## 3.1 Day 1 Assignment: Following Questions 1 – 5 plus Textbook P 105 #1-12, 17, 18

For problems 1 – 5, use the function $f(x)=\dfrac{x}{x+2}$.

1. Find $f'(x)$ by finding $\displaystyle\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$.
2. Find the slope of the tangent line drawn to the graph of *f*(*x*) at *x* = –2.
3. Find the slope of the tangent line drawn to the graph of *f*(*x*) at *x* = –1.
4. Find the equation of the tangent line drawn to the graph of *f*(*x*) at *x* = –1.
5. Find $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, where *a* = –1.

### 3.1 Day 1 Solutions

1. $\displaystyle\lim_{h\to0}\frac{\frac{x+h}{x+h+2}-\frac{x}{x+2}}{h}$

$\displaystyle\lim_{h\to0}\frac{(x+h)(x+2)-x(x+h+2)}{(x+h+2)(x+2)}\cdot\frac{1}{h}$

$\displaystyle\lim_{h\to0}\frac{\cancel{x^2}+\cancel{2x}+xh+2h-\cancel{x^2}-xh-\cancel{2x}}{h(x+h+2)(x+2)}$

$\displaystyle\lim_{h\to0}\frac{\cancel{h}(x+2-x)}{\cancel{h}(x+h+2)(x+2)}$

$\displaystyle\lim_{h\to0}\frac{2}{(x+h+2)(x+2)}$

$\dfrac{2}{(x+0+2)(x+2)}$

$\boxed{f'(x)=\dfrac{2}{(x+2)^2}}$

2. $f'(-2)=\dfrac{2}{(-2+2)^2}=\dfrac{2}{0}$

$=\boxed{\text{undefined}}$

3. $f'(-1)=\dfrac{2}{(-1+2)^2}=\dfrac{2}{1}=\boxed{2}$

4. $f(-1)=\dfrac{-1}{-1+2}=\dfrac{-1}{1}=-1$

$f'(-1)=2$

$\boxed{y+1=2(x+1)}$

5. $\displaystyle\lim_{x\to-1}\frac{\frac{x}{x+2}-(-1)}{x+1}$

$\displaystyle\lim_{x\to-1}\frac{\frac{x}{x+2}+\frac{x+2}{x+2}}{x+1}$

$\displaystyle\lim_{x\to-1}\frac{2x+2}{x+2}\cdot\frac{1}{x+1}$

$\displaystyle\lim_{x\to-1}\frac{2\cancel{(x+1)}}{x+2}\cdot\frac{1}{\cancel{x+1}}=\frac{2}{-1+2}$

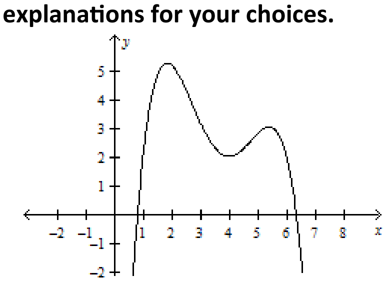
$=\dfrac{2}{1}$

$=\boxed{2}$

## 3.1 Day 2 Assignment: Following Questions & Textbook Questions P105 #13-16, 21, 22, 24, 25, 26, 31

1. The line defined by the equation $2y+3=-\frac{2}{3}(x-3)$ is tangent to the graph of *g*(*x*) at *x* = –3. What is the value of $\displaystyle\lim_{x\to-3}\frac{g(x)-g(-3)}{x+3}$? Show your work and explain your reasoning.

**Use the graph of *f*(*x*) pictured to the right to perform the actions in exercises 2 – 6. Give written explanations for your choices.**



2. Label a point, A, on the graph of *y* = *f*(*x*) where the derivative is negative.
3. Label a point, B, on the graph of *y* = *f*(*x*) where the value of the function is negative.
4. Label a point, C, on the graph of *y* = *f*(*x*) where the derivative is greatest in value.
5. Label a point, D, on the graph of *y* = *f*(*x*) where the derivative is zero.
6. Label two different points, E and F, on the graph of *y* = *f*(*x*) where the values of the derivative are opposites.

