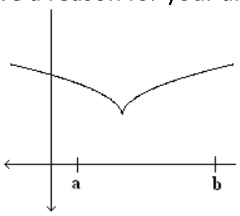
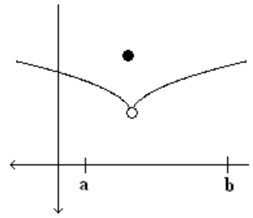
## 5.1 Assignment: Textbook Page 198 #1-10, 11-13, 15, 17, 19, 20, 22, 23, 26, 30, 35, 40, 43 Plus Additional Questions 1-12 Below

1. For which of the following functions is the Extreme Value Theorem NOT APPLICABLE on the interval [*a*, *b*]? Give a reason for your answer.

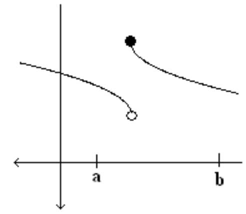
Graph I



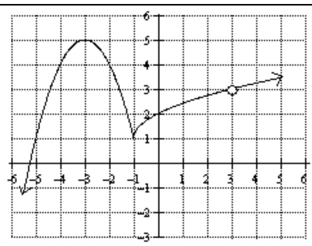
Graph II



Graph III



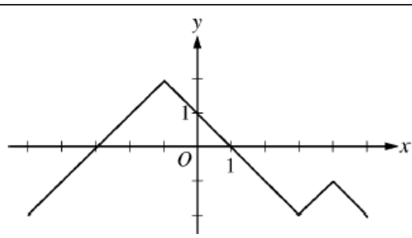
For exercises 2 – 4, determine the critical numbers for each of the functions below.

| | | |
|---|---|---|
| 2.  | 3. $g(x) = \ln(x^2 + 4)$ | 4. $h(x) = \sqrt[3]{x+3}$ |

5. The function $g$ is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of $g$ on the closed interval $[-2, 1]$?

(A) $-7$ (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

6.

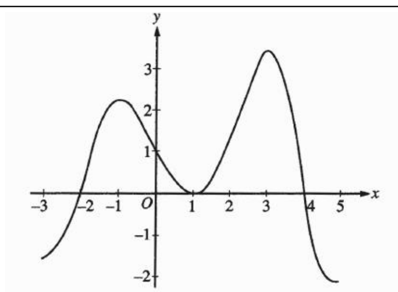


Graph of $f'$

The graph of $f'$, the derivative of $f$, is shown in the figure above. The function $f$ has a local maximum at $x =$

(A) $-3$ (B) $-1$ (C) 1 (D) 3 (E) 4

7.

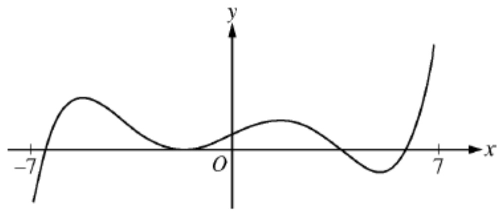


Graph of $f'$

The graph of the derivative of a function $f$ is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of $x$ does $f$ have a relative maximum?

(A) $-2$ only (B) 1 only (C) 4 only (D) $-1$ and 3 only (E) $-2$, 1, and 4

8.




Graph of $f'$

The figure above shows the graph of $f'$, the derivative of the function $f$, on the open interval $-7 < x < 7$. If $f'$ has four zeros on $-7 < x < 7$, how many relative maxima does $f$ have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

**The following questions are calculator active:**

9. . If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then $f$ has a local maximum at $x =$

(A) $-2.314$ (B) $-1.332$ (C) $0.350$ (D) $0.829$ (E) $1.234$

10. The derivative of the function $f$ is given by $f'(x) = x^3 - 4\sin\left(x^2\right) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of $x$ does $f$ have a relative minimum'

(A) $-1.970$ and $0$

(B) $-1.467$ and $1.075$

(C) $-0.475$, $0.542$, and $1.396$

(D) $-0.475$ and $1.396$ only

(E) $0.542$ only

11. Let $f$ be the function with first derivative given by $f'(x) = \left(3 - 2x - x^2\right)\sin(2x - 3)$. How many relative extrema does $f$ have on the open interval $-4 < x < 2$?

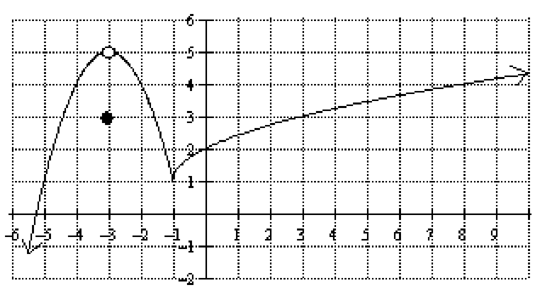
(A) Two (B) Three (C) Four (D) Five (E) Six

**Solutions to 5.1 Extra Questions**

1. Graphs II and III are not continuous on the interval [a, b]. Therefore, the Extreme Value Theorem is not applicable.
2. x = -3, x = -1, x = 3 3. X = 0 4. X = -3
5. A 6. C 7. C 8. A 9. C 10. D 11. E

**5.2 Day 1 Assignment: Textbook Page 206 #1-14, 53, 56**
**Additional Questions Below 1-13**
**OPTIONAL: Rolle's Theorem Questions p 42 of Duo Tang**

1. Using the graph of the function, $f(x)$, pictured below, and given the intervals [–1, 8], determine if the Mean Value Theorem can be applied or not. Give SPECIFIC reasons for your answer.



