5.5 Assignment P246 #1-19 & Questions Below *(INTEGRAL CALCULUS 30L QUESTIONS)*

Linear Approximation Extra Questions

1. Let $f(x) = \ln x$.

a. Write an equation of the tangent line to *f(x)* at $x = 1$.

b. Use the tangent line to approximate $f(1.1) \approx$

c. Is the approximation in part (b) an under or over approximation? *Explain why using the graph of*

f(x).

2. Let y be a function such that $\frac{dy}{dx} = e^x + 1$ and y(0)=1

a. Write an equation of the tangent line to y at $x = 0$.

b. Use the tangent line to approximate $y(0.2) \approx$

3. Approximate $\sqrt[3]{7.9}$ using a linearization.

4. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

SOLUTIONS TO 5.5 EXTRA QUESTIONS:

1. a) L(x)=x – 1 b) 0.1 c) Over approximation

2. a) L(x) = 2x + 1 b) 1.4

3. $\frac{239}{120}$

4. a) L(x) = -3(x – 1); $-\frac{3}{5}$

5. a) L(x) = 44x + 1400; 1411 tons

5.4 CALC 30L ASSIGNMENT FOR OPTOMIZATION & THE THREE BIG THEOREMS

YOU MUST USE CALCULUS IN YOUR ANALYSIS AND JUSTIFICATION FOR EACH OF THE FOLLOWING

1. During the time interval $0 \le t \le 12$ hours, the amount of heating oil in a tank is given by

$$H(t) = \frac{1}{6}t^3 - \frac{11}{3}t^2 + 20t + 125$$

gallons, where t is measured in hours. To the nearest whole number, what is the least amount of heating oil in the tank? Show the Calculus analysis that leads to your conclusion.

2. The size of a population of bacteria introduced to a food grows according to the formula $P(t) = \dfrac{6000t}{60+t^2}$, where t is measured in weeks and $0 \le t \le 20$. Determine when the bacteria will reach its maximum size. What is the maximum size of the population? Justify your answer.

3. The rate at which people enter an auditorium for a rock concert is modeled by the function $R(t) = 1240t^2 - 625t^3$ for $0 \le t \le 1.5$ hours, where $R(t)$ is measured in people per hour. Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

4. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every 3 hours for a 24-hour period.

T (hours)	0	3	6	9	12	15	18	21	24
R(*t*) (gallons per hour)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a. Estimate the value of $R'(5)$, indicating correct units of measure. Explain what this value means about $R(t)$.

b. Using correct units of measure, find the average rate of change of $R(t)$ from $t = 3$ to $t = 18$.

c. Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

5. The total order and transportation cost $C(x)$, measured in dollars, of bottles of Pepsi Cola is approximated by the function $C(x) = 10{,}000\left(\dfrac{1}{x} + \dfrac{x}{x+3}\right)$, where x is the order size in number of bottles of Pepsi Cola in hundreds. Answer the following questions.

a. Is there guaranteed a value of r on the interval $0 \le r \le 3$ such that the average rate of change of cost is equal to $C'(r)$? Give a reason for your answer.

b. Is there a value of r on the interval $3 \le r \le 6$ such that $C'(r) = 0$. Give a reason for your answer and if such a value of r exists, then find that value of r.

c. For $3 \le x \le 9$, what is the greatest cost for order and transportation?

6. A car company introduces a new car for which the number of cars sold, S, is modeled by the function $S(t) = 300\left(5 - \dfrac{9}{t+2}\right)$, where t is the time in months.

a. Find the value of $S'(2.5)$. Using correct units, explain what this value represents in the context of this problem.

b. Find the average rate of change of cars sold over the first 12 months. Indicate correct units of measure and explain what this value represents in the context of this problem.

c. Is it possible that a value of c for $0 \le c \le 12$ exists such that $S'(c)$ is equal to the average rate of change? Give a reason for your answer and if such a value of c exists, find the value.

7. Blood pressure in a patient will drop by an amount $D(x)=0.025x^2(30-x)$, where x is the amount of drug injected in cm^3 and $0\le x\le 25$. Find the dosage that provides the greatest drop in blood pressure. What is the drop in blood pressure? Justify your answer.

