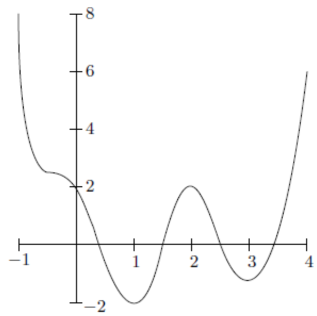
2.1 Day 1 Duo Tang Assignment: Following #207 -214 (plus Textbook Exercises p66 #1-4)

207. The position $p(t)$ is given by the graph at the right.

a) Find the average velocity of the object between times $t = 1$ and $t = 4$.

b) Find the equation of the secant line of $p(t)$ between times $t = 1$ and $t = 4$.

c) For what times t is the object's velocity positive? For what times is it negative?



208. Suppose $f(1) = 2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

209. The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t) = 3t^2 + 1$.

a) Find the change in position between times $t = 1$ and $t = 3$.

b) Find the average velocity of the object between times $t = 1$ and $t = 4$.

c) Find the average velocity of the object between any time t and another time $t + \Delta t$.

210. Let $f(x) = x^2 + x - 2$.

a) Find the average rate of change of $f(x)$ between times $x = -1$ and $x = 2$.

b) Draw the graph of f and the graph of the secant line through $(-1, -2)$ and $(2, 4)$.

c) Find the slope of the secant line graphed in part b) and then find an equation of this secant line.

d) Find the average rate of change of $f(x)$ between any point x and another point $x + \Delta x$.

FIND THE AVERAGE RATE OF CHANGE OF EACH FUNCTION OVER THE GIVEN INTERVALS.

211. $f(x) = x^3 + 1$ over a) $[2, 3]$; b) $[-1, 1]$

212. $R(x) = \sqrt{4x+1}$ over a) $[0, \frac{3}{4}]$; b) $[0, 2]$

213. $h(t) = \dfrac{1}{\tan t}$ over a) $[\frac{\pi}{4}, \frac{3\pi}{4}]$; b) $[\frac{\pi}{6}, \frac{\pi}{3}]$

214. $g(t) = 2 + \cos t$ over a) $[0, \pi]$; b) $[-\pi, \pi]$

2.1 Solutions

207. a) $\frac{8}{3}\frac{\text{distance unit}}{\text{time unit}}$ b) * y=8/3(x-1)-2

c) Note: t>0: Negative Velocity: $[0,1]\cup[2,3]$

Positive Velocity: $[1,2]\cup[3,4]$

208. a) f(5)=14

209. a) $\frac{12}{1}\frac{\text{distance unit}}{\text{time unit}}$ b) $\frac{15}{1}\frac{\text{distance unit}}{\text{time unit}}$

c) $\frac{6x+\Delta x}{1}\frac{\text{distance unit}}{\text{time unit}}$

210. (a) 2 (c) $y = 2x$ (d) $2x + 1 + \Delta x$

211. (a) 19 (b) 1

212. (a) $\frac{4}{3}$ (b) 1

213. (a) $-\frac{4}{\pi}$ (b) $-\frac{4\sqrt{3}}{\pi}$

214. (a) $-\frac{2}{\pi}$ (b) 0

2.1 Day 2 Assignment: DUO TANG Following #1, 2, 8-13 (Plus Textbook P66 #35, 43-50, 57, 58, 60)

Below are tables of values for given types of functions. For each table, the type of function represented by the table is given. Use your knowledge of the numerical behavior of each type of function to find the indicated limits. For limits that do not exist, write D.N.E.

1. Exponential Function

x	−7	−4	−1	2	5	8	11
$H(x)$	−125	−13	1	2.75	2.969	2.996	2.999

a) $\lim\limits_{x\to-\infty} H(x) =$ b) $\lim\limits_{x\to-1} H(x) =$ c) $\lim\limits_{x\to\infty} H(x) =$

2. Rational Function

x	–1000	–2.001	–2	–1.999	0.999	1	1.001	1000
$G(x)$	0.998	0.333	Undefined	0.333	–1999	Undefined	2001	1.002