The table below represents values on the graph of a cubic polynomial function, $h(x)$. Use the table to complete exercises 7-9.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|---|
| $h(x)$ | –24 | 0 | 8 | 6 | 0 | –4 | 18 |

7. Two of the zeros of $h(x)$ are listed in the table. Between which two values of $x$ does the Intermediate Value Theorem guarantee that a third value of $x$ exists such that $h(x) = 0$? Explain your reasoning.

8. Estimate the value of $h'(1.5)$. Based on this value, describe the behavior of $h(x)$ at $x = 1.5$. Justify your reasoning

9. Estimate the value of $h'(-1.75)$. Based on this value, describe the behavior of $h(x)$ at $x = -1.75$. Justify your reasoning.

## 3.2 Assignment: P114 #1-33 Odds, 41-45 Odds (Optional Twinning Activity by BP)

## 3.3 Day 1 Assignment: Questions Below: Part A: 1-5 odds, 7, 8, 11 Part B: 1-5

**Part A:**

1. Write each function as the sum/difference of terms of the form $cx^n$, and find the derivative. You may leave negative exponents in your answer and radicals do not have to be rationalized.

(a) $f(x)=3x^2-7x+11$

(b) $y=-4x^3+6x^2-3x+2$

(c) $y=\frac{3}{4}x^4-\frac{5}{3}x^3+\frac{7}{2}x^2-x-6$

(d) $f(x)=\frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+\frac{2}{3}x+\frac{3}{4}$

(e) $y=6x^{2/3}-4x^{1/2}+3\pi$

(f) $f(x)=4-\frac{2}{x}+\frac{5}{x^2}-\frac{7}{x^3}$

(g) $f(x)=(x-8)^2$

(h) $y=(2x+1)^3$

(i) $y=(3x-4)(2x+5)$

(j) $f(x)=x^3(2x-1)(2x+1)$

(k) $f(x)=\frac{2x^4-3x^3}{x}$

(l) $f(x)=\frac{x^2+4x-8}{2x^4}$

(m) $y=8\sqrt{x}-6\sqrt[3]{x}-4\sqrt[4]{x}+2\sqrt{2}$

(n) $y=\sqrt{2x}+\sqrt[3]{3x}+\sqrt[4]{5x}+\sqrt[5]{7}$

(o) $y=\sqrt{\frac{x}{2}}-\sqrt[3]{\frac{x}{3}}$

(p) $f(x)=\frac{3}{x^3}+\frac{8}{\sqrt{x}}-\sqrt{3x}$

2. (a) If $y=m^4-6m^2-8$, find $\frac{dy}{dm}$.

(b) If $V=\frac{4}{3}\pi r^3$, find $\frac{dV}{dr}$.

(c) If $f(x)=\frac{1}{x^4}-\frac{2}{x^3}+\frac{3}{x^2}$, find $\frac{d}{dx}f(x)$.

(d) If $y=2a^{-2}+3a-4$, find $y'$.

(e) Find $\frac{d}{dc}\left(2c^{10}-5c^2\right)$.

(f) If $x=4y^2-6y+11$, find $\frac{dx}{dy}$.

3. Find the slope of the tangent line to each function at the given point.

(a) $f(x)=2x^2-5x+3$ at $x=-2$

(b) $y=\frac{1}{6}x^3-\frac{3}{4}x^2+2x-7$ at $x=4$

(c) $y=3\sqrt{x}-2\sqrt[3]{x}+6$ at $x=64$

(d) $f(x)=\frac{x-3}{x}$ at $x=-3$

4. Find the equation of the tangent line to the given curve at the given point. You may leave your answers in slope-intercept form.
   (a) $f(x) = -x^3 + 8x^2$ at $(5, f(5))$
   (b) $y = \frac{3}{x} - \frac{2}{x^2} + 7$ at $(-2, f(-2))$
   (c) $y = (x^2 - 2x + 3)^2$ at $(-1, f(-1))$
5. Find the coordinates of the point(s), if any, at which the tangent line is horizontal.
   (a) $f(x) = 3x^2 - 12x + 5$
   (b) $y = x^3 - 6x^2 + 9x - 1$
   (c) $f(x) = 2x^3 + 3x^2 + 30x - 40$
6. Find the derivatives of $f(x) = 2x - 3$ and $g(x) = (2x-3)^2$. How does $g'(x)$ compare with $f(x)$?
7. Find the equations of two lines, each of slope 6, that are tangent to the curve $f(x) = x^3 + 3x + 1$.
8. What is the smallest slope that a tangent line to the curve $y = x^3 - 6x$ will ever have?
9. The slope of the tangent line drawn at the vertex of a parabola will always be 0. Based on this, show that the $x$-coordinate of the vertex of the parabola $f(x) = ax^2 + bx + c$ is $x = -\frac{b}{2a}$.
10. Tangent lines are drawn to the function $f(x) = x^2 - \frac{4}{x}$ at the points $(-1, 5)$ and $(1, -3)$. Find the coordinates of the point at which the tangent lines intersect.
11. A line is drawn tangent to the curve $y = \sqrt{x}$ at the point $(4, 2)$. How high above the curve is the tangent line at $x = 16$?

## Part B:

1. For what value(s) of *x* will the slope of the tangent line to the graph of $h(x) = 4\sqrt{x}$ be 2? Find the equation of the line tangent to *h*(*x*) at this/these *x* – values. Show your work.
2. Find the equation of the line tangent to the graph of $g(x) = \frac{2}{\sqrt[4]{x^3}}$ when *x* = 1.
3. The line defined by the equation $\frac{1}{2}x + 3 = -2(y - 3)$ is the line tangent to the graph of a function *f*(*x*) when *x* = *a*. What is the value of $f\ '(a)$? Show your work and explain your reasoning.
4. The line defined by the equation $y - 3 = -\frac{2}{3}(x + 3)$ is the line tangent to the graph of a function *f*(*x*) at the point (–3, 3). What is the equation of the normal line when *x* = –3. Explain your reasoning.
5. Determine the value(s) of *x* at which the function $f(x) = x^4 - 8x^2 + 2$ has a horizontal tangent.
6. For what value(s) of *k* is the line *y* = 4*x* – 9 tangent to the graph of *f*(*x*) = *x*² – *kx*?

**SOLUTIONS: Part A**

1. (a) $6x - 7$ (b) $-12x^2 + 12x - 3$ (c) $3x^3 - 5x^2 + 7x - 1$ (d) $x^4 - x^3 + x^2 - x + \frac{2}{3}$ (e) $4x^{-1/3} - 2x^{-1/2}$
(f) $2x^{-2} - 10x^{-3} + 21x^{-4}$ (g) $2x - 16$ (h) $24x^2 + 24x + 6$ (i) $12x + 7$ (j) $20x^4 - 3x^2$ (k) $6x^2 - 6x$
(l) $-x^{-3} - 6x^{-4} + 16x^{-5}$ (m) $4x^{-1/2} - 2x^{-2/3} - x^{-3/4}$ (n) $\frac{\sqrt{2}}{2}x^{-1/2} + \frac{\sqrt[3]{3}}{3}x^{-2/3} + \frac{\sqrt[4]{5}}{4}x^{-3/4}$
(o) $\frac{1}{2\sqrt{2}}x^{-1/2} - \frac{1}{2\sqrt[3]{3}}x^{-2/3}$ (p) $-9x^{-4} - 4x^{-3/2} - \frac{\sqrt{3}}{2}x^{-1/2}$ 2. (a) $4m^3 - 12m$ (b) $4\pi r^2$
(c) $-4x^{-5} + 6x^{-4} - 6x^{-3}$ (d) $-4a^{-3} + 3$ (e) $20c^9 - 10c$ (f) $8y - 6$ 3. (a) $-13$ (b) 4 (c) $\frac{7}{48}$ (d) $\frac{1}{3}$
4. (a) $y = 5x + 50$ (b) $y = -\frac{5}{4}x + \frac{5}{2}$ (c) $y = -48x - 12$ 5. (a) $(2, -7)$ (b) $(1, 3)$ and $(3, -1)$
(c) no such point 6. $f'(x) = 2$, $g'(x) = 8x - 12 = 4(2x - 3)$; $g'(x) = 4f(x)$ 7. $y = 6x - 1$ and $y = 6x + 3$
8. $-6$ 9. $f'(x) = 2ax + b$. At the vertex the tangent line slope is 0. Thus $2ax + b = 0$. Solving, $x = -\frac{b}{2a}$.
10. $(4, 15)$ 11. 1 unit 12. $82.9°$ at $x = \pm 2$; $76.0°$ at $x = 0$ 13. If $F(x) = f(x) - g(x)$, then