2. The function $f$ is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

A) There exists $c$, where $-2 < c < 1$, such that $f(c) = 0$.
B) There exists $c$, where $-2 < c < 1$, such that $f'(c) = 0$.
C) There exists $c$, where $-2 < c < 1$, such that $f(c) = 3$.
D) There exists $c$, where $-2 < c < 1$, such that $f'(c) = 3$.
E) There exists $c$, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all $x$ on the closed interval $-2 \le x \le 1$.

3. Let $f$ be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one $c$ between $-3$ and $6$

(C) $-1 \le f(x) \le 3$ for all $x$ between $-3$ and $6$

(D) $f(c) = 1$ for at least one $c$ between $-3$ and $6$

(E) $f(c) = 0$ for at least one $c$ between $-1$ and $3$

4. Let $f$ be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. $f$ has at least 2 zeros.
II. The graph of $f$ has at least one horizontal tangent.
III. For some $c$, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

5. THE FOLLOWING QUESTION IS CALCULATOR ACTIVE:

Let f be the function defined by $f(x) = x + \ln(x)$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

(A) 0.456 (B) 1.244 (C) 2.164 (D) 2.342 € 2.452

6. If $f$ is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number $c$ in the open interval $(a, b)$ such that $f(c) = 0$.

(B) There is a number $c$ in the open interval $(a, b)$ such that $f(a) < f(c) < f(b)$.

(C) There is a number $c$ in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all $x$ in $[a, b]$.

(D) There is a number $c$ in the open interval $(a, b)$ such that $f'(c) = 0$.

(E) There is a number $c$ in the open interval $(a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

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7. If $f(x) = \sin\left(\dfrac{x}{2}\right)$, then there exists a number $c$ in the interval $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be $c$ ?

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) $\pi$ (E) $\dfrac{3\pi}{2}$

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8. A polynomial $p(x)$ has a relative maximum at $(-2, 4)$, a relative minimum at $(1, 1)$, a relative maximum at $(5, 7)$ and no other critical points. How many zeros does $p(x)$ have?

(A) One (B) Two (C) Three (D) Four (E) Five

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9.

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $f(x)$ | 20 | 17 | 12 | 16 | 20 |

The function $f$ is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of $f$ on this interval. Which of the following statements must be true?

I. The minimum value of $f$ on $[3, 7]$ is 12.

II. There exists $c$, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

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10. Let $f$ be the function given by $f(x) = x^3 - 3x^2$. What are all values of $c$ that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

11. **2011 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

| $t$ (seconds) | 0 | 10 | 40 | 60 |
|---|---|---|---|---|
| $B(t)$ (meters) | 100 | 136 | 9 | 49 |
| $v(t)$ (meters per second) | 2.0 | 2.3 | 2.5 | 4.6 |

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function $B$ models Ben's position on the track, measured in meters from the western end of the track, at time $t$, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times $t$.

(c) For $40 \le t \le 60$, must there be a time $t$ when Ben's velocity is 2 meters per second? Justify your answer.

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12. **2016 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

| $t$ (hours) | 0 | 1 | 3 | 6 | 8 |
|---|---|---|---|---|---|
| $R(t)$ (liters / hour) | 1340 | 1190 | 950 | 740 | 700 |

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where $t$ is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where $R$ is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(d) For $0 \le t \le 8$, is there a time $t$ when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

## Solutions to 5.2 Day 1 Extra Questions

1. Since f(x) is continuous on [-1, 8] and differentiable on (-1, 8) the Mean Value Theorem can be applied.)

2. B 3. D 4. E 5. C 6. C 7. D 8. B 9. B 11. B

11. (c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time $t$, $40 < t < 60$, such that $v(t) = 2$.

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{cases}$$

12. (d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one time $t$, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

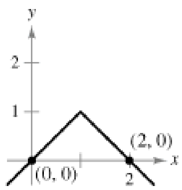
For this value of $t$, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

$$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$$

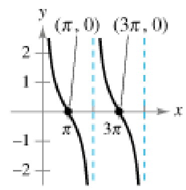
# OPTIONAL: Rolle's Theorem Questions

**In Exercises 1 and 2, explain why Rolle's Theorem does not apply to the function even though there exist $a$ and $b$ such that $f(a) = f(b)$.**

**1.** $f(x) = 1 - |x - 1|$




**2.** $f(x) = \cot \frac{x}{2}$




**In Exercises 3–6, find the two $x$-intercepts of the function $f$ and show that $f'(x) = 0$ at some point between the two intercepts.**

**3.** $f(x) = x^2 - x - 2$

**4.** $f(x) = x(x - 3)$

**5.** $f(x) = x\sqrt{x + 4}$

**6.** $f(x) = -3x\sqrt{x + 1}$

**In Exercises 7–20, determine whether Rolle's Theorem can be applied to $f$ on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of $c$ in the open interval $(a, b)$ such that $f'(c) = 0$.**

**7.** $f(x) = x^2 - 2x, [0, 2]$

**8.** $f(x) = x^2 - 5x + 4, [1, 4]$

**9.** $f(x) = (x - 1)(x - 2)(x - 3), [1, 3]$

**10.** $f(x) = (x - 3)(x + 1)^2, [-1, 3]$

**11.** $f(x) = x^{2/3} - 1, [-8, 8]$

**12.** $f(x) = 3 - |x - 3|, [0, 6]$

**13.** $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, [-1, 3]$

**14.** $f(x) = \dfrac{x^2 - 1}{x}, [-1, 1]$

**15.** $f(x) = \sin x, [0, 2\pi]$

**16.** $f(x) = \cos x, [0, 2\pi]$

**17.** $f(x) = \dfrac{6x}{\pi} - 4\sin^2 x, \left[0, \dfrac{\pi}{6}\right]$

**18.** $f(x) = \cos 2x, \left[-\dfrac{\pi}{12}, \dfrac{\pi}{6}\right]$

**19.** $f(x) = \tan x, [0, \pi]$

**20.** $f(x) = \sec x, \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

**In Exercises 21–24, use a graphing utility to graph the function on the closed interval $[a, b]$. Determine whether Rolle's Theorem can be applied to $f$ on the interval and, if so, find all values of $c$ in the open interval $(a, b)$ such that $f'(c) = 0$.**

**21.** $f(x) = |x| - 1, [-1, 1]$

**22.** $f(x) = x - x^{1/3}, [0, 1]$

**23.** $f(x) = 4x - \tan \pi x, \left[-\frac{1}{4}, \frac{1}{4}\right]$

**24.** $f(x) = \dfrac{x}{2} - \sin \dfrac{\pi x}{6}, [-1, 0]$

**25.** ***Vertical Motion*** The height of a ball $t$ seconds after it is thrown upward from a height of 32 feet and with an initial velocity of 48 feet per second is $f(t) = -16t^2 + 48t + 32$.

(a) Verify that $f(1) = f(2)$.

(b) According to Rolle's Theorem, what must be the velocity at some time in the interval $(1, 2)$? Find that time.

**SOLUTIONS**

1. f is not differentiable @ x = 1
3. @ x = 1/2, x ints are (-1, 0) (2, 0)
5. @ x = -8/3, x ints are (-4, 0) (0, 0)
7. c = 1
9. c = (6 ±√3) / 3
11. f is not differentiable, Rolle's Theorem Does not apply
13. c = - 2 + √5
15. c = π/2, 3π/3
17. c = 0.2489
19. f is not continuous, Rolle's Theorem does not apply
21. f is not differentiable, Rolle's Theorem does not apply
23. c = ±0.1533 radians
25. v(t) must be 0, t = 3/2 seconds

## 5.2 Day 2 Assignment: Textbook Page 206 #15-28, 30, 54
## Additional Questions 1 Below

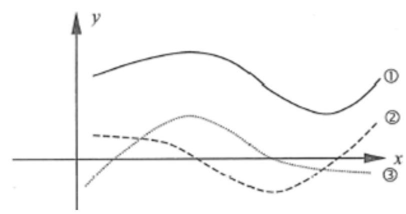
| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $g'(x)$ | 2 | 3 | 0 | −3 | −2 | −1 | 0 | 3 | 2 |

1. The derivative $g'$ of a function $g$ is continuous and has exactly two zeros. Selected values of $g'$ are of given in the table above. If the domain of $g$ is the set of all real numbers, then $g$ is decreasing on which the following intervals? Show work as well & then select the correct answer from the list below.

A. $-2 < x < 2$ only
B. $-1 \le x \le 1$ only
C. $x \ge -2$
D. $x \ge 2$ only
E. $x \le -2$ or $x \ge 2$

## 5.3 Day 1 Assignment: Textbook Page 206 # 40a, 44 (See Ex 8 p205) & Page 219 #1-6, 21-24 Additional Questions 1-5 Below

1. Shown below are the graphs of $f(x)$, $f'(x)$, and $g(x)$ (which is not $f'(x)$). Which is which? Explain your reasoning.




2. For the function $h(x)=\dfrac{x^2-3x-4}{x-2}$, determine the open intervals on which the given function is increasing or decreasing and the *x*–values of any relative extrema. Show your analysis and explain your reasoning.