a) $\lim\limits_{x\to -\infty} G(x) =$

b) $\lim\limits_{x\to -2^-} G(x) =$

c) $\lim\limits_{x\to -2^+} G(x) =$

d) $\lim\limits_{x\to -2} G(x) =$

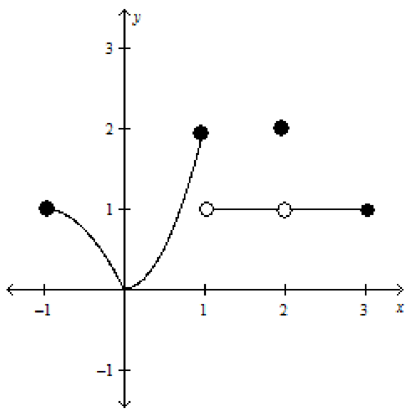
e) $\lim\limits_{x\to 1^-} G(x) =$

f) $\lim\limits_{x\to 1^+} G(x) =$

g) $\lim\limits_{x\to 1} G(x) =$

h) $\lim\limits_{x\to \infty} G(x) =$

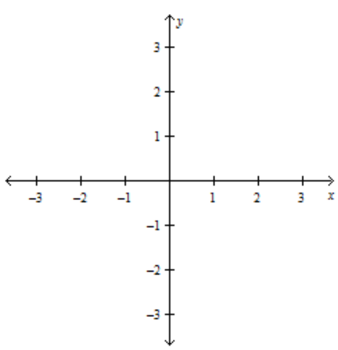
Given the graph of the function, $g(x)$, below, determine if the statements are true or false. For statements that are false, explain why.



8. $\lim\limits_{x\to 1} g(x) = 2$
9. $\lim\limits_{x\to c} g(x)$ exists for every value of c on the interval $(-1, 1)$.
10. $\lim\limits_{x\to 2} g(x)$ does not exist.

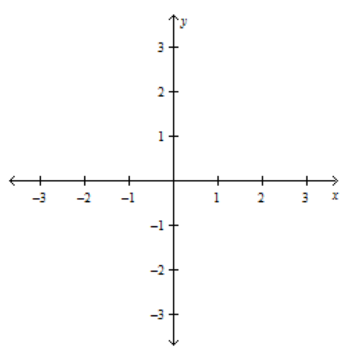
Sketch a graph of a function that fits the requirements described below.

11. $\lim\limits_{x\to 1^-} f(x) = 3$ $\quad \lim\limits_{x\to 1^+} f(x) = -1$ $\quad f(1) = 1$



12. $\lim\limits_{x\to -2^-} f(x) = -\infty$ $\quad \lim\limits_{x\to -2^+} f(x) = \infty$

$f(2)$ is undefined but $\lim\limits_{x\to 2} f(x)$ exists.



13. In exercise 11, does $\lim\limits_{x\to 1} f(x)$ exist? Explain why or why not.

2.1 Day 2 Solutions

1. a) $\lim_{x\to-\infty} H(x) = -\infty$ b) $\lim_{x\to-1} H(x) = 1$ c) $\lim_{x\to\infty} H(x) = 3$

2. a) $\lim_{x\to-\infty} G(x) = 1$ b) $\lim_{x\to-2^-} G(x) = \frac{1}{3}$ c) $\lim_{x\to-2^+} G(x) = \frac{1}{3}$

d) $\lim_{x\to-2} G(x) = \frac{1}{3}$ e) $\lim_{x\to1^-} G(x) = -\infty$ f) $\lim_{x\to1^+} G(x) = \infty$

g) $\lim_{x\to1} G(x) =$ D.N.E. h) $\lim_{x\to\infty} G(x) = 1$

8. $\lim_{x\to1} g(x) = 2$

False b/c $\lim_{x\to1} g(x)$ DNE since $\lim_{x\to1^-} g(x) \neq \lim_{x\to1^+} g(x)$.

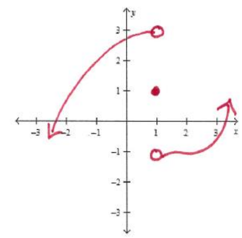
9. $\lim_{x\to c} g(x)$ exists for every value of c on the interval $(-1, 1)$.

True

10. $\lim_{x\to2} g(x)$ does not exist.

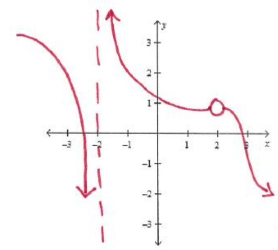
False b/c $\lim_{x\to2^-} g(x) = \lim_{x\to2^+} g(x) = 1$.

11. $\lim_{x\to1^-} f(x) = 3$ $\lim_{x\to1^+} f(x) = -1$ $f(1) = 1$



12. $\lim_{x\to-2^-} f(x) = -\infty$ $\lim_{x\to-2^+} f(x) = \infty$

$f(2)$ is undefined but $\lim_{x\to2} f(x)$ exists.



13. In exercise 11, does $\lim_{x\to1} f(x)$ exist? Explain why or why not.

$\lim_{x\to1} f(x)$ does not exist b/c $\lim_{x\to1^-} g(x) \neq \lim_{x\to1^+} g(x)$.

2.1 Day 3 Assignment: As Follows

Evaluate each limit, if it exists.