**SOLUTIONS: Part B**

1. $h(x) = 4x^{1/2}$
$h'(x) = 2x^{-\frac{1}{2}}$
$h'(x) = \frac{2}{\sqrt{x}}$
$\frac{2}{\sqrt{x}} = -2$
$-2\sqrt{x} = 2$
$\sqrt{x} = -1$
No solution
Since $h(x)$ is always increasing, the slope of the tangent line will never be negative.

2. $g(1) = \frac{2}{\sqrt[4]{1^3}} = 2$
POT : $(1, 2)$
SOT : $-\frac{3}{2}$
$\boxed{y-2 = -\frac{3}{2}(x-1)}$
$g(x) = 2x^{-3/4}$
$g'(x) = -\frac{3}{2}x^{-7/4}$
$g'(x) = -\frac{3}{2\sqrt[4]{x^7}}$
$g'(1) = -\frac{3}{2\sqrt[4]{1^7}} = -\frac{3}{2}$

3. $f'(a)$ is the slope of the tangent line so the slope of $\frac{1}{2}x+3 = -2(y-3)$ is $f'(a)$. $f'(a) = -\frac{1}{4}$
$\frac{1}{2}x+3 = -2y+6$
$-\frac{1}{2}[-2y = \frac{1}{2}x - 3$
$y = -\frac{1}{4}x + \frac{3}{2}$

4. $\boxed{x=0 \quad x=-2 \quad x=2}$

5. $f'(x) = 2x - K = 4$
$2x - 4 = K$
$x^2 - Kx = 4x - 9$
$x^2 - x(2x-4) = 4x - 9$
$x^2 - 2x^2 + 4x = 4x - 9$
$-x^2 = -9$
$x^2 = 9$
$x = \pm 3$
$x = 3$: $K = 2(3) - 4$, $K = 6 - 4$, $\boxed{K = 2}$
$x = -3$: $K = 2(-3) - 4$, $K = -6 - 4$, $\boxed{K = -10}$

**3.3 Day 2 Assignment: QUESTIONS BELOW Part A: 1-20 EVEN, 21-24 Part B: a, b**

**Part A:**

Find the derivative of each of the following functions *using the product rule in each case*. Write each answer in simplified form.

1. $y = (9x+11)(3x-4)$
2. $f(x) = (x^2-4x)(2x+7)$
3. $f(x) = (x^2-6x+5)(x^2-3x-2)$
4. $y = (6x^4)(5x^2+8x-4)$
5. $y = (-3x^{-2})(2x^5+4x^3)$
6. $f(x) = 6x$ (Think of $6x$ as the product of 6 and $x$.)
7. $f(x) = (x-x^{-1})(x+x^{-1})$
8. $y = (2x)(3x-4)(5x+2)$
9. $y = x^2(2x+3)(x^2-x)$
10. $f(x) = (x^2-1)^2$ (Think of $(x^2-1)^2$ as $(x^2-1)(x^2-1)$.)

Find the derivative of each of the following functions using the product rule. **Do not simplify** your answer.

11. $f(x) = (2x-1)(x^2+3)(2x^4)$
12. $f(x) = (2x^2-6x-1)(x^2+4x+8)$
13. $y = (4x^{1/2})(2x^3-x^{-3})(5x^{-1/3}+2x^2)$
14. $y = (6-\sqrt{x})(2x+9\sqrt[3]{x})(-x^2+\sqrt[4]{x^3})$
15. $f(x) = \left(\frac{x}{4}-\frac{4}{x}\right)\left(\frac{x^2}{6}+\frac{6}{\sqrt[3]{x}}\right)$

Find the slope of the tangent line to each function at the given value of $x$. Use the product rule in each question.