3. On the interval 0 < *x* <10, how many relative minimums does the graph of *g*(*x*) have if $g'(x)=\dfrac{\sin x}{x+2}$? This question is **Calculator Active.**

A. 0 B. 1 C. 2 D. 3 E. 4

4. The table below gives various points for a continuous and differentiable function f(x). Use the chart to answer the following questions.

| x | -3 | -1 | 0 | 2 | 4 | 7 |
|---|---|---|---|---|---|---|
| f(x) | 7 | 10 | 2 | -1 | 0 | 2 |

A) Can you guarantee an x value on the interval (4, 7) such that f(x) = 1? Justify your answer.

B) On the interval (-3, 7) is there guaranteed an x-value such that $f'(x)=\frac{-1}{2}$? Justify your answer.

C) True or False: At x = -1, f(x) = 10 is a relative maximum. Explain your reasoning.

D) What is the minimum number of relative extrema that f(x) must have on the interval (-3, 7)? Explain your reasoning.

5. If $F'(x)=(x-1)^2(x-2)(x-4)$, where is the graph of *F*(*x*) increasing, decreasing, and/or reaching a relative maximum or minimum? Show your work and justify your reasoning.

### Solutions to 5.3 Day 1 Extra Questions

1 So ① is $f(x)$, ② is $f'(x)$, and ③ is $g(x)$.

2. h(x) is increasing on $(-\infty,\infty)$ (except for a finite number of points where F'(x)=0) h'(x)>0 for all points on the domain of h(x). h(x) is never decreasing since h'(x) is never less than zero. Since h'(x) never changes signs there are no relative extrema.

3. B

4. a) Yes. Since the function is stated as continuous and differentiable on [4, 7], and f(4)=0 and f(7)=2, by the Intermediate Value Theorem f(x) takes on every value between 0 and 2 and there must exist a value of x=c on [4, 7] such that f(c)=1.

b) f(-3)=7 and f(7)=2, therefore the slope between (-3,7) and (7,2) is $-\frac{1}{2}$. Since f(x) is continuous on [-3 $f'(c)=-\frac{1}{2}$, 7] and differentiable on (3, 7) and $\frac{f(7)-f(-3)}{7-(-3)}=-\frac{1}{2}$, by the Mean Value Theorem there exists a value x=c on (3,7) such that

c) Since we do not have all values of f(x), we can not determine for certain that f(-1) is greater than all nearby values of f(x), we must say that the conclusion stated (that there is a maximum of 10 when x = -1) is false

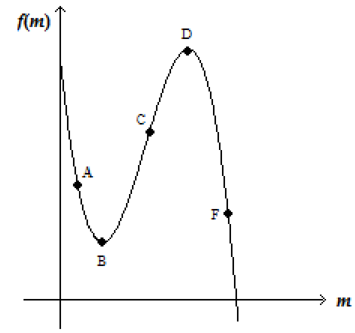
d) f(x) is differentiable and continuous on [-3,7]. Given the table values, the Mean Value theorem implies that f'(x)>0 for some x in (-3, -1) and some x in (2, 4). Similarly, f'(x)<0 for some x in (-1, 2). Therefore, the Intermediate Value Theorem implies that f'(x)=0 for at least two values of x in [-3, 7] (see 2008 AB2c

5. (Draw Sign analysis). F(x) is increasing on $(-\infty,2]\cup[4,\infty)$ because F'(x)>0 on those intervals). F(x) is decreasing on [2, 4] because F'(x) <0 on that interval. F(x) has a relative maximum at x=2 because F'(x) changes from positive to negative at x=2. F(x) has a relative minimum at x=4 because F'(x) changes from negative to positive at x = 4.

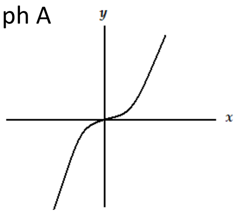
## 5.3 Day 2 Assignment: Textbook Page 219 #7-20
## Additional Questions 1-15 Below

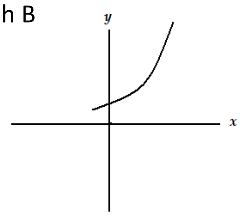
1. Pictured to the right is a function, $f(m)$. Complete the chart below indicating the sign (+ or – or 0) for $f(m)$, $f'(m)$ and $f''(m)$ at each of the indicated points.

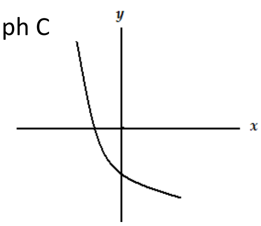
| Point | $f(m)$ | $f'(m)$ | $f''(m)$ |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |
| F | | | |



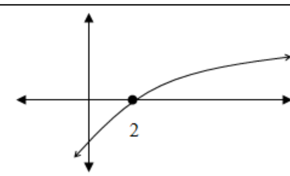

2. If, for all real numbers $x$, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of $f(x)$? Explain your reasoning FOR EACH GRAPH.