1. $\lim_{x\to-1}\left(x^3 - x^2 - x\right)$

2. $\lim_{x\to6}\frac{x^2+36}{x+3}$

3. $\lim_{x\to-2}\frac{x^2-2x-8}{2x+1}$

4. $\lim_{x\to3}\frac{2^{x-4}}{x-1}$

5. $\lim_{x\to5} x$

6. $\lim_{x\to-4} 8$

7. $\lim_{x\to\pi/4}\sin^2 x$

8. $\lim_{x\to8}\left[\left(\log_2 x\right)\left(2^{10-x}\right)\right]$

9. $\lim_{x\to\pi/3}\left[\frac{3x}{\pi}\left(\tan^4 x\right)\right]$

10. $\lim_{x\to2^+}\frac{x+4}{x-2}$

11. $\lim_{x\to2^-}\frac{x+4}{x-2}$

12. $\lim_{x\to2}\frac{x+4}{x-2}$

13. $\lim_{x\to-2^-}\frac{x}{(x+2)^3}$

14. $\lim_{x\to0^+}\frac{\cos x}{x}$

15. $\lim_{x\to1^-}\frac{10^x}{\log x}$

16. $\lim_{x\to3^-}\frac{x+3}{x^2-4x+3}$

17. $\lim_{x\to1}\frac{2x-1}{(x-1)^3}$

18. $\lim_{x\to-5}\frac{x^3}{(x+5)^2}$

19. $\lim_{x\to4}\frac{x-4}{x^2-4x}$

20. $\lim_{x\to0}\frac{6x}{x^2+3x}$

21. $\lim_{x\to-5}\frac{x+5}{x^2-25}$

22. $\lim\limits_{x\to 1}\dfrac{x^2+4x-5}{x^2+x-2}$

23. $\lim\limits_{w\to -2}\dfrac{3w^2+4w-4}{2w^2+7w+6}$

24. $\lim\limits_{s\to 2}\dfrac{s-2}{s^3-8}$

25. $\lim\limits_{v\to 5/3}\dfrac{27v^3-125}{3v-5}$

26. $\lim\limits_{t\to -1}\dfrac{t^4-3t^2+2}{t+1}$

27. $\lim\limits_{x\to 1}\dfrac{x^7-1}{x-1}$

28. $\lim\limits_{x\to 0}\dfrac{(x+1)^2-1}{x}$

29. $\lim\limits_{h\to 0}\dfrac{(3+h)^2-2(3+h)-3}{h}$

30. $\lim\limits_{h\to 0}\dfrac{(1+h)^3-2(1+h)+1}{h}$

31. $\lim\limits_{x\to 4}\dfrac{\frac{1}{x-2}-\frac{1}{2}}{x-4}$

32. $\lim\limits_{x\to 1}\dfrac{\frac{1}{2x+1}-\frac{1}{3}}{x-1}$

33. $\lim\limits_{a\to 0}\dfrac{\frac{1}{(a+2)^2}-\frac{1}{4}}{a}$

34. $\lim\limits_{x\to 9}\dfrac{x-9}{\sqrt{x}-3}$

35. $\lim\limits_{y\to 0}\dfrac{\sqrt{y+2}-\sqrt{2}}{y}$

36. $\lim\limits_{x\to 1}\dfrac{2-\sqrt{5-x}}{1-x}$

37. $\lim\limits_{x\to 5}\dfrac{x-5}{\sqrt{x+4}-3}$

38. $\lim\limits_{q\to 0}\dfrac{\frac{1}{\sqrt{4+q}}-\frac{1}{2}}{q}$

39. $\lim\limits_{x\to 4}\dfrac{8\sqrt{x}-x^2}{2-\sqrt{x}}$

Hint: "rationalize" both numerator and denominator.

PART 2: Operations with Limits

If $\lim\limits_{x\to 3} f(x)=2$ and $\lim\limits_{x\to 3} g(x)=-4$, find each of the following limits. Show your analysis applying the properties of limits.

3. $\lim\limits_{x\to 3}\left[\dfrac{5f(x)}{g(x)}\right]$	4. $\lim\limits_{x\to 3}\left[f(x)+2g(x)\right]$	5. $\lim\limits_{x\to 3}\sqrt{4f(x)}$
6. $\lim\limits_{x\to 3}\dfrac{g(x)}{8}$	7. $\lim\limits_{x\to 3}\left[3f(x)-g(x)\right]$	8. $\lim\limits_{x\to 3}\left[\dfrac{f(x)g(x)}{12}\right]$

If $\lim\limits_{x\to 4} f(x)=0$ and $\lim\limits_{x\to 4} g(x)=3$, find each of the following limits. Show your analysis applying the properties of limits.

9. $\lim\limits_{x\to 4}\left[\dfrac{g(x)}{f(x)-1}\right]$	10. $\lim\limits_{x\to 4} xf(x)$
11. $\lim\limits_{x\to 4}\left[g(x)+3\right]$	12. $\lim\limits_{x\to 4} g^2(x)$

2.1 Day 2 Solutions

1. -1 2. 8 3. 0 4. $\frac{1}{4}$ 5. 5 6. 8 7. $\frac{1}{2}$ 8. 12 9. 9 10. ∞ 11. $-\infty$ 12. does not exist 13. ∞ 14. ∞

15. $-\infty$ 16. $-\infty$ 17. does not exist 18. $-\infty$ 19. $\frac{1}{4}$ 20. 2 21. $-\frac{1}{10}$ 22. 2 23. 8 24. $\frac{1}{12}$ 25. 75 26. 2

27. 7 28. 2 29. 4 30. 1 31. $-\frac{1}{4}$ 32. $-\frac{2}{9}$ 33. $-\frac{1}{4}$ 34. 6 35. $\frac{\sqrt{2}}{4}$ 36. $-\frac{1}{4}$ 37. 6 38. $-\frac{1}{16}$ 39. 24 40. 0

Part 2: (worked out solutions on google classroom)

3. -5/2	4. -6	5. $2\sqrt{2}$	6. -1/2	7. 10
8. -2/3	9. -3	10. 0	11. 6	12. 9

2.1 Day 4 Assignment: **Textbook P66 #17, 18, 20, 31-33, 57, 59, 65-68 (Show all steps!)**
Part A Below: #1, 2 & 3
LEAVE UNTIL CHAPTER 9: Part B Below: any of the additional questions as needed

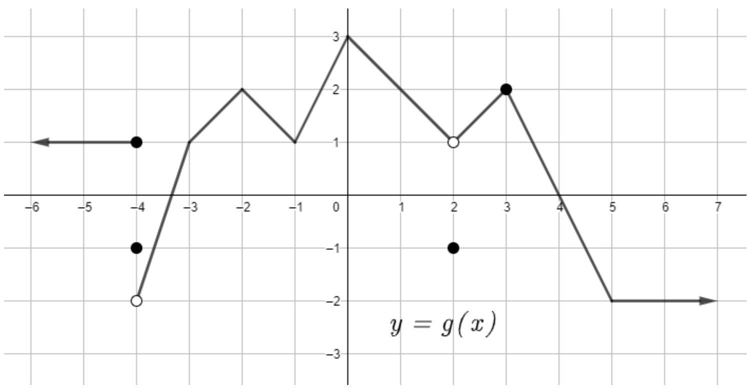
PART A:

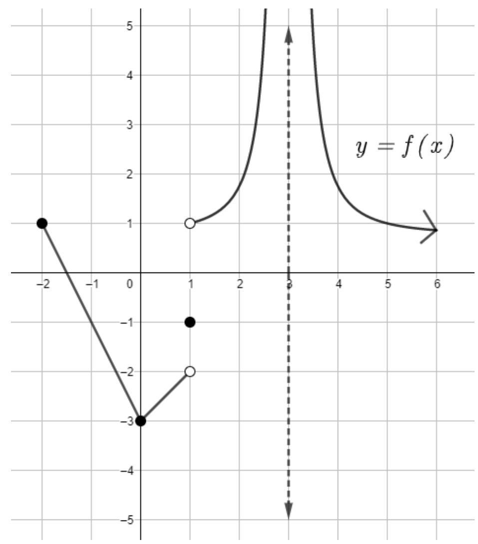
For the following problems, evaluate the limit using the squeeze theorem. Use a calculator to graph the functions $f(x)$, $g(x)$, and $h(x)$ when possible.

1. True or False? If $2x - 1 \le g(x) \le x^2 - 2x + 3$, then $\lim_{x \to 2} g(x) = 0$.

2. True or False? $\lim_{\theta \to 0} \theta^2 \cos\left(\frac{1}{\theta}\right)$

3.





$a.\ \lim_{x \to 2} f(g(x)) =$

$b.\ \lim_{x \to 1} [f(x) + g(x-5)] =$

$c.\ \lim_{x \to 1} g(f(x)) =$

$d.\ \lim_{x \to 3} g(f(x)) =$

$e.\ \lim_{x \to 4} [f(x-3)g(x)] =$

$f.\ \lim_{x \to -3} f(g(x)) =$

PART B: LEAVE UNTIL CHAPTER 9

PLEASE NOTE

Evaluate each of the following limits.

1. $\lim_{x \to 0} \frac{\sin 10x}{10x}$

2. $\lim_{x \to 0} \frac{\cos x - 1}{x}$

3. $\lim_{x \to 0} \frac{\sin 3x}{x}$

4. $\lim_{x \to 0} \frac{\sin x}{5x}$

5. $\lim_{x \to 0} \frac{2\sin x}{7x}$

6. $\lim_{x \to 0} \frac{4x}{\cos 4x}$

7. $\lim_{x \to 0} \frac{\tan 2x}{2x}$

8. $\lim_{x \to 0} \frac{\sin 3x}{\sin 7x}$

9. $\lim_{x \to 0} \frac{\sin 5x}{\sin 2x}$

10. $\lim_{x \to 0} \frac{4\sin 10x}{25\sin 8x}$

11. $\lim_{x \to 0} \frac{\sin 3x}{\cos 4x}$

12. $\lim_{x \to 0} \frac{\sin x}{x^3 - 4x}$

13. $\lim_{x \to 0} \frac{\sin^3 x}{x^3}$ (Hint: $\frac{\sin^3 x}{x^3} = \left(\frac{\sin x}{x}\right)^3$)

14. $\lim_{x \to 0^-} \frac{\cos x}{x}$ (Hint: perform a sign analysis.)

15. $\lim_{x \to 0} \frac{\sin 2x}{\sin x}$ (Hint: $\sin 2x = 2\sin x \cos x$.)

16. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ (Hint: multiply numerator and denominator by $1 + \cos x$.)

17. $\lim_{x \to 0} \frac{4 - 4\cos x}{x}$

18. $\lim_{x \to 0} \frac{\tan 5x}{\tan 2x}$

19. $\lim_{x \to 0} \frac{6x - \sin 2x}{2x}$

20. $\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2 - x}$

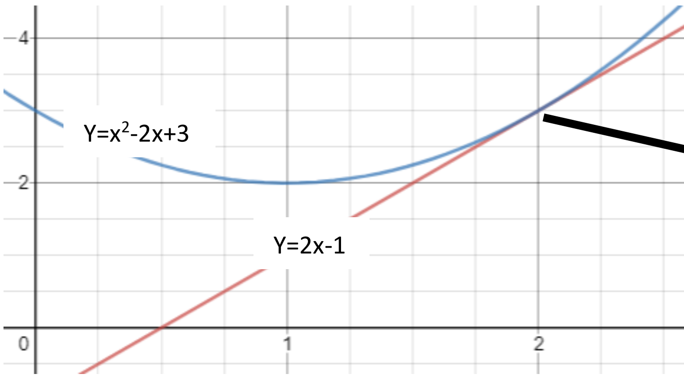
21. $\lim_{x\to 0}\frac{(1-\cos x)^4}{x^3}$ 22. $\lim_{x\to 0}\frac{x^4}{\sin x}$ 23. $\lim_{x\to 0}\frac{1-\cos x}{\sin x}$ 24. $\lim_{x\to \pi/2}\frac{2x-\pi/2}{\sin x}$

25. $\lim_{x\to 0}\frac{\cos 2x}{\cos 3x}$ 26. $\lim_{x\to 0}\frac{x}{\sin(x/2)}$ 27. $\lim_{x\to \infty}\frac{\sin x}{x}$ 28. $\lim_{x\to \infty}\frac{\cos 2x}{2x}$

2.1 Day 4 SOLUTIONS:

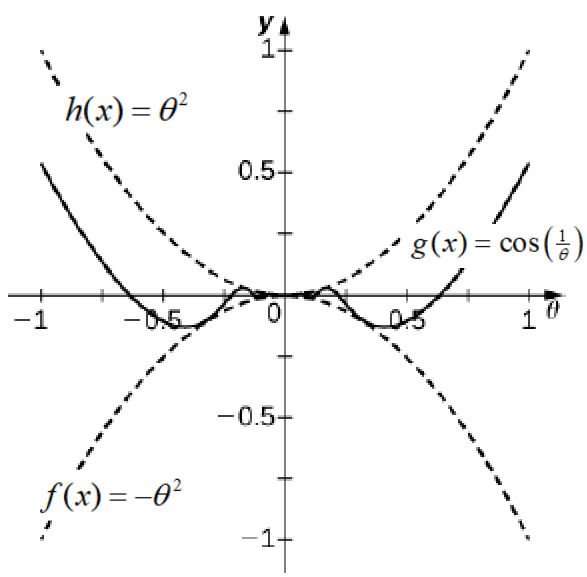
PART A:

1. FALSE



When we graph both the end functions, we know that g(x) is some other function whose graph is squeezed between these two functions. Since the limit of the end functions will be three as x approaches 2, $\lim_{x\to 2} g(x)=3$. The original statement that $\lim_{x\to 2} g(x)=0$ is therefore false.

2. The limit is zero.



- If we first graph $\cos(\frac{1}{\theta})$, we can see that $-1\le\cos(\frac{1}{\theta})\le 1$
- Multiplying the above inequality by θ^2 , we get $-1(\theta^2)\le\theta^2\cos(\frac{1}{\theta})\le 1(\theta^2)$ which is $-\theta^2\le\theta^2\cos(\frac{1}{\theta})\le\theta^2$
- We now take the limit of each: $\lim_{\theta\to 0}(-\theta^2)\le\lim_{\theta\to 0}\theta^2\cos(\frac{1}{\theta})\le\lim_{\theta\to 0}\theta^2$
- Now we algebraically (or graphically) determine each end limits
- $\lim_{\theta\to 0}(-\theta^2)\le\lim_{\theta\to 0}\theta^2\cos(\frac{1}{\theta})\le\lim_{\theta\to 0}\theta^2$

$\lim_{\theta\to 0}(-\theta^2)=0$ $\lim_{\theta\to 0}(\theta^2)=0$

- Since each end limit is zero, and our functions' limit is between them, it also must be zero. Graphically, if you graph all three separate functions you can see that as $\theta\to 1$, the limit squeezes to 0

3. a) 1 b) -1 c) 2 d) -2 e) 0 f) DNE

PART B: LEAVE UNTIL CHAPTER 9

1. 1 2. 0 3. 3 4. 1/5 5. 2/7 6. 0 7. 1 8. 3/7 9. 5/2 10. 1/5 11. 0 12. −1/4 13. 1 14. −∞ 15. 2
16. 1/2 17. 0 18. 5/2 19. 2 20. 0 21. 0 22. 0 23. 0 24. $\pi/2$ 25. 1 26. 2 27. 0 28. 0 29. 2 30. 0

2.2 Day 1 Assignment: Textbook P 76 #3, 6, 9, 13,15, 19,21, 27, 30, 55 & the Following Questions

For Questions 40 -45, evaluate the limits:

40. $\lim_{x\to\infty}\frac{6}{3x-2}$

41. $\lim_{x\to\infty}\frac{2x+5}{x+1}$

42. $\lim_{x\to\infty}\frac{6x^2-1}{2x^2+3x}$

43. $\lim_{x\to-\infty}\frac{-4x^3}{x^3-2x^2}$

44. $\lim_{x\to-\infty}\frac{(x+2)(2x-1)}{x^2+4x+1}$

45. $\lim_{x\to\infty}\frac{2x^2}{x-1}$

SOLUTIONS: 40. 0 41. 2 42. 3 43. -4 44. 2 45. ∞

2.2 Day 2 Assignment: P 76 #35-38 & the Following Questions

46. $\lim_{x\to-\infty}\frac{2x^2}{x-1}$

47. $\lim_{x\to-\infty}\frac{\sqrt{4x^2-x}}{x-2}$

48. $\lim_{g\to\infty}\frac{2g+5}{\sqrt{g^2+6g}}$

49. $\lim_{x\to-\infty}\frac{\sqrt{x^2-4x+4}}{5-x}$

50. $\lim_{x\to\infty}\frac{\sqrt{x^2-2x}}{x^2}$

51. $\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right)$
Hint: rationalize the numerator.

52. Find each of the following limits at infinity. What do the results show about the existence of a horizontal asymptote? Justify your reasoning.

a) $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-x}}$ b) $\lim_{x\to\infty}\frac{-2x^2+x}{\sqrt{2x^2-3}}$

SOLUTIONS: 46. $-\infty$ 47. -2 48. 2 49. 1 50. 0 51. 2

52. a $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-x}}$ $\sqrt{x^2}=-x$, for $x<0$

$\lim_{x\to-\infty}\frac{\frac{2x}{-x}+\frac{1}{-x}}{\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}}$

$\lim_{x\to-\infty}\frac{-2-\frac{1}{x}}{\sqrt{1-\frac{1}{x}}}=\frac{-2-0}{\sqrt{1-0}}$

$=\boxed{-2}$

Since $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-x}}=-2$, the graph has a horizontal asymptote at $y=-2$ the graph approaches to the left.

b $\lim_{x\to\infty}\frac{-2x^2+x}{\sqrt{2x^2-3}}$ $\sqrt{x^2}=x$, for $x>0$

$\lim_{x\to\infty}\frac{-\frac{2x^2}{x}+\frac{x}{x}}{\sqrt{\frac{2x^2}{x^2}-\frac{3}{x^2}}}$

$\lim_{x\to\infty}\frac{-2x+1}{\sqrt{2-\frac{3}{x^2}}}=\frac{-2(\infty)+1}{\sqrt{2-0}}$

$=\boxed{-\infty}$

Since $\lim_{x\to\infty}\frac{-2x^2+x}{\sqrt{2x^2-3}}\neq$ a real number, the graph does not have a horizontal asymptote the graph approaches on the right side.

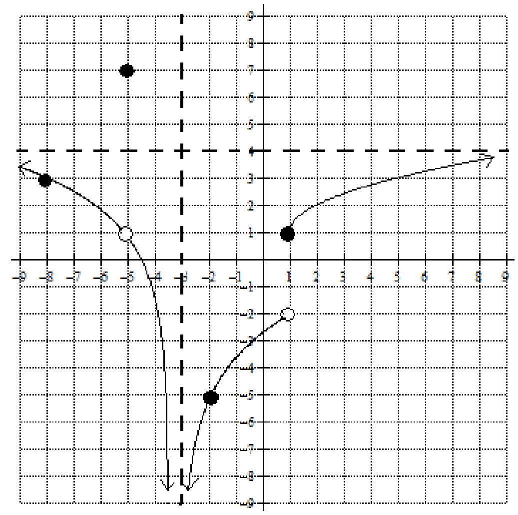
2.3 Assignment: Questions below & Textbook P84 #1, 2, 5-17, 19-29 Odds

1. Determine, using the intermediate value theorem, if the function $F(x) = x^3 + 2x - 1$ has a zero on the interval [0, 1]. Justify your answer and find the indicated zero, if it exists.
2. Determine, using the intermediate value theorem, if the function $g(\vartheta) = \vartheta^2 - 2 - \cos\vartheta$ has a zero on the interval $[0, \pi]$. Justify your answer and find the indicated zero, if it exists.

For exercises 3 – 5, first, verify that the I.V.T. is applicable for the given function on the given interval. Then, if it is applicable, find the value of the indicated *c*, guaranteed by the theorem.

3. $f(x) = x^2 - 6x + 8$ Interval: [0, 3] $f(c) = 0$
4. $g(x) = x^3 - x^2 + x - 2$ Interval: [0, 3] $g(c) = 4$
5. $h(x) = \dfrac{x^2 + x}{x - 1}$ Interval: $\left[\frac{5}{2}, 4\right]$ $h(c) = 6$

6. Given the graph of a function, determine if the function is continuous at x = 5, x = 1 and x = -2. JUSTIFY by using the three part definition of continuity to perform your analysis.



2.3 SOLUTIONS (Continued on next page)

3. $f(x) = x^2 - 6x + 8$
Interval: [0, 3] $f(c) = 0$

① f(x) is a quadratic function that is always continuous.
② $f(0) = 8$ $f(3) = -1$
Since $f(3) < f(c) = 0 < f(0)$, then there exists a c on the interval (0, 3).

$c^2 - 6c + 8 = 0$
$(c-4)(c-2) = 0$
$\cancel{c = 4}$ $\boxed{c = 2}$

4. $g(x) = x^3 - x^2 + x - 2$
Interval: [0, 3] $g(c) = 4$

① g(x) is a cubic function that is always continuous.
② $g(0) = -2$ $g(3) = 19$
Since $g(0) < g(c) = 4 < g(3)$, then there exists a c on the interval (0, 3).

$c^3 - c^2 + c - 2 = 4$
$c^3 - c^2 + c - 6 = 0$
$\boxed{c = 2}$

5. $h(x) = \dfrac{x^2 + x}{x - 1}$
Interval: $\left[\frac{5}{2}, 4\right]$ $h(c) = 6$

① h(x) is only discontinuous at x = 1 which is not on the interval.
② $h(5/2) = 5.833$
$h(4) = 6.667$
Since $h(5/2) < h(c) = 6 < h(4)$, then there exists an x = c on (5/2, 4).

$$\frac{c^2 + c}{c - 1} = \frac{6}{1}$$
$c^2 + c = 6c - 6$
$c^2 - 5c + 6 = 0$
$(c-3)(c-2) = 0$
$\boxed{c = 3}$ $\cancel{c = 2}$

2.3 SOLUTIONS (Continued)

6.

a) $x=-5$	b) $x=1$	c) $x=-2$
I $f(-5)$ is defined and its value is 7. II $\lim_{x\to -5^-} f(x) = 1$ $\lim_{x\to -5^+} f(x) = 1$ $\lim_{x\to -5} f(x)$ exists and equals 1 since $\lim_{x\to -5^-} f(x) = \lim_{x\to -5^+} f(x)$. III $f(-5) = 7 \neq \lim_{x\to 5} f(x)$. ∴, $f(x)$ is not continuous at $x=-5$.	I $f(1)$ is defined and its value is 1 II $\lim_{x\to 1^-} f(x) = -2$ $\lim_{x\to 1^+} f(x) = 1$ Since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$, then $\lim_{x\to 1} f(x)$ DNE. ∴, $f(x)$ is not continuous at $x=1$.	I $f(-2)$ is defined and its value is -5. II $\lim_{x\to -2^-} f(x) = -5$ $\lim_{x\to -2^+} f(x) = -5$ Since $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$, $\lim_{x\to -2} f(x)$ exists and equals -5. III $f(-2) = \lim_{x\to -2} f(x) = -5$. ∴, $f(x)$ is continuous at $x=-2$.

2.1-2.2 AP PRACTICE QUESTIONS: (Printable Version with Extra Space will be posted on Google Classroom)

Multiple Choice Practice

1. $\lim_{x\to 0} \frac{4x-3}{7x+1} =$

   A. ∞ B. −∞ C. 0 D. $\frac{4}{7}$ E. −3

2. $\lim_{x\to \frac{1}{3}} \frac{9x^2-1}{3x-1} =$

   A. ∞ B. −∞ C. 0 D. 2 E. 3

3. $\lim_{x\to 2} \frac{x^3-8}{x^2-4} =$

   A. 4 B. 0 C. 1 D. 3 E. 2

4. The function $G(x) = \begin{cases} x-3, & x<2 \\ -5, & x=2 \\ 3x-7, & x>2 \end{cases}$ is not continuous at x = 2 because…

   A. G(2) is not defined B. $\lim_{x\to 2} G(x)$ does not exist C. $\lim_{x\to 2} G(x) \neq G(2)$

   D. Only reasons B and C E. All of the above reasons.

5. $\lim\limits_{x\to\infty} \dfrac{-3x^2+7x^3+2}{2x^3-3x^2+5} =$

A. ∞ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$

6. $\lim\limits_{x\to-2} \dfrac{\sqrt{2x+5}-1}{x+2} =$

A. 1 B. 0 C. ∞ D. $-\infty$ E. Does Not Exist

7. If $f(x)=3x^2-5x$, then find $\lim\limits_{h\to0} \dfrac{f(x+h)-f(x)}{h}$.

A. $3x-5$ B. $6x-5$ C. $6x$ D. 0 E. Does not exist

8. $\lim\limits_{x\to-\infty} \dfrac{2-5x}{\sqrt{x^2+2}} =$

A. 5 B. -5 C. 0 D. $-\infty$ E. ∞

9. The function $f(x)=\dfrac{2x^2+x-3}{x^2+4x-5}$ has a vertical asymptote at $x = -5$ because…

A. $\lim\limits_{x\to-5^+} f(x)=\infty$

B. $\lim\limits_{x\to-5^-} f(x)=-\infty$

C. $\lim\limits_{x\to-5^-} f(x)=\infty$

D. $\lim\limits_{x\to\infty} f(x)=-5$

E. $f(x)$ does not have a vertical asymptote at $x = -5$

10. Consider the function $H(x)=\begin{cases}3x-5, & x<3\\ x^2-2x, & x\geq 3\end{cases}$. Which of the following statements is/are true?

I. $\lim\limits_{x\to3^-} H(x)=4$.

II. $\lim\limits_{x\to3} H(x)$ exists.

III. $H(x)$ is continuous at $x = 3$.

A. I only B. II only C. I and II only

D. I, II and III E. None of these statements is true

2.1-2.2 AP PRACTICE QUESTIONS: CONTINUED (Printable Version with Extra Space will be posted on Google Classroom)

Free Response Practice #1
Calculator Permitted

Consider the function $h(x) = \dfrac{-2x - \sin x}{x - 1}$ to answer the following questions.

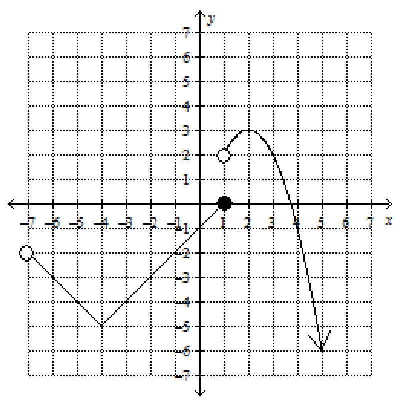
a. Find $\lim\limits_{x \to 1^+} h(x)$. Show your numerical analysis that leads to your answer and explain what this result implies graphically about $h(x)$ at $x = 1$.

b. Find $\lim\limits_{x \to \frac{\pi}{2}} \left[h(x) \cdot (2x - 2)\right]$. Show your analysis.

c. Explain why the Intermediate Value Theorem guarantees a value of c on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find c.

2.1-2.2 AP PRACTICE QUESTIONS: CONTINUED (Printable Version with Extra Space will be posted on Google Classroom)

Free Response Practice #2
Calculator NOT Permitted



Graph of $g(x)$

$$f(x) = \begin{cases} ax + 3, & x < -3 \\ x^2 - 3x, & -3 \le x < 2 \\ bx - 5, & x \ge 2 \end{cases}$$

Equation of $f(x)$

Pictured above is the graph of a function $g(x)$ and the equation of a piece-wise defined function $f(x)$. Answer the following questions.

a. Find $\lim\limits_{x \to 1^+} \left[2g(x) - f(x) \cdot \cos \pi x\right]$. Show your work applying the properties of limits.

b. On its domain, what is one value of x at which $g(x)$ is discontinuous? Use the three part definition of continuity to explain why $g(x)$ is discontinuous at this value.

c. For what value(s) of a and b, if they exist, would the function $f(x)$ be continuous everywhere? Justify your answer using limits.