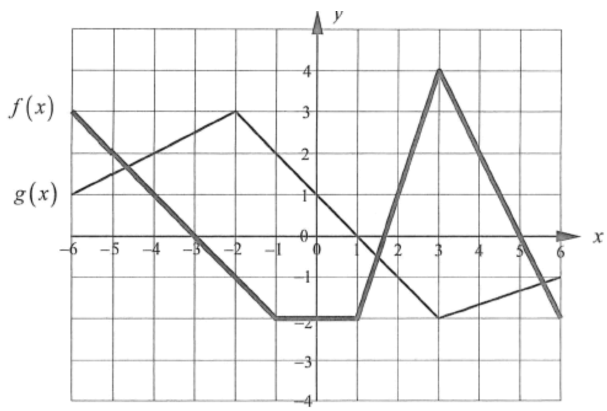
16. $y = (6x^2-5x-4)(x^3+4x^2)$; $x=1$
17. $y = (x-3)(2x^2-x-1)(x^3+3x^2+2)$; $x=0$
18. $f(x) = (x-2)(x-1)(x)(x+3)$; $x=-1$

Find the equation of the tangent line to each function at the given point. Leave your answers in slope-intercept form.

19. $f(x) = -x^2(x^2+4x)(3x-2)$; $x=-1$

20. $y = \sqrt{x}\left(5-3\sqrt[3]{x}\right)\left(\frac{1}{x}-2\right)$; $x = 1$

Shown at right is a graph of the functions $f(x)$ and $g(x)$. Assume that $F(x) = f(x)\cdot g(x)$. By studying the graph and using the product rule, determine the value of each of the following.




21. $F'(2)$
22. $F'(-4)$
23. $F'(0)$
24. $F'(3)$
25. If $f(x) = (x^2+3x)^5$, use the product rule to show that $f'(x) = 5(x^2+3x)^4(2x+3)$.

**Part B:** Use the table to answer the following questions:

| x | f(x) | f ′(x) | g(x) | g′(x) |
|---|---|---|---|---|
| –2 | 1 | –1 | 2 | 4 |
| –1 | 3 | –2 | 1 | 1 |
| 0 | –1 | 2 | –2 | –3 |

a) If $H(x) = 2f(x)\cdot g(x)$, what is the value of $\lim_{x\to -2} \frac{H(x)-H(-2)}{x+2}$?

b) If $K(x) = (4x - f(x))(2g(x)-2)$, what is the slope of the normal line when $x = -2$?

**SOLUTIONS Part A:** 1. $54x-3$ 2. $6x^2-2x-28$ 3. $4x^3-27x^2+42x-3$ 4. $180x^5+240x^4-96x^3$ 5. $-18x^2-12$ 6. 6
7. $2x+2x^{-3}$ 8. $90x^2-56x-16$ 9. $10x^4+4x^3-9x^2$ 10. $4x^3-4x$
11. $2(x^2+3)(2x^4)+(2x-1)(2x)(2x^4)+(2x-1)(x^2+3)(8x^3)$
12. $(4x-6)(x^2+4x+8)+(2x^2-6x-1)(2x+4)$
13. $(2x^{-1/2})(2x^3-x^{-3})(5x^{-1/3}+2x^2)+(4x^{1/2})(6x^2+3x^{-4})(5x^{-1/3}+2x^2)+(4x^{1/2})(2x^3-x^{-3})\left(-\frac{5}{3}x^{-4/3}+4x\right)$
14. $\left(-\frac{1}{2}x^{-1/2}\right)(2x+9\sqrt[3]{x})(-x^2+\sqrt[4]{x^3})+(6-\sqrt{x})(2+3x^{-2/3})(-x^2+\sqrt[4]{x^3})+(6-\sqrt{x})(2x+9\sqrt[3]{x})\left(-2x+\frac{3}{4}x^{-1/4}\right)$
15. $\left(\frac{1}{4}+4x^{-2}\right)\left(\frac{x^2}{6}+\frac{6}{\sqrt[3]{x}}\right)+\left(\frac{x}{4}-\frac{4}{x}\right)\left(\frac{1}{3}x-2x^{-4/3}\right)$ 16. 2 17. 4 18. 16 19. $y=49x+34$ 20. $y=-2x$
21. $F'(2)=f'(2)g(2)+f(2)g'(2)=(3)(-1)+(1)(-1)=-4$ 22. $-\frac{3}{2}$ 23. 2 24. $F'(3)$ does not exist since $f'(3)$ and $g'(3)$ do not exist.
25. Let $g(x)=x^2+3x$. Then $g'(x)=2x+3$.
Now $f(x)=(x^2+3x)^5=[g(x)]^5=[g(x)][g(x)][g(x)][g(x)][g(x)]$.
$\therefore f'(x)=[g'(x)][g(x)][g(x)][g(x)][g(x)]+[g(x)][g'(x)][g(x)][g(x)][g(x)]+\ldots$
$+[g(x)][g(x)][g(x)][g(x)][g'(x)]$ (there are 5 such terms)
$=5[g(x)]^4g'(x)=5(x^2+3x)^4(2x+3)$