Graph A



Graph B


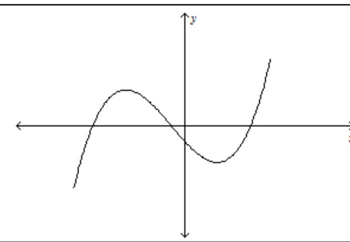

Graph C



3. The graph of a twice differentiable function is shown to the right. Order the values of $f(2)$, $f'(2)$ and $f''(2)$ in order from least to greatest. Explain your reasoning.

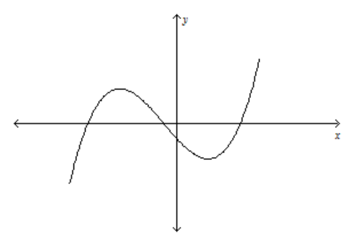



**The graph of $f'(x)$, the derivative of $f(x)$ is shown in each of the following questions. Answer the questions 4 – 6 using this graph.**

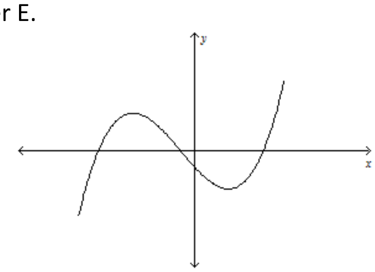
4. How many relative maximums does $f(x)$ have? Label these $x$ values with the letter C. Explain your reasoning.



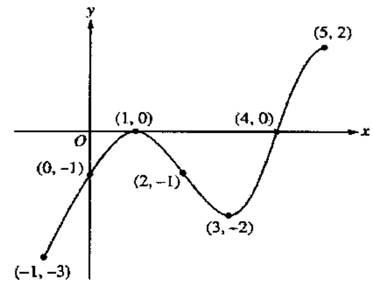

5. How many relative minimums does $f(x)$ have? Label these $x$ values with the letter D. Explain your reasoning.




6. How many points of inflection does the graph of $f(x)$ have? Label these $x$ values with the letter E. Explain your reasoning.



**Pictured to the right is the graph of $f'(x)$. Use the graph to answer questions 7 – 12.**




7. What are the value(s) of *x* where *f*(*x*) has a relative maximum? Explain your reasoning.

8. What are the value(s) of *x* where *f*(*x*) has a relative minimum? Explain your reasoning.

9. On what interval(s) is the graph of *f*(*x*) increasing? Explain your reasoning.

10. At what value(s) of *x* does the graph of *f*(*x*) have a point of inflection? Explain your reasoning.

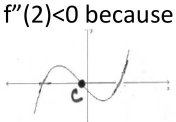
11. On what interval(s) is the graph of *f*(*x*) concave up or concave down? Explain your reasoning.

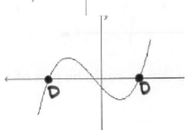
12. If *f*(2) = 4, what is the equation of the normal line to the graph of *f*(*x*) when *x* = 2?

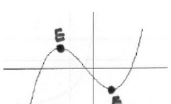
## Solutions to 5.3 Day 2 Extra Questions

1.

| Point | *f*(*m*) | $f'(m)$ | $f''(m)$ |
|---|---|---|---|
| A | + | - | + |
| B | + | 0 | + |
| C | + | + | 0 |
| D | + | 0 | - |
| F | + | - | - |

2. If f'(x) <0 then f(x) is always decreasing. If f''(x)>0 then f is always concave up.
Graph A: No because Graph A is always increasing and partially concave up so it does not fit the criteria.
Graph B: No because Graph B is always increasing so does not fit the criteria
Graph C: Yes because Graph C is always decreasing and always concave up
3. f(2) = 0 because when x = 2, y = 0.
f'(2)>0 because f(x) is increasing when x =2
f"(2)<0 because f(x) is concave down when x=2 Therefore f"(2)<f(2)<f'(2)
4. 

f(x) has one relative maximum because f'(x) changes from positive to negative only once
5. 

f(x) has two relative minimums because f'(x) changes from negative to positive twice.
6. 

f(x) has a point of inflection anytime that f"(x) changes signs. When f"(x) changes signs, then f'(x) has a maximum or minimum. This occurs twice.
7. There is no value of x where f'(x) changes from positive to negative, therefore f(x) has no relative maximum.
8. x=4 is a relative minimum of f(x) because f'(x) changes from negative to positive
9. f(x) is increasing on [4, 5] because f'(x)>0 on that interval
10. f(x) has a point of inflection when f"(x) changes signs/when f'(x) has a relative maximum or minimum . Therefore the points of inflection occur when x = 1 and x = 3.
11. f(x) is concave up when f"(x)>0 and when f'(x) is increasing. Therefore, f(x) is concave up on (-1, 1) U (3, 5). f(x) is concave down when f"(x)<0 and f'(x) is decreasing. There, f(x) is concave down on (1, 3)
12. y=-1(x-2)+4

## 5.3 Day 3 Assignment: Textbook Page 219 #21-24, 39, 40, 40
## Additional Questions 1-7

1. If $h(x) = \dfrac{x^2 - 3x - 4}{x - 2}$ and If *h*(*x*) is a twice differentiable function such that $h(x) < 0$ for all values of *x*, then at what value(s) does the graph of *g*(*x*) have a relative maximum if $g'(x) = (9 - x^2) \cdot h(x)$ ?

**A function, *F*, is continuous on its domain of [–2, 4]. Additionally, *F*(–2) = 5, *F*(4) = 1 with $F'$ and $F''$ have the properties shown in the table below. Use this information to answer questions 2-4.**

| *x* | –2 < *x* < 0 | *x* = 0 | 0 < *x* < 2 | *x* = 2 | 2 < *x* < 4 |
|---|---|---|---|---|---|
| $F'(x)$ | Positive | Does not exist | Negative | 0 | Negative |
| $F''(x)$ | Positive | Does not Exist | Positive | 0 | Negative |

2. At what value(s) of *x* does *F* have relative extrema? Classify the extrema by type and give a reason for your answer.
3. At what value(s) of *x* does *F* have a point of inflection? Justify your answer.
4. On what interval(s) is the graph of *F* increasing, decreasing, concave up or concave down? Justify your reasoning.
5. **2001 AP® CALCULUS AB**
**Question 4**

Let $h$ be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of $h$ is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of $x$ for which the graph of $h$ has a horizontal tangent, and determine whether $h$ has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

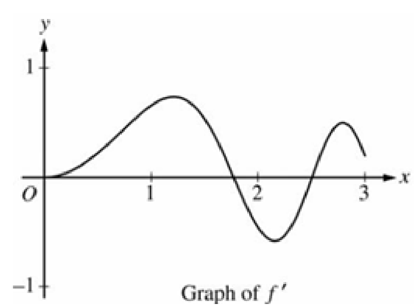
(b) On what intervals, if any, is the graph of $h$ concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of $h$ at $x = 4$.

(d) Does the line tangent to the graph of $h$ at $x = 4$ lie above or below the graph of $h$ for $x > 4$? Why?

**6.**

**2006 AP® CALCULUS AB**
**(Form B)**
**Question 2** **(A Modified Version)**




Let *f* be the function defined for *x* ≥ 0 with *f*(2) = 5.623 and $f'$, the first derivative of *f*, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $f'$ is shown to the right.

(a) Use the graph of $f'$ to determine whether the graph of *f* is concave up, down or neither on the interval 1.7 < *x* < 1.9. Explain your reasoning.

(b) On the interval 0 < *x* < 3, at what *x* – value(s) does the graph of *f* have a relative maximum? A relative minimum? Justify your answers.

(c) Write an equation for the tangent line to the graph of *f* at *x* = 2. Will the tangent line be above or below the graph of *f* for 1.5 < *x* < 2? Give a reason for your answer.

**7.**

## 2007 AP® CALCULUS AB
### Question 6

Let $f$ be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where $k$ is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant $k$ does $f$ have a critical point at $x = 1$? For this value of $k$, determine whether $f$ has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant $k$, the graph of $f$ has a point of inflection on the $x$-axis. Find this value of $k$.

**Solutions to 5.3 Day 3 Extra Questions:**

**1.** Show your sign analysis and then the concluding statement will be:
g(x) has a relative maximum at x=-3 because g'(x) changes from positive to negative at x = -3

2. F(x) has a relative maximum at x =0 because f'(x) changes from positive to negative at x=0

3. F(x) has a point of inflection when F"(x) changes signs which occurs at x=2

4. F(x) is increasing on [-2, 0] because F'(x)>0 on that interval and F(x) is decreasing on [0, 4] because F'(x) <0 on that interval (except for a finite number of points where F'(x)=0). F(x) is concave up on (-2, 0) U (0, 2) because F"(x) >0 on those intervals. F(x) is concave down on (2, 4) because F"(x) <0 on that interval.

5.

| | |
|---|---|
| (a) $h'(x) = 0$ at $x = \pm\sqrt{2}$<br>$h'(x)$: $-$ 0 $+$ und $-$ 0 $+$<br>$x$: $-\sqrt{2}$, $0$, $\sqrt{2}$<br>Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$ | $4: \begin{cases} 1: x = \pm\sqrt{2} \\ 1: \text{analysis} \\ 2: \text{conclusions} \\ \quad <-1> \text{ not dealing with discontinuity at } 0 \end{cases}$ |
| (b) $h''(x) = 1 + \frac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of $h$ is concave up for all $x \neq 0$. | $3: \begin{cases} 1: h''(x) \\ 1: h''(x) > 0 \\ 1: \text{answer} \end{cases}$ |
| (c) $h'(4) = \frac{16-2}{4} = \frac{7}{2}$<br>$y + 3 = \frac{7}{2}(x-4)$ | 1 : tangent line equation |
| (d) The tangent line is below the graph because the graph of $h$ is concave up for $x > 4$. | 1 : answer with reason |

**6. a)** Since the provided graph of f'(x) is decreasing on (1.7, 1.9), then f"(x) will be <0 on that interval. When f"(x) <0, f(x) will be concave down.

b) f(x) has a relative maximum at x =1.772 because f'(x) changes from positive to negative at x=1.772. f(x) has a relative minimum at x=2.507 because f'(x) changes from negative to positive at x=2.507.

c) Point of tangency (2, 5.623) Slope of Tangency: f'(2)=-0.459 Equation: y=-0.459(x-2)+5.623
Since f'(x) is decreasing at x=2, then f"(2)<0 and f(x) is concave down at x=2. When f(x) is concave down, the tangent line will be above the graph of f(x).

## Solutions to 5.3 Day 3 Extra Questions CONT:

**7.**

| Solution | Scoring |
|---|---|
| (a) $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$<br><br>$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$ | $2: \begin{cases} 1: f'(x) \\ 1: f''(x) \end{cases}$ |
| (b) $f'(1) = \frac{1}{2}k - 1 = 0 \Rightarrow k = 2$<br>When $k = 2$, $f'(1) = 0$ and $f''(1) = -\frac{1}{2} + 1 > 0.$<br>$f$ has a relative minimum value at $x = 1$ by the Second Derivative Test. | $4: \begin{cases} 1: \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1: \text{solves for } k \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$ |
| (c) At this inflection point, $f''(x) = 0$ and $f(x) = 0.$<br>$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$<br>$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$<br>Therefore, $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$<br>$\Rightarrow 4 = \ln x$<br>$\Rightarrow x = e^4$<br>$\Rightarrow k = \frac{4}{e^2}$ | $3: \begin{cases} 1: f''(x) = 0 \text{ or } f(x) = 0 \\ 1: \text{equation in one variable} \\ 1: \text{answer} \end{cases}$ |

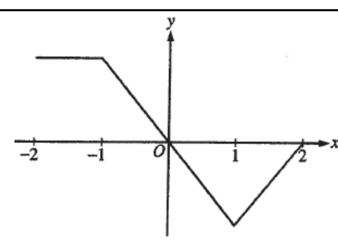
## 5-1 - 5.3 REVIEW Assignment: Textbook Page 260: 1-18, 31-38
## Plus the following Questions 1-14

Unless otherwise indicated, all problems are NONCALCULATOR ACTIVE

1. Let *f* be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the *x* – coordinates of the points of inflection of the graph of *f* ?

   A. 0 only  B. 3 only  C. 0 and 6 only D. 3 and 6 only E. 0, 3, and 6

2. The graph of $f'$, the derivative of the function *f*, is shown to the right. Which of the following statements is true about *f* ?

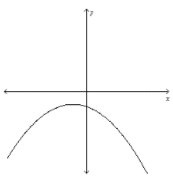


Graph of $f'$

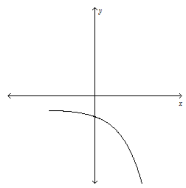
   A. *f* is decreasing for $-1 \le x \le 1$.
   B. *f* is increasing for $-2 \le x \le 0$.
   C. *f* is increasing for $1 \le x \le 2$.
   D. *f* has a local minimum at $x = 0$.
   E. *f* is not differentiable at $x = -1$ and $x = 1$.

3. The function *f* has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values of *x*. Which of the following could be the graph of *f* ?

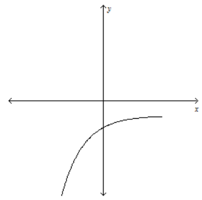
A.



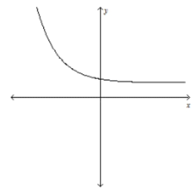
B.



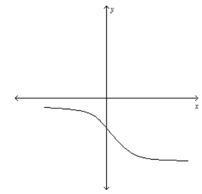
C.



D.



E.



4. Which of the following statements is true of *f*(*x*) = $-x^3-6x^2-9x-2$?

A. *f* is increasing on (–3, –1)
B. *f* is increasing on $(-\infty,-3)\cup(-1,\infty)$
C. *f* is increasing on $(-\infty,5)$
D. *f* is increasing on $(-2,\infty)$
E. *f* is never increasing.

5. The function *g*(*x*) is continuous on the closed interval [–2, 0] and twice differentiable on the open interval (–2, 0). If $g'(-1)=-2$ and $g''(x)>0$ on the open interval (–2, 0), which of the following could be a table of values of *g*?

A.

| $x$ | $g(x)$ |
|---|---|
| –2 | 5 |
| –1 | 3 |
| 0 | 1 |

B.

| $x$ | $g(x)$ |
|---|---|
| –2 | 5 |
| –1 | 4 |
| 0 | 1 |

C.

| $x$ | $g(x)$ |
|---|---|
| –2 | 5 |
| –1 | 6 |
| 0 | 9 |

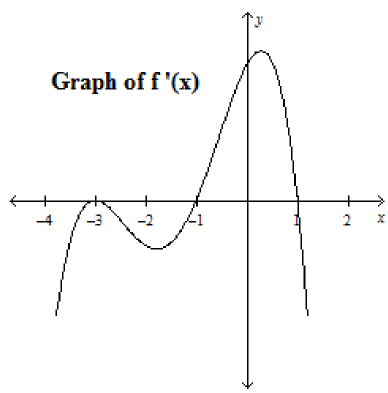
D.

| $x$ | $g(x)$ |
|---|---|
| –2 | 5 |
| –1 | 7 |
| 0 | 9 |

E.

| $x$ | $g(x)$ |
|---|---|
| –2 | 7.5 |
| –1 | 4 |
| 0 | 3.5 |

6. Pictured to the right is the graph of $f'$, the first derivative of *f*. At which of the following value(s) of *x* does the graph of *f* have a horizontal tangent but NOT a relative maximum or minimum?



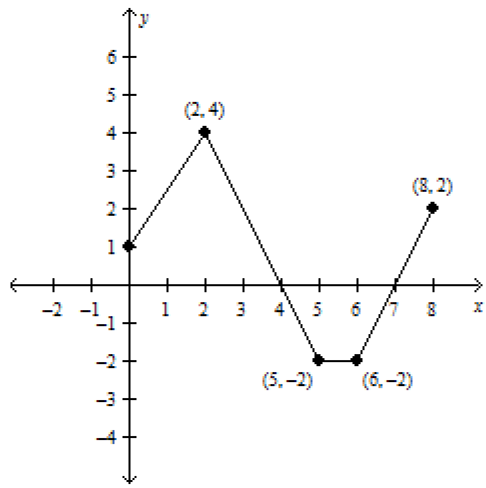

I. $x = -3$ II. $x = -1$ III. $x = 1$

A. I only
B. I and III only
C. II only
D. II and III only
E. I, II , and III

7. The function *f* has a first derivative given by $f'(x)=\dfrac{\sqrt{x}}{1+x+x^3}$. What is the *x*–coordinate of the point of inflection of the graph of *f* ? (CALCULATOR PROBLEM)

A. 1.008
B. 0.473
C. 0
D. –0.278
E. The graph has no points of inflection.

The graph given to the right is the graph of $f\,'$, the first derivative of a differentiable function, *f*. Use the graph to answer the questions below.




8. On the interval [0, 8], are there any values where *f*(*x*) is not differentiable? Justify..

9. On what interval(s) is $f\,''>0$? $<0$? Justify.

10. At what value(s) of *x* does the graph of *f* have a horizontal tangent? Give a reason for your answer.

11. What is the value of $f\,''(4)$? Explain your reasoning.

12. What is the value of $f\,''(8)$? Explain your reasoning

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13. If $g(x)=e^{2x}\cdot f(x)$ and *f*(2) = –3, what is the equation of the normal line to the graph of *g* at *x* = 2?

14. Consider the function $f(x)=2x^3+ax^2+bx+5$. Given the table of information below, answer the questions that follow.

| *x* | < –1 | –1 | $-1<x<\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}<x<2$ | 2 | > 2 |
|---|---|---|---|---|---|---|---|
| $f\,'$ | Positive | 0 | Negative | Negative | Negative | 0 | Positive |
| $f\,''$ | Negative | Negative | Negative | 0 | Positive | Positive | Positive |

a. Determine intervals of increasing and decreasing values of *f*. Justify your answers.

b. Determine and classify all *x* – values of relative extrema of *f*. Justify your answers.

c. Determine the intervals of concavity of *f*. Justify your answer.

d. Determine the values of *a* and *b* in the equation of *f*. Show your work.

**Solutions to Review Extra Questions:**

1. D 2. B 3. B 4. A 5. E 6. A 7. B

8. f'(x) is defined for all values on [0, 8]. Thus, f(x) is differentiable for all values on [0, 8]

9. f"(x)>0 on (0, 2) U (6, 8) because f'(x) is increasing on those intervals. f"(x)<0 on (2, 5) because f'(x) is decreasing on that interval

10. f(x) is increasing on [0, 4] U [7, 8] because f'(x) >0 on those intervals. f(x) is decreasing on [4, 7] because f'(x)<0 on that interval

11. $f''(x)=\dfrac{4-(-2)}{1-5}=-2$. f"(x) represents the slope of the tangent line to the graph of f'(x) at x=4.

12. at x = 8, f'(x) is not differentiable since the provided graph of f'(x) has a cusp at x=8. If f'(8) is not differentiable, then f"(8) is undefined

13. $y=\dfrac{1}{2e^4}(x-2)-3e^4$