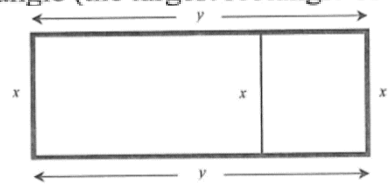
SOLUTIONS TO 5.4 Calc 30L Assignment

1. On [0, 12] The minimum amount of heating oil in the tank is 123 gallons (Fully Justify your work & full sentence)
2. The max number of bacteria is 387.298 bacteria when t=7.750 weeks (Fully Justify your work & full sentence)
3. The rate at which people enter the auditorium is a maximum at t = 1.323 hr(Fully Justify your work & full sentence)
4. a) $R'(5)\approx 0.133$. Since R'(5)>0, the rate at which water is flowing out of the pipe at t=5 hours is increasing
   b) 0.05 gallons/hour^2 c) Since R(t) is differentiable on (0,24) then it is also continuous on [0,24]. The MVT guarantees there is at least one point c in (0, 24) at which $f'(c)=\dfrac{f(24)-f(0)}{24-0}=0$
5. Since C(0) is undefined, then C(x) is not continuous on [0, 3], ∴ the MVT does not guarantee a value of r on (0, 3) such that $C'(r)=\dfrac{C(3)-C(0)}{3-0}$. B) C(x) is continuous on [3,6] and differentiable on (3,6). The MVT guarantees there is at least one point r in (3, 6) at which $C'(r)=\dfrac{C(6)-C(3)}{6-3}=0$. The value of r=4.098 when C(x) has a relative minimum. C) On [3,9], the greatest cost is \$8611.11 when x=9
6. a) s'(2.5)=133.333. After 2.5 months, the cars are being sold at a rate of 133 cars per month. b) The average rate of change on [0,12] is 96.429. During the first 12 months an average of96.429 cars are sold per month (*3 decimal answers!) c) s(t) is continuous on [0, 12] and differentiable on (0, 12), ∴ the MVT guarantees a value of c on (0, 12) such that $S'(c)=\dfrac{S(12)-S(0)}{12-0}$. That value is 3.292 months.
7. The greatest drop in blood pressure was 100 units when x=20 cm^3 (Fully Justify your work & full sentence)

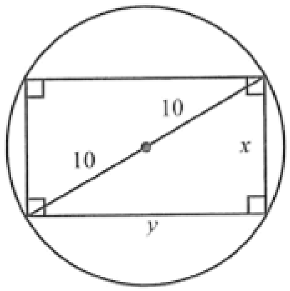
NOTE: THIS MAY BE TAUGHT AFTER THE AP EXAM EXCEPT WHERE TIME PERMITS OTHERWISE
5.4 REGULAR CALC 30 Assignment P230 #1b, 5, 7, 14, 16, 32, 40, 56 PLUS THE FOLLOWING

2. One number is 6 larger than another. Find these numbers if their product is to be a minimum. What is the minimum product?
3. One number is 10 larger than another. Find these numbers if the sum of their squares is to be a minimum. What is the minimum sum of squares?
4. Two numbers have a product of 16. Find these numbers if the sum of their squares is to be a minimum. What is the minimum sum of squares?
5. Two numbers have a sum of 50. What is their maximum product?
6. What number exceeds its positive square root by the least possible amount? For example, 100 exceeds its square root of 10 by 90 whereas 9 exceeds its square root of 3 by only 6.
7. Two nonnegative numbers have a sum of 21. Find these numbers if the product of one of the numbers with the square of the other is to be a maximum. What is this maximum product?
8. Two nonnegative numbers have a sum of 20. Find the numbers if the product of the cube of one of them with the square of the other is to be a maximum. What is this maximum product?
9. Two nonnegative numbers have a product of 100. What is their minimum sum?
10. Two nonnegative numbers have a sum of 10. What is the least possible sum of their reciprocals?
11. What is the maximum area a rectangle can enclose if its perimeter is 400 metres?
12. What is the minimum perimeter of a rectangle whose area is 400 m^2?

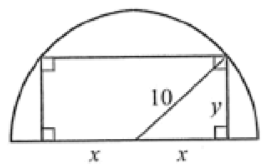
14. A farmer has 600 metres of fencing with which he wants to make two adjacent rectangular pens as shown in the figure below. What is the maximum total area that can be enclosed? What are the dimensions of the outside rectangle (the largest rectangle of the three in the figure)?



15. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius 10 cm.

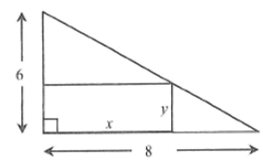


Question 15



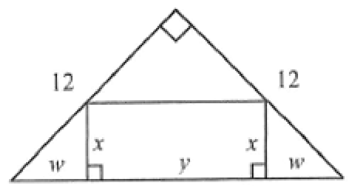
Question 16

16. Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 10 cm.
17. Find the dimensions of the rectangle of largest area that can be inscribed in a right triangle with legs of length 6 cm and 8 cm as shown in the figure.