**SOLUTIONS Part B:** a) 4 b) 1/62

## 3.3 Day 3 Assignment: QUESTIONS BELOW Part A: 1-14 Even, 17-19 Part B: a & b

## Part A:

Use the quotient rule to find the derivative of each of the following functions.

1. $y=\dfrac{2x}{x+1}$
2. $y=\dfrac{x^2}{2x-3}$
3. $f(x)=\dfrac{x+4}{x-4}$
4. $f(x)=\dfrac{2x-3}{2x+3}$
5. $y=\dfrac{x-4}{x^2}$
6. $y=\dfrac{1}{x^2+2x+3}$
7. $f(x)=\dfrac{4-2x}{1-x}$
8. $f(x)=\dfrac{x^2-4}{x^2+1}$
9. $f(x)=\dfrac{2}{\sqrt{x}+1}$
10. $y=\dfrac{\sqrt{x}}{x+2}$
11. $y=\dfrac{x}{\sqrt{x}+1}$
12. $f(x)=\dfrac{\sqrt{x}+2}{\sqrt{x}-2}$

Find the equation of the tangent line drawn to the given curve at the given point. Give your answers in the general form $Ax+By+C=0$.

13. $y=\dfrac{x^2}{x+2}$, $(2,1)$
14. $f(x)=\dfrac{\sqrt{x}}{2-x}$, $(4,-1)$
15. $y=\dfrac{12}{x^2+2}$, $(-1,4)$

In questions 16 and 17, find the derivative in two different ways. First, simplify the expression and then differentiate. Second, use the quotient rule.

16. $y=\dfrac{15x^6}{3x^2}$
17. $f(x)=\dfrac{x^3-8}{x-2}$

18. Find the coordinates of two points on the graph of the function $f(x)=\dfrac{10x}{x^2+1}$ at which the tangent line is horizontal. This curve is known as a *serpentine*.

19. Find the equations of two lines tangent to the curve $y=\dfrac{x}{x+1}$ that are parallel to the line $x-4y-6=0$. Write your answers in slope-intercept form.

## Part B:

Use the table below to complete exercises 8 – 10.

| *x* | *f*(*x*) | *f* ′(*x*) | *g*(*x*) | *g*′(*x*) |
|---|---|---|---|---|
| –2 | 1 | –1 | 2 | 4 |
| –1 | 3 | –2 | 1 | 1 |
| 0 | –1 | 2 | –2 | –3 |

a) If $H(x)=\dfrac{2f(x)}{g(x)}$, what is the equation of the tangent line when $x = -1$?

10. If $K(x)=\dfrac{4x+f(x)}{3-g(x)}$, what is the slope of the tangent line when $x = -2$?

**SOLUTIONS Part A:**

1. $\dfrac{2}{(x+1)^2}$ 2. $\dfrac{2x^2-6x}{(2x-3)^2}$ 3. $\dfrac{-8}{(x-4)^2}$ 4. $\dfrac{12}{(2x+3)^2}$ 5. $\dfrac{8-x}{x^3}$ 6. $\dfrac{-2x-2}{(x^2+2x+3)^2}$ 7. $\dfrac{2}{(x-1)^2}$ 8. $\dfrac{10x}{(x^2+1)^2}$

9. $\dfrac{-1}{\sqrt{x}(\sqrt{x}+1)^2}$ 10. $\dfrac{2-x}{2\sqrt{x}(x+2)^2}$ 11. $\dfrac{\sqrt{x}+2}{2(\sqrt{x}+1)^2}$ 12. $\dfrac{-2}{\sqrt{x}(\sqrt{x}-2)^2}$ 13. $3x-4y-2=0$

14. $3x-8y-20=0$ 15. $8x-3y+20=0$ 16. $20x^3$ 17. $2x+2$ 18. $(1,5)$ and $(-1,-5)$ 19. $y=\frac{1}{4}x+\frac{1}{4}$, tangent at $(1,1/2)$ and $y=\frac{1}{4}x+\frac{9}{4}$, tangent at $(-3,3/2)$ 20. $y=x^{-n}=\frac{1}{x^n}$. Using the quotient rule, $\frac{dy}{dx}=\frac{0(x^n)-(1)(nx^{n-1})}{(x^n)^2}=\frac{-nx^{n-1}}{x^{2n}}=-nx^{n-1-2n}=-nx^{-n-1}$ 21. **(a)** $T(0)=100$, $T(1)=80$, $T(5)=37.5$;

**SOLUTIONS Part B:** a) $\boxed{y-6=-10(x+1)}$

b) 25

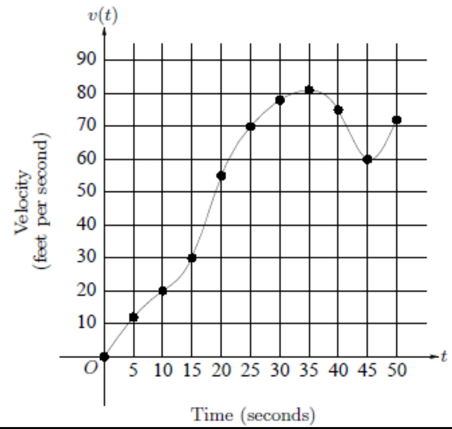
## 3.3 Day 4 Assignment: Textbook P 124 #24, 25, 30, 32-36, 39

## 3.4 Assignment: Textbook P 135 #1, 8, 9, 10a, 13, 15, 16, 19, 24, 34 & Questions 1 & 2 Below

**1. 1998 AP Calculus AB #3 (Modified)**

The graph of the velocity $v(t)$, in feet per second, of a car traveling on a straight road, for $0 \le t \le 50$ is shown below. A table of values for $v(t)$, at 5 second intervals of time, is also shown to the right of the graph.

a. During what interval(s) of time is the acceleration of the car positive? Give a reason for your answer.




| $t$ (seconds) | $v(t)$ (feet per second) |
|---|---|
| 0 | 0 |
| 5 | 12 |
| 10 | 20 |
| 15 | 30 |
| 20 | 55 |
| 25 | 70 |
| 30 | 78 |
| 35 | 81 |
| 40 | 75 |
| 45 | 60 |
| 50 | 72 |

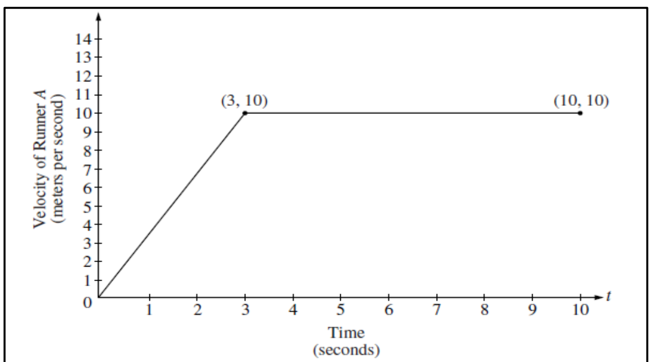
b. Find the average acceleration of the car over the interval $0 \le t \le 50$. Indicate units of measure.

c. Find one approximation for the acceleration of the car at $t = 40$. Show the computations you used to arrive at your answer. Indicate units of measure.

d. Is the speed of the car increasing or decreasing at $t = 40$? Give a reason for your answer.

**2. 2000 AP Calculus AB #2 (Partial)**

Two runners, $A$ and $B$, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner $A$. The velocity, in meters per second, of Runner $B$ is given by the function $v$ defined by $v(t) = \frac{24t}{2t+3}$.




a. Find the velocity of Runner A and the velocity of Runner B at $t = 2$ seconds. Indicate units of measure.

b. Find the acceleration of Runner $A$ and the acceleration of Runner $B$ at time $t = 2$ seconds. Indicate units of measure.

**SOLUTIONS**

1a If $a(t) > 0$, then $v(t)$ must be increasing which occurs on $0 < t < 35$ and $45 < t < 50$.

b) Average Acceleration $= \frac{v(0) - v(50)}{0 - 50}$

$= \frac{0 - 72}{0 - 50} = \frac{-72}{-50} = \boxed{\frac{36}{25} \text{ ft/sec}^2}$

c) $v'(40) = a(40) \approx \frac{v(35) - v(45)}{35 - 45} \approx \frac{81 - 60}{-10} \approx \boxed{\frac{21}{-10} \text{ ft/sec}^2}$

d) $v(40) = 75 > 0$. Since $v(t)$ is decreasing at $t = 40$, then $v'(40) = a(40) < 0$. Since $v(40)$ and $a(40)$ have opposite signs, then the speed is decreasing at $t = 40$.

**SOLUTIONS to 3.4 Cont:**

2 a) Runner A : $v(t) = \frac{10}{3}t + 0$ $v(2) = \frac{10}{3}(2) = \boxed{\frac{20}{3}\ m/sec}$

Runner B : $v(2) = \frac{24(2)}{2(2)+3} = \boxed{\frac{48}{7}\ m/sec}$

b) Runner A : $v(t) = \frac{10}{3}t$

$v'(t) = \frac{10}{3}$

$v'(2) = \frac{10}{3}$

$a(2) = \boxed{\frac{10}{3}\ m/sec^2}$

Runner B: $v'(t) = \frac{(2t+3)(24) - 24t(2)}{(2t+3)^2}$

$v'(2) = \frac{(2\cdot 2+3)(24) - 24(2)(2)}{(2\cdot 2+3)^2}$

$a(2) = \frac{168-96}{49}$

$a(2) = \boxed{\frac{72}{49}\ m/sec^2}$

## 3.5 Assignment: P146 #1, 2, 4, 6, 9, 10, 21, 29, 35, 37, 46-48 Plus Questions 1-3 Below

1. Determine the value(s) of $\vartheta$ at which the function $f(\theta) = \sqrt{3}\theta + 2\cos\theta$ has a horizontal tangent on the interval $[0, 2\pi)$.

2. Find the equation of the line tangent to the graph of $g(x) = x^2 \cos x$ when $x = \frac{\pi}{2}$.

3. Use the table to answer the following question: . If $J(x) = g(x)\cdot\sin x$, what is the value of $J'(0)$?

| *x* | *f(x)* | *f ′(x)* | *g(x)* | *g′(x)* |
|---|---|---|---|---|
| –2 | 1 | –1 | 2 | 4 |
| –1 | 3 | –2 | 1 | 1 |
| 0 | –1 | 2 | –2 | –3 |

**SOLUTIONS**

1. $f(\theta) = \sqrt{3}\theta + 2\cos\theta$

$f'(\theta) = \sqrt{3} - 2\sin\theta = 0$

$-2\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{3} \text{ and } \frac{2\pi}{3}}$

2. $g(\frac{\pi}{2}) = (\frac{\pi}{2})^2 \cdot \cos\frac{\pi}{2}$

$g(\frac{\pi}{2}) = \frac{\pi^2}{4}\cdot 0 = 0$

$g'(x) = x^2 \cdot -\sin x + 2x \cdot \cos x$

$g'(\pi/2) = (\frac{\pi}{2})^2 \cdot -\sin\frac{\pi}{2} + 2\cdot\frac{\pi}{2}\cdot\cos\frac{\pi}{2}$

$g'(\pi/2) = \frac{\pi^2}{4}\cdot -1 + 0$

$g'(\frac{\pi}{2}) = -\frac{\pi^2}{4}$

$\boxed{y - 0 = -\frac{\pi^2}{4}(x - \frac{\pi}{2})}$

3. $J'(x) = g'(x)\cdot\sin x + g(x)\cdot\cos x$

$J'(0) = g'(0)\sin 0 + g(0)\cdot\cos 0$

$J'(0) = -3(0) + (-2)(1)$

$\boxed{J'(0) = -2}$