKE A RATIO BY DIVIDING THE OPPOSITE SIDE LENGTH BY THE HYPOTENUSE
20°			
30°			

## The Sine of an Angle

- When we divide the opposite side by the hypotenuse in a right triangle, it finds for us a special ratio called THE SINE RATIO. (Mathematicians are lazy are actually short it to call it SIN although they still pronounce this as SINE)
- Because all right triangles with the same angles are similar, finding the SINE Ratio for any sized triangle with the same angles will always give us the same decimal value of the ratio.
- Someone had to find all the decimals of every sine ratio for angles between 1 and 90°

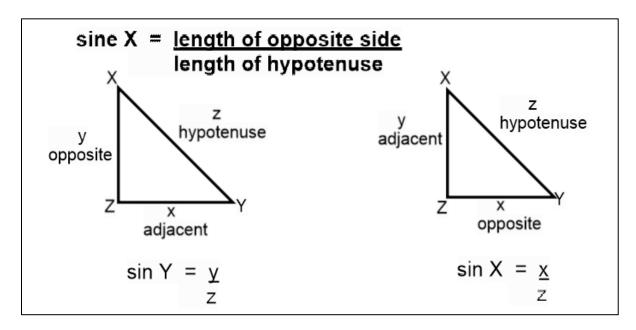
Angle	Sine	Angle	Sine	Angle	Sine	Angle	Sine
1° 2° 3° 4° 5°	.0175 .0349 .0523 .0698 .0872	26° 27° 28° 29° 30° 31°	.4384 .4540 .4695 .4848 .5000 .5150	46° 47° 48° 49° 50°	.7193 .7314 .7431 .7547 .7660	71° 72° 73° 74° 75° 76°	.9455 .9511 .9563 .9613 .9659
6° 7° 8° 9° 10°	.1045 .1219 .1392 .1564 .1736	32° 33° 34° 35°	.5299 .5446 .5592 .5736	51° 52° 53° 54° 55°	.7771 .7880 .7986 .8090 .8192	76° 77° 78° 79° 80°	.9703 .9744 .9781 .9816 .9848
11° 12° 13° 14° 15°	.1908 .2079 .2250 .2419 .2588	36° 37° 38° 39° 40°	.5878 .6018 .6157 .6293 .6428	56° 57° 58° 59° 60°	.8290 .8387 .8480 .8572 .8660	81° 82° 83° 84° 85°	.9877 .9903 .9925 .9945 .9962
16° 17° 18° 19° 20°	.2756 .2924 .3090 .3256 .3420	41° 42° 43° 44° 45°	.6561 .6691 .6820 .6947 .7071	61° 62° 63° 64° 65°	.8746 .8829 .8910 .8988 .9063	86° 87° 88° 89°	.9976 .9986 .9994 .9998
21° 22° 23° 24° 25°	.3584 .3746 .3907 .4067 .4226			66° 67° 68° 69°	.9135 .9205 .9272 .9336		

## Table of Trigonomotrie Detion

- When I was in high school we carried around a long list of these numbers to four decimal places. Now they are built into calculators.
  - Get your calculator out.
  - Be sure it is in the correct mode (or every answer will be wrong!). Your calculator should say D or Deg on the screen. If it says R or Rad or G or Grad you will need to change the mode – call me over and I can help!
  - $\circ$  Check to see if you have a "forward" calculator or a "reverse" calculator. Test it by finding  $\sqrt{9}$ . If you press the  $\sqrt{}$  button first and then 9, you have a forward calculator. If you have to press the 9 button first and then the  $\sqrt{}$  second , you have a reverse calculator.
  - Find the sine ratio for 20° we will use the shorthand version of this which is sin20°. If you have a forward calculator, you will find the button SIN, press it and then type in 20. If you have a reverse calculator, you will type in the 20 first and then press the SIN button. The screen should fill up with decimals. In truth, the decimals for most SIN ratios will go on forever like PI!
  - We usually use round these answers to FOUR decimal places. Check the list above to see if you get the same answer as the Sine of 20° above!
  - Repeat for SIN30°

#### Discussion

- 1. When the class has filled in the overhead chart, compare the sine ratios. What do you notice about the values of sin A as A increases from 10° to 80°?
- 2. What is the smallest value the sine of an acute angle of a right triangle can have? How about the largest?



- It is VERY important to begin by correctly labelling your triangle. The opposite side we use in the formula is always the side opposite to the ANGLE YOU ARE USING AS YOUR STARTING POINT.
- Sometimes we don't use our usual letters to name our angles, we use the Greek alphabet instead. The most common you might see is  $\theta$  is a greek letter called THETA

$$Sin\theta = \frac{Opposite}{Hypotenuse} = \frac{Opp}{Hyp}$$

**Example #1:** Find the following to four decimal places using your calculator. Check your answers on the list of trigonometric values on the previous page

1) sin 10° =

2) sin 35° =

Example #2: Use the table on the previous page to find the angle whose sin ratio is 0.8290

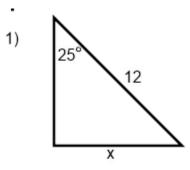
- How would you rewrite this question using mathematical notation only?
- Do you think you could use your calculator to find this answer? Write down the buttons you need to press on your calculator.
- Repeat the process to find out the angle A where sinA=0.1219

Example #3 : Label the following triangles using the words HYPOTENUSE (HYP), OPPOSITE (OPP), ADJACENT (ADJ)



**Example #4:** Use the formula  $Sin\theta = \frac{Opposite}{Hypotenuse} = \frac{Opp}{Hyp}$  to find out the length of the unknown side in the following diagrams

following diagrams.

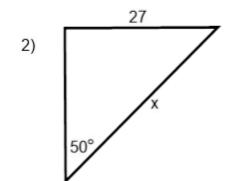


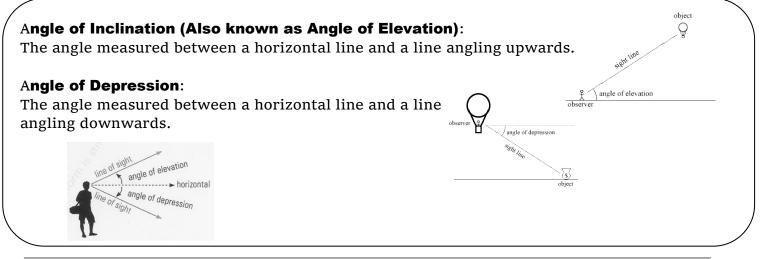
## STEPS:

- 1. Name the given angle using a letter such as X or A or  $\theta$ .
- 2. Label the hypotenuse as HYP, label the side across from the given angle as OPP and name the third side ADJ
- 3. Use the numbers and letters from the diagram, fill in the

formula 
$$\sin \theta = \frac{opp}{hyp}$$

4. Put the left side of this formula over 1 and cross multiply to find the unknown side length

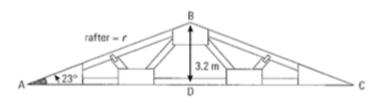




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## Example 1:

Helene is building a garage on her farm near Stavely, Alberta. She knows that the angle of elevation of the roof must be 23° for the peak of the roof to be 3.2 metres above the ends of the rafters, as shown in the diagram. How long is each rafter?



#### Example 2:

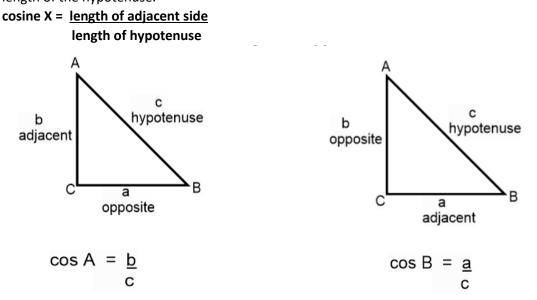
From the top of a cliff by the ocean, Cedric sights a boat at an angle of depression of 48°. If the top of the cliff is 73 m above the water, and Cedric is 2 m tall, how far is Cedric from the boat? Draw a diagram.

## 7.2 Assignment : WORKBOOK Build Your Skills – Page 298 #1 - 3 Page 300 #4 - 12 Practice Your Skills Page P 306 – 307 #1-5 Plus The Following Two Questions #1 & 2

- Downstream from the confluence of the North and South Saskatchewan Rivers, archaeologists are excavating the site of a former Cree settlement. They find a stone circle that represents where a tipi once stood. The average tipi had about 17 poles, which ranged from 10 to 24 feet long. An archaeologist determines that the tipi was 15 feet high at its peak and 12 feet wide. What would the length of the tipi poles be?
- 2. Laiwan, who lives in Grand Forks, BC, must have a wheel chair ramp built to her front porch. The porch is 1.9 m above the ground level and the steepest angle of elevation allowed by the building code is 6°.
  a) What is the shortest ramp that Laiwan can have installed?
  b) About how many metres (to one decimal) from the base of the porch must the ramp start?

## 7.3 – The COSINE Ratio

A second trigonometric ratio is cosine. Cosine in a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse.

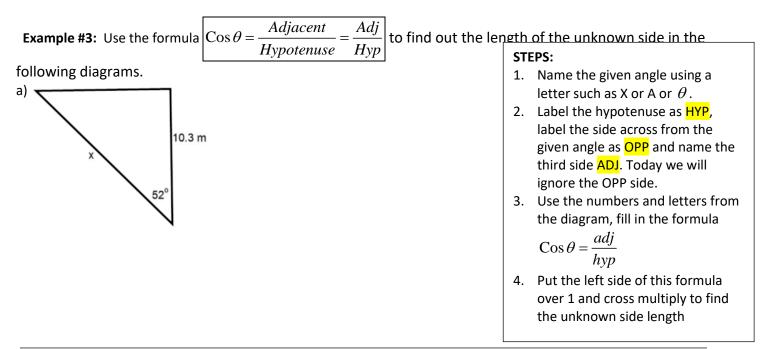


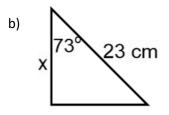
**Example #1:** Use your calculator to find the following to four decimal places(you can verify your findings by using the table on page 20)

a) cos 38° b) cos 75°

**Example #2:** Use the table on page to find the angle whose cosine is 0.1564. The mathematical way to write this question is to find  $\theta$  if  $\cos \theta = 0.1564$ 

• Can you find this answer on your calculator?





#### Example #2: Math on the Job Page 293

Richard McCaffrey works at an auto parts warehouse in Burnaby, BC. His work involves keeping inventory of what parts are in stock, ordering new parts when stock is low, and delivering parts to various dealers in the Lower Mainland. Stock is often moved by a conveyor belt to different levels for storage.

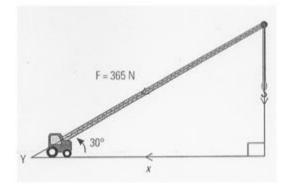
The conveyor belt needs replacing, and Richard must determine approximately what length of belt to order. He knows that the angle of depression from the upper floor along the conveyor belt is 38°. The belt reaches the lower level at a point 6.1 m further along the floor. How can Richard use similar triangles with a scale diagram to determine the length of belt he must order?

#### Example #3:

Given PQR where angle Q equals 90°, q equals 4.3 cm, and R equals 51°, solve the triangle. Draw a sketch.

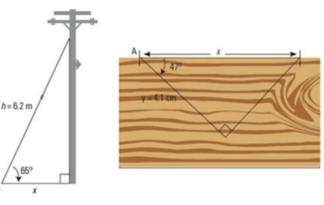
### Example 3:

In construction, Marie knows that a force acting at an angle can be broken up into a vertical force and a horizontal force. If a force of 365 Newtons is exerted diagonally downward at an angle of 30° to the horizontal, what forces will be applied horizontally?



## 7.3 Assignment : WORKBOOK Build Your Skills - Page 310 #1 & 2, P 312 #3 & 4 Page 314 #6-8 Practice Your Skills Page P 315 – 317 #1 – 6 Plus The Following Two Questions #1-2

- 1. Refer to the diagrams to the right.
  - a) How far from the base of a pole must a 6.2 m long guy wire be attached if the angle of elevation is 65°?
  - b) A notch is cut from a block of wood as indicated. What is the width of the opening of the cut-out portion?



2. Totem poles are almost always erected by being pulled upright with ropes into a wooden scaffold support until they are stable. Suppose that two of the ropes attached to a pole are at angles of elevation of 47° and 57° respectively. If the base of the ropes is approximately 26 m from the base 47° 57°

26 m

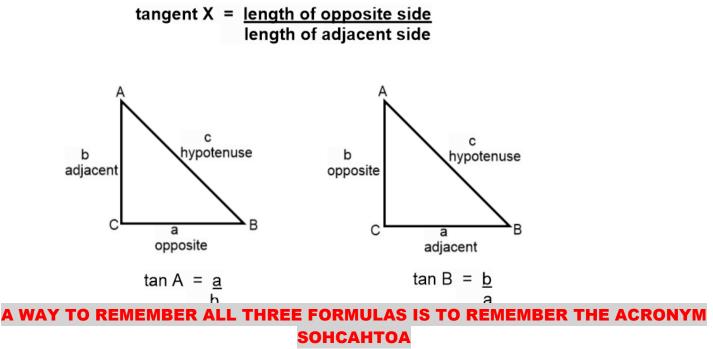
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of the totem pole, how long is each rope?

26 m

## 7.4 – The TANGENT Ratio

A third trigonometric ratio is tangent. **Tangent** in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side.

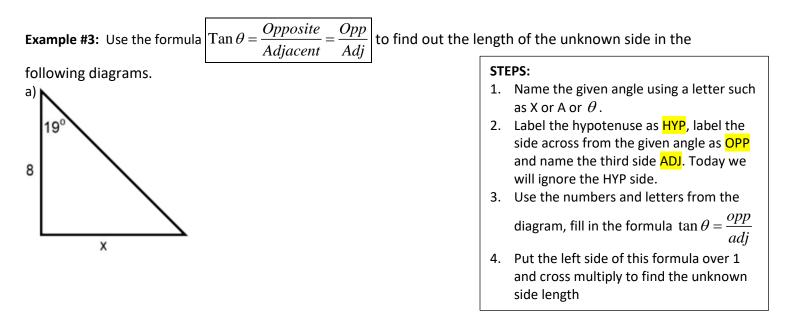


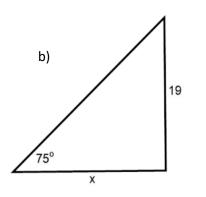
**Example #1:** Use your calculator to find the following to four decimal places(you can verify your findings by using the table on page 20)

a) tan 38° b) tan 75°

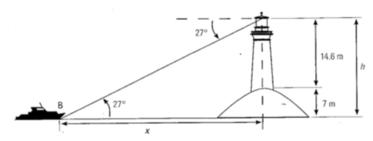
**Example #2:** Use the table on page to find the angle whose tangent is 0.5543. The mathematical way to write this question is to find  $\theta$  if tan  $\theta = 0.5543$ 

• Can you find this answer on your calculator?

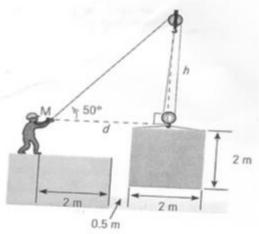




**Example #4:** Gull Harbour Lighthouse is located on Manitoba's Lake Winnipeg. Assume the lighthouse is 14.6 m tall and stands 7 m above the surface of the lake. If the angle of depression to a boat on Lake Winnipeg is measured at 27°, approximately how far away from the base of the lighthouse is the boat?

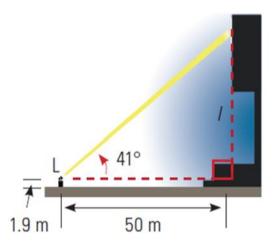


**Example 5:** A dockworker pulls a light crate (measuring 2 m x 2 m x 2 m) up to the dock using a pulley system. The angle of elevation of the rope is 50°. The man is 2 m from the edge of the pier and the bundle clears the pier by 0.5 m. How close are the pulleys to each other when the bottom pulley is at hand level?



## 7.4 Assignment : WORKBOOK Build Your Skills – Page 320 – 321 #1-3 Practice Your Skills Page P 321 – 323 #1 – 6 Plus The Following Three Questions #1-3

- Near Estevan, Saskatchewan, Mary and James like to lie on their backs in an open field to watch planes landing. One day as they watch, a helicopter approaches and hovers over a building 1 km away from them. If the angle of elevation is 25°, how high above the ground is the helicopter? Draw a diagram.
- 2. Mike and Lianne are lighting technicians who work for a special events company. Their current job is to set up the spotlights for an outdoor music festival. The performers want to suspend a banner with the name of their troupe directly above the stage, as high as possible, with a spotlight shining on it. Mike and Lianne have only one spotlight left. It is 50 m away from the stage and mounted on a stand 1.9 m high. It has a maximum angle of elevation of 410. How high will the performers be able to suspend the banner?



## 3. Math on the Job Page 301

When Chris Haika, of Calgary, AB, was in elementary school, he knew he wanted to fly airplanes. He got his pilot's licence in his last semester of high school and went on to Mount Royal College to get his aviation diploma. After getting his diploma, Chris worked as a customer agent for an airline in Grande Prairie, AB. After seven months, he was promoted to pilot. When flying into Calgary, Chris has been at an elevation of 28 000 ft. His descent to the airport is at an angle of depression of 3°. At what horizontal distance from the airport must he begin his descent? Draw a diagram.

## 7.5– Finding Angles & Solving Right Triangles

As we saw in the earlier sections, we can find the size of an angle for any given ratio (decimal ratio answer).

**Example #1**: Use the table on page 20 to find the angles for the following:

a)  $\sin\theta = 0.2588$ 

c)  $\sin\theta = 4.3315$ 

- To find these answers on the calculator, we use the buttons above the sin, cos, tan buttons. These buttons are sin<sup>-1</sup>, cos<sup>-1</sup>, tan<sup>-1</sup>. We read these as "sine to the minus one" or "arcsine", "cosine to the minus one" or "arccosine" and "tangent to the minus one" or "arctangent"
- each of these is an **inverse of the function**. Inverse function means it "undoes" the function

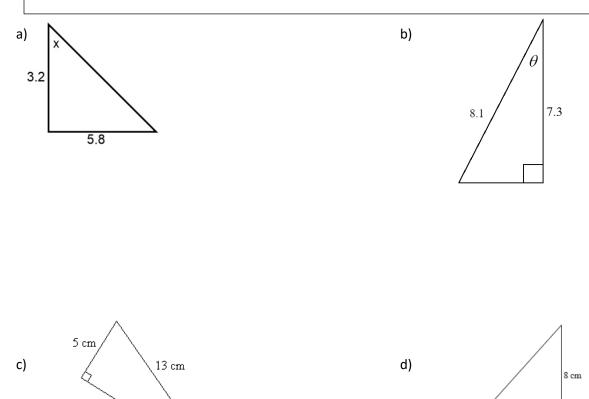
b)  $\cos\theta = 0.7771$ 

• These buttons will give the value of the angle if you know the value of the ratio of that trigonometric function

#### **Example #2**: Use the following steps to find the value of the unknown angle.

#### STEPS:

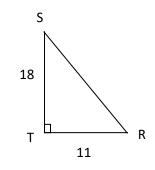
- 1. Decide which angle to use as your "Starting Point". Name this angle using a letter such as X or A or  $\theta$ .
- 2. Label, your hypotenuse, your opposite and your adjacent side.
- 3. In order to find an angle, you only need to have TWO sides that have lengths. Ignore the side that has no given length (if you are given all three lengths, you may choose to ignore any of them)
- 4. Using the sides that you have lengths for, choose the formula that contains those side names.
  - If you have the Opposite and Hypotenuse, you will use  $\sin \theta = \frac{opp}{hvp}$
  - If you have the Adjacent and Hypotenuse, you will use  $\cos \theta = \frac{adj}{hyp}$
  - If you have the Opposite and Adjacent, you will use  $\tan \theta = \frac{opp}{adi}$
- 5. Fill in the lengths into the formula you have chosen
- 6. Divide your fraction. Leave your answer to 4 decimal places.
- 7. Use either sin<sup>-1</sup>, cos <sup>-1</sup>, or tan<sup>-1</sup> of the four decimal answer to find your missing angle. You could also use the table (but it doesn't contain all the decimals so it isn't the best option)
- 8. You may leave your final answer to two decimals or use the instructions in the question.



12 cm

5 cm

**Example #3**: Determine the measures of  $\measuredangle$  R and  $\measuredangle$  S to the nearest tenth of a degree



Solving a right triangle means that at the end of each question you will know the length of all three sides and the size of all three angles. We can use the following to help us solve the triangles:

$$\operatorname{SinA} = \frac{\operatorname{opp}}{\operatorname{hyp}}, \quad \operatorname{Cos} A = \frac{\operatorname{adj}}{\operatorname{hyp}}, \quad \operatorname{Tan} A = \frac{\operatorname{opp}}{\operatorname{adj}}, \quad a^2 + b^2 = c^2, \quad \measuredangle A + \measuredangle B + \measuredangle C = 180^\circ$$

**Example #4:** Solve the following triangles. Find side lengths to the nearest tenth and angles to the nearest degree.

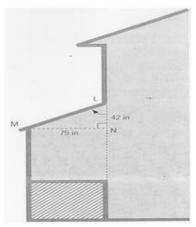


**Example 5:** Determine the angle indicated in each of the following. Draw a sketch for a and b.

a) What angle is created by a guy wire 8.5 m long attached 5.7 m from the base of a pole.

b) The angle of depression from a point 10.1 m down a hill if the horizontal distance is 6.9 m.

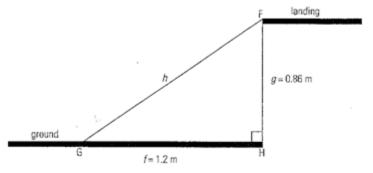
c) The angle between the side of a house and the glass roof of a small bay window, if the bay window is 75 inches deep and the vertical displacement of the roof is 42 inches.



#### Example 6:

The Pulaarvik Kablu Friendship Centre in Rankin Inlet, Nunavut, is a place where elders share their skills and knowledge with young people. Tagak is one of the maintenance people who cares for the centre. Her current job is to replace the centre's front steps. She knows that the distance between the ground and the landing is 0.86 m and the stairs end at a point 1.2 m from the edge of the landing.

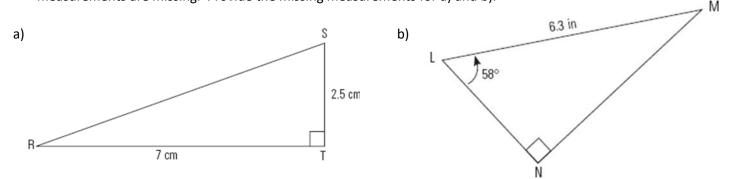
a) What will be the angle of elevation from the bottom to the landing?



b) What is the distance between the bottom of the stairs and the landing?

# 7.5 Assignment : WORKBOOKBuild Your Skills – Page 325 #1-3Page 327 # 4 – 6Page 329 #7 - 12Practice Your Skills Page P 333 - 335 #1 – 6Plus The Following Four Questions #1-4

 Emile is cutting pieces of stained glass to replace a window in his local church. He's been given rough diagrams of the pieces he needs to cut, but some of the measurements are missing. Provide the missing measurements for a) and b).



- Heather is working on a brochure about hiking trails in Prince Albert National Park.
   One of the hills on a trail has an angle of elevation of 15°, with a viewpoint 100 m from its base. imagine that you walk along the trail from the base to the viewpoint.
  - a) How much altitude would you gain?
  - b) What horizontal distance would you cover?
- 3. A 15 m ladder is placed against the side of an apartment and reaches a widow sill that is 12 m above the ground.a) What is the angle of elevation of the ladder?
  - b) How far from the base of the apartment is the ladder?

## 4. Activity 7.7 Page 311Rock Band Lighting

You and your partner are working as lighting technicians for a rock band, L & N. The band has asked that you position lights off the floor 10 m from the lead singer. The red, blue, and green lights are to be placed at heights of 10 m, 9 m, and 7 m respectively. Determine the angle of elevation at which to set each light so it lights up the lead singer.

## Chapter 7 Review : WORKBOOK Page 336 – 339 #1 - 11

# Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
<b>1°</b>	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
<b>6</b> °	.1045	.9945	.1051	51°	.7771	.6293	1.2349
<b>7</b> °	.1219	.9925	.1228	52°	.7880	.6157	1.2799
<b>8</b> °	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000				