Question 17

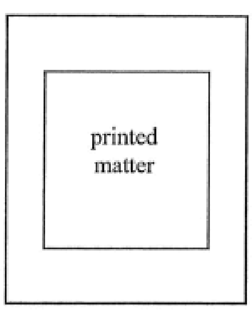
18. Find the dimensions of the rectangle of largest area that can be inscribed in an isosceles right triangle with legs of length 12 cm as shown in the figure.



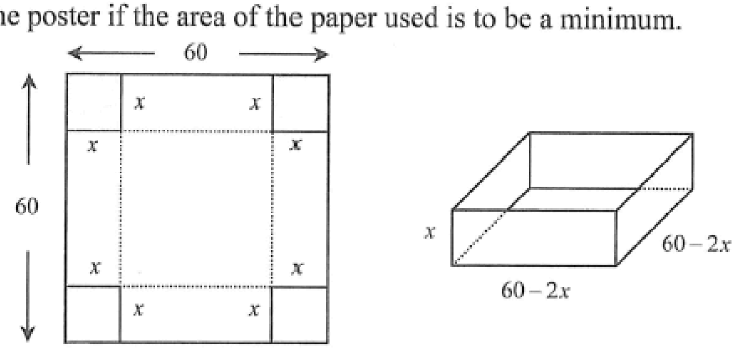
Question 18

19. Find the maximum area of an isosceles triangle whose legs are 20 cm.

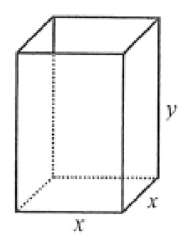
20. A rectangular paper poster must contain 450 cm^2 of printed matter. The printed area is to be surrounded by a border of width 6 cm on the top and bottom and 3 cm on the left and right. Find the outside dimensions of the poster if the area of the paper used is to be a minimum.



Question 20



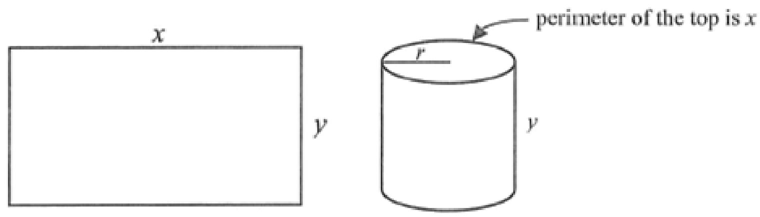
Question 21



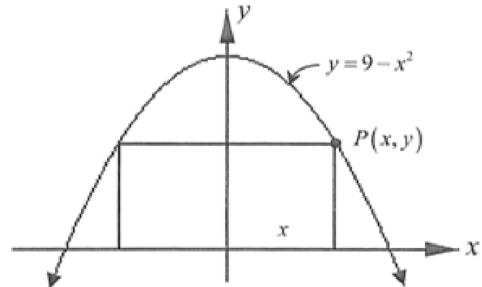
Question 23

21. A piece of cardboard, 60 cm by 60 cm is to be transformed into a box with an open top by cutting squares of the same size from each corner and folding up the flaps. Find the dimensions of the cut-out squares so that the volume of the box can be maximized.
22. Repeat question 21 if the piece of cardboard has dimensions 24 cm by 45 cm.
23. A box-shaped storage bin with an open top and a square base (bottom) is to be built to hold 32 m^3. Find its dimensions in order to minimize the area of the material used in making the box. See the figure above.

24. A rectangle has a perimeter of 216 cm. The rectangle is to be rolled into a cylindrical tube with hollow ends. Find the dimensions of the rectangle in order to maximize the volume of the tube. See the figure below.



Question 24



Question 25

25. Find the area of the largest rectangle bounded by the curve $y = 9 - x^2$ above and the x-axis below. See the figure above.

SOLUTIONS TO EXTRA QUESTIONS:
(May not be to 3 decimal AP Standard)

1. **(a)** 7 and 8 **(b)** 7.5 and 7.5 **(c)** 56.25 **(d)** part (a) considers only integer possibilities, whereas part (b) broadens the search to include real number possibilities. **2.** −3 and 3; −9 **3.** −5 and 5; 50 **4.** 4 and 4; 32 **5.** 625 **6.** 1/4 **7.** 14 and 7 (14 is the number to be squared) **8.** 12 and 8 (12 is the number to be cubed) **9.** 20 **10.** 2/5 **11.** 10 000 m^2 **12.** 80 m **13.** 128 m^2 **14.** 15 000 m^2; 100 m by 150 m **15.** $10\sqrt{2}$ cm by $10\sqrt{2}$ cm **16.** $10\sqrt{2}$ cm by $5\sqrt{2}$ cm **17.** 3 cm by 4 cm **18.** $6\sqrt{2}$ cm by $3\sqrt{2}$ cm **19.** 200 cm^2 **20.** 21 cm by 42 cm **21.** 10 cm by 10 cm **22.** 5 cm by 5 cm **23.** 4 m by 4m by 2 m **24.** 72 cm by 36 cm **25.** $12\sqrt{3}$ u^2 **26.** 250 000 cm^2 **27.** $13; $84 500; 6500 fans **28.** $\sqrt{3}$ cm^2

5.6 RELATED RATES

QUESTIONS 9-20 REQUIRE PROPER SENTENCES (USING POSITIVE QUANTITIES WITH EITHER INCREASING OR DECREASING DESCRIPTIONS)

1. The radius r and area A of a circle are related by the equation $A = \pi r^2$. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$.
2. The radius r and surface area S of a sphere are related by the equation $S = 4\pi r^2$. Write an equation that relates $\frac{dS}{dt}$ to $\frac{dr}{dt}$.

The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$. Use this relationship to answer questions 3 – 5.

3. How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ if h is constant?
4. How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ if r is constant?
5. How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither r nor h is constant?

Use the following information to solve problems 6 – 8. The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When l = 12 cm and w = 5 cm, find each of the rates of change of each quantity indicated below.

6. Area
7. Perimeter
8. Length of a diagonal of the rectangle

Use the following information to solve problems 10 – 11. A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

9. How fast is the balloon's radius increasing when the radius is 5 feet?
10. How fast is the surface area increasing when the radius is 5 feet?
11. A conical tank with the vertex down is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

A ladder is 25 feet long and is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Use this information to complete exercises 12 – 14.

12. How fast is the top of the ladder moving down the wall when its base is 15 feet from the wall?
13. Consider the triangle formed by the side of the house, the ladder and the ground. Find the rate at which the area of the triangle is changing when the ladder is 9 feet from the wall.
14. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 7 feet from the wall.

Water is flowing at a rate of 50 cubic meters per minute from a concrete conical reservoir. The radius of the reservoir is 45 m and the height is 6 m. Use this information to complete exercises 16 and 17.

15. How fast is the water level falling when the water is 5 meters deep?
16. How fast is the radius of the water's surface changing when the water is 5 meters deep?
17. A man 6 feet tall walks at a rate of 5 feet per second toward a street light that is 16 feet tall. At what rate is the length of his shadow changing when he is 10 feet from the base of the light?

18. If the volume of a cube is increasing at a rate of 24 in^3/min and each edge is increasing at 2 in/min, what is the length of each edge of the cube?
19. If the volume of a cube is increasing at a rate of 24 in^3/min and the surface area is increasing at 12 in^2/min, what is the length of each edge of the cube?
20. On a morning when the sun will pass directly overhead, the shadow of an 80—foot building on level ground is 60 feet long. At the moment in question, the angle ϑ the sun makes with the ground is increasing at the rate of 0.27 radians per minute. At what rate is the length of the shadow decreasing?

SOLUTIONS TO 5.6 RELATED RATES:

1. $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$
2. $\frac{ds}{dt}=8\pi r\frac{dr}{dt}$
3. $\frac{dV}{dt}=2\pi hr\frac{dr}{dt}$
4. $\frac{dV}{dt}=\pi r^2\frac{dh}{dt}$
5. $\frac{dV}{dt}=2\pi rh\frac{dr}{dt}+\pi r^2\frac{dh}{dt}$
6. $\frac{dA}{dt}=14\, {}^{cm^2}\!/_{s}$
7. $\frac{dP}{dt}=0\, {}^{cm}\!/_{s}$
8. $\frac{dx}{dt}=\frac{-14}{13}\, {}^{cm}\!/_{s}$
9. $\frac{dr}{dt}=1\, {}^{ft}\!/_{\min}$
10. $\frac{dA}{dt}=40\pi\, {}^{ft^2}\!/_{\min}$
11. $\frac{dh}{dt}=\frac{9}{10\pi}\, {}^{ft}\!/_{\min}$
12. $\frac{dx}{dt}=-\frac{3}{2}\, {}^{ft}\!/_{s}$
13. $\frac{dA}{dt}=19.851\, {}^{ft^2}\!/_{s}$
14. $\frac{d\theta}{dt}=-\frac{1}{12}\, {}^{radians}\!/_{s}$
15. $\frac{dh}{dt}=\frac{-8}{225\pi}\, {}^{m}\!/_{\min}$
16. $\frac{dr}{dt}=\frac{-4}{15\pi}\, {}^{m}\!/_{\min}$
17. $\frac{dx}{dt}=-3\, {}^{ft}\!/_{s}$
18. e = 2 in
19. e = 8 in
20. $\frac{dx}{dt}=-33.75\, {}^{ft}\!/_{\min}$

NOTE: The answers to these questions can be found on AP Central (or, for example, just google AP Calculus 2005 free response answers and it will take you to the official AP scoring guideline document)

2005 AP® CALCULUS AB
(Form B)
Question 5

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

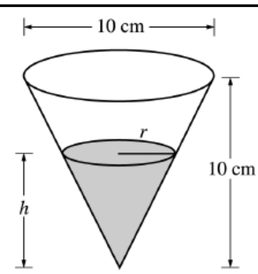
(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

2002 AP® CALCULUS AB

- **Question #5**



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth, h, is changing at the rate of $-\frac{3}{10}$ centimeters per hour.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

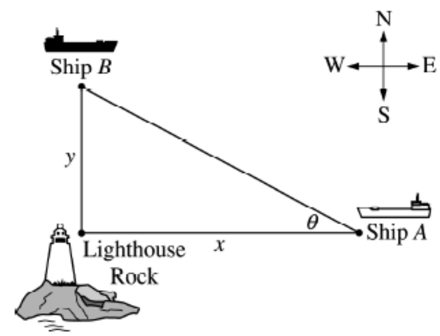
(a) Suppose you were asked to find the rate of change of the volume of water in the container when the depth of the water is 5 cm. Explain why the formula $V = \frac{1}{3}\pi r^2 h$ could not be implicitly differentiated with respect to time in order to determine this rate? What would have to be done in order to find this rate?

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate correct units of measure.

(c) The surface area of the exposed water is changing at a rate of 9π cm^2 per hour when the depth of the water is 4 cm. At what rate is the radius of the exposed water changing at this point in time? Indicate correct units of measure.

2002 AP® CALCULUS AB (Form B)

Question #6



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

Extra MULTIPLE CHOICE Practice – Implicit Differentiation and Related Rates

1.	Sand is deposited into a pile with a circular base. The volume V of the pile is given by $V = \frac{r^3}{3}$, where r is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of 5π feet per hour. When the circumference of the base is 8π feet, what is the rate of change of the volume of the pile, in cubic feet per hour? (A) $\frac{8}{\pi}$ (B) 16 (C) 40 (D) 40π (E) 80π
2.	If $e^{xy} - y^2 = e - 4$, then at $x = \frac{1}{2}$ and $y = 2$, $\frac{dy}{dx} =$ (A) $\frac{e}{4}$ (B) $\frac{e}{2}$ (C) $\frac{4e}{8-e}$ (D) $\frac{4e}{4-e}$ (E) $\frac{8-4e}{e}$
3.	If $y^3 + y = x^2$, then $\frac{dy}{dx} =$ (A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1+3y^2}$

4.	The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown below. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall? (A) $-\frac{2}{9}$ (B) $-\frac{1}{6}$ (C) $-\frac{2}{25}$ (D) $\frac{2}{25}$ (E) $\frac{1}{9}$
5.	Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$? (A) $y+3=-11(x-1)$ (B) $y+3=-\frac{7}{3}(x-1)$ (C) $y+3=\frac{1}{3}(x-1)$ (D) $y+3=\frac{7}{3}(x-1)$ (E) $y+3=\frac{11}{3}(x-1)$
6.	If $\ln(2x+y)=x+1$, then $\frac{dy}{dx}=$ (A) -2 (B) $2x+y-2$ (C) $2x+y$ (D) $4x+2y-2$ (E) $y-\frac{y}{x}$
7.	The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere? (The volume V of a sphere with radius r is $V=\frac{4}{3}\pi r^3$.) (A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm
8.	If $(x+2y)\cdot\frac{dy}{dx}=2x-y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$? (A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined
9.	A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening? (A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec
10.	An ice sculpture in the form of a sphere melts in such a way that it maintains its spherical shape. The volume of the sphere is decreasing at a constant rate of 2π cubic meters per hour. At what rate, in square meters per hour, is the surface area of the sphere decreasing at the moment when the radius is 5 meters? (Note: For a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.) (A) $\frac{4\pi}{5}$ (B) 40π (C) $80\pi^2$ (D) 100π

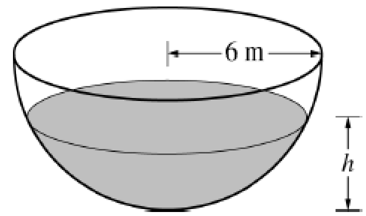
11. A hemispherical water tank, shown above, has a radius of 6 meters and is losing water. The area of the surface of the water is $A = 12\pi h - \pi h^2$ square meters, where h is the depth, in meters, of the water in the tank. When $h = 3$ meters, the depth of the water is decreasing at a rate of $\frac{1}{2}$ meter per minute. At that instant, what is the rate at which the area of the water's surface is decreasing with respect to time?

(A) 3π square meters per minute

(B) 6π square meters per minute

(C) 9π square meters per minute

(D) 27π square meters per minute



Extra SHORT ANSWER Practice –Related Rates

1. A conical paper cup 3 inches across the top and 4 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is 3 inches deep.

2. Sand is dumped off of a conveyer belt into a pile at the rate of 2 cubic feet per minute. The sand pile is shaped like a cone whose height and base diameter are always equal. At what rate is the height of the pile growing when the pile is 5 feet high?

3. A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a rate of 8 inches per second, how fast is the area of the triangle formed by the ladder, the building and the ground changing at the instant when the top of the ladder is 12 feet above the ground?

4. A ladder 13 feet long is leaning against the side of a building If the foot of the ladder is pulled away from the building at a rate of 2 inches per second, how fast is the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 feet from the ground?

SOLUTIONS TO 5.6 MULTIPLE CHOICE & EXTRA SHORT ANSWER QUESTIONS

MC: 1. C 2. C 3. E 4. A 5. E 6. B 7. E 8. A 9. B 10. A 11.A

SHORT ANSWER: 1. $-\frac{128}{81\pi}$ inches per minute 2. $\frac{8}{25\pi}$ feet per minute 3. $\frac{119}{36}$ feet^2 per second

4 $-\frac{1}{72}$ radians per second

PARTICLE MOTION DUO TANG ASSIGNMENT: P 55-56 #1 – 20 & Three AP FRQ P 56-57

Work these on **notebook paper**. Use your calculator only on part (f) of problems 1. Do **not** use your calculator on the other problems. Write your justifications in a sentence.

1. A particle moves along a horizontal line so that its position at any time is given by

$s(t)=t^3-12t^2+36t,\ t\geq 0,$ where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at time t and at $t = 3$ seconds.

(b) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.

(c) Find the displacement of the particle after the first 8 seconds. Explain the meaning of your answer.

(d) Find the total distance traveled by the particle during the first 8 seconds.

(e) Find the acceleration of the particle at time t and at $t = 3$ seconds.

(f) Graph the position, velocity, and acceleration functions for $0\leq t\leq 8$.

(g) When is the particle speeding up? Slowing down? Justify your answers.

---

2. The maximum acceleration attained on the interval $0\leq t\leq 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

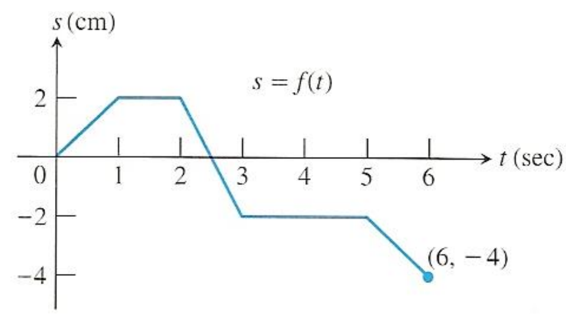
3. The figure on the right shows the position s of a particle moving along a horizontal line.

(a) When is the particle moving to the left? moving to the right? standing still? Justify your answer.

(b) For each of $v(1.5)$, $v(2.5)$, $v(4)$, and $v(5)$, find the value or explain why it does not exist.

(c) Graph the particle's velocity.

(d) Graph the particle's speed.



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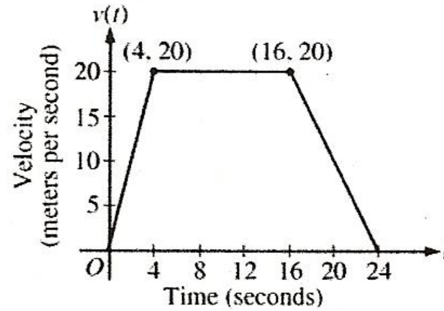
4. (2005) A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph on the right.



(a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(c) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

5. (Modification of 2009 Form B, Problem 6)

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above.

(a) Use data from the table to estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(c) Based on the values in the table, what is the smallest number of instances at which the velocity $v(t)$ could equal $-9\,\text{m}/\text{sec}$ on the interval $0 < t < 40$? Justify your answer.

---

6. A particle moves along a horizontal line so that its position at any time $t \ge 0$ is given by $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at any time t and when $t = 2$.

(b) Find the acceleration of the particle at any time t and when $t = 2$.

(c) When is the particle at rest? When is moving to the right? To the left? Justify your answers.

(d) Find the displacement of the particle during the first two seconds.

(e) Find the total distance traveled by the particle during the first two seconds.

(f) Are the answers to (d) and (e) the same? Explain.

(g) When is the particle speeding up? Slowing down? Justify your answers.

---

7. The position of a particle at time t seconds, $t \ge 0$, is given by $s(t) = t^2 - \sin t,\ 0 \le t \le 3$, where t is measured in seconds and s is measured in meters. Find the particle's acceleration each time the velocity is zero.

8. A particle's velocity at time t seconds, $t \geq 0$, is given by $v(t) = \cos(t^2) + t,\ 0 \leq t \leq 2$, where t is measured in seconds and v is measured in meters/second. Find the velocity of the particle each time the acceleration is zero.

9. (2004) A particle moves along the y-axis so that its velocity at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$.

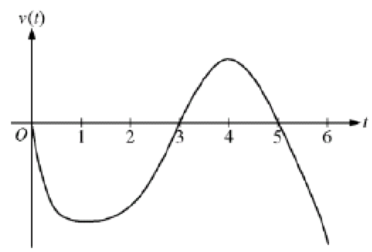
(a) Find the acceleration of the particle at time $t = 2$.
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

10. (Modification of 2005 Form B, Problem 3)

A particle moves along the x-axis so that its velocity at time t, for $0 \leq t \leq 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$

(a) Find the acceleration of the particle at time $t = 4$.
(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left? Justify your answer.
(c) Find the average rate of change of $v(t)$ on $1.5 \leq t \leq 3.2$.

11. (Modification of 2008, Problem 4)



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$.

(a) On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
(b) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
(c) During what intervals, if any, is the acceleration of the particle negative? Justify

SOLUTIONS TO PARTICLE MOTION QUESTIONS

1. (a) $3t^2 - 24t + 36,\ -9\text{m/sec}$ (b) At rest at $t = 2$ because $v(t) = 0$ there. Moving right for [0, 2) and $(6, \infty)$ because $v(t) > 0$. Moving left for (2, 6) because $v(t) < 0$. (c) 32 meters (d) 96 meters

(e) $6t - 24,\ -6\text{m/sec}^2$ (f) Graph (g) Speeding up on (2, 4) because vel. and acc. are both neg. there and on $(6, \infty)$ because vel. and acc. are both pos. there. Slowing down on [0. 2) because vel. is pos. and acc. is neg. and on (4, 6) because vel. is neg. and acc. is pos.

SOLUTIONS TO PARTICLE MOTION QUESTIONS CONT:

2. D 3. (a) Moving left on (2, 3) and (5, 6) because $v(t) < 0$. Moving right on (0, 1) because $v(t) > 0$. Standing still on (1, 2) and (3, 5) because $v(t) = 0$ there. (b) 0, - 4, 0, dne because graph of s has a sharp turn there

(c) and (d) Graphs

4. (2005 AB 5) (a) $v'(4)$ does not exist because the graph of $v(t)$ has a sharp turn at $t = 4$. $v'(20) = -\frac{5}{2}\,\mathrm{m/sec^2}$.

$$\boxed{\text{(b)}\ a(t) = \begin{cases} 5, & 0 < t < 4 \\ 0, & 4 < t < 16 \\ -\frac{5}{2}, & 16 < t < 24 \end{cases}}$$

(c) Ave. rate of change = $-\frac{5}{6}\,\mathrm{m/sec^2}$. No, the MVT does not apply for 8 < *c* < 20 because the graph of $v(t)$ is not differentiable at $t = 16$.

5. (2009 Form B, Problem 6) (a) $\frac{11}{8}\frac{\mathrm{m}}{\mathrm{sec}^2}$ (b) The particle changes direction on (8, 20) because $v(8) = 5$ and $v(20) = -10$. The particle also changes direction on (32, 40) because $v(32) = -4$ and $v(40) = 7$.

(c) $v(t)$ must equal $-9\frac{\mathrm{m}}{\mathrm{sec}}$ at least two times on (0, 40). Since $v(t)$ is differentiable, it must be continuous. $v(8) = 5,\ v(20) = -10$, and -9 lies between 5 and -10 so $v(t)$ must equal -9 for some t between 8 and 20. Similarly, since $v(20) = -10,\ v(25) = -8$, and -9 lies between -10 and -8 so $v(t)$ must equal -9 for some t between 20 and 25 by the IVT>

6. (a) $-3t^2 + 14t = 14,\ 2\,\mathrm{m/sec}$ (b) $-6t + 14,\ 2\,\mathrm{m/sec^2}$

(c) At rest at $t = 1.451$ and $t = 3.215$ because $v(t) = 0$ there. Moving left for [0, 1.451) and $(3.215, \infty)$ because $v(t) < 0$. Moving right for (1.451, 3,215) because $v(t) > 0$. (d) – 8 m (e) 9.262 m (f) No, the displacement and distance are not the same because the particle changed direction at $t = 1.451$. (g) Slowing down on (0, 1.451) and (2.333, 3.215) because vel. and acc. have opposite signs. Speeding up on (1.451, 2.333) and $(3.215, \infty)$ because vel. and acc. have the same sign.

7. $a(0.45018\ldots) = 2.435\,\mathrm{m/sec^2}$ 8. 1.600 $\frac{\mathrm{m}}{\mathrm{sec}}$, 0.730 $\frac{\mathrm{m}}{\mathrm{sec}}$

9. (a) – 0.133 (b) – 0.436. Speed is increasing at $t = 2$ because $v(t)$ and $a(t)$ are both negative.

(c) $v(t) = 0$ when $t = 0.443$. This is the only critical number. $v(t) > 0$ for (0, 0.443) and $v(t) < 0$ for $(0.443, \infty)$ so the particle reaches its highest point at $t = 0.443$.

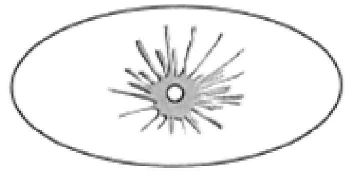
10. (a) 0.714 (b) The particle changes direction at $t = 1$ and at $t = 2$ because $v(t)$ changes from positive to negative or vice versa there. The particle travels to the left on (2, 3) because $v(t) < 0$ there. (c) 0.929

11. (2008) (a) On (3, 4) $v(t) > 0$ and $v(t)$ is increasing so $v'(t) = a(t) > 0$. Therefore, the speed is increasing on (3, 4).

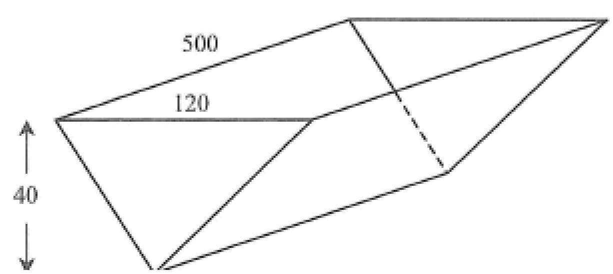
(b) On (2, 3), $v(t) < 0$ and $v(t)$ is increasing so $v'(t) = a(t) > 0$. Therefore, the speed is decreasing on (2, 3).

(c) The acceleration is negative on (0, 1) and (4, 6) because the velocity is decreasing there.

Related Rates Review Questions (ignore the fact that some questions are circled)



1. On the seventh hole, Steve's golf ball landed in the water hazard. The radius of the circular outer ripple created by the splash grew at a rate of 25 cm/s. At what rate was the area of the circle increasing when the radius of the circle was 200 cm?
2. Marina was pulling an air seeder with her tractor at a speed of 80m/minute in a straight-line path. If the seeder is 13 m wide, what is the rate of change in the area of the rectangle being seeded?
3. The radius of a healing circular skin burn is shrinking at a rate of 2 mm/day. At what rate is the circumference of the burn decreasing when the radius of the burn is 2.5 cm? Give your answer in mm/day.
4. The length of a rectangle is increasing at a rate of 10 cm/s while its width is decreasing at a rate of 15 cm/s. How is the area changing when the length is 30 cm and the width is 22 cm?
5. The legs of an isosceles triangle are increasing at a rate of 16 cm/s while the base of the triangle remains constant at 60 cm. At what rate is the area of the triangle increasing when each leg is 50 cm?
6. An ice-cube is melting at a rate of 15 mm^3/s. If each dimension is melting uniformly, at what rate is an edge of the ice-cube shrinking when it is 30 mm long? Give your answer in mm/s.
7. A spherical snowball rolls down a hill causing the radius to increase at a rate of 3 cm/s. When the radius is 20 cm, find the rate at which:
   (a) the surface area of the snowball is increasing.
   (b) the volume of the snowball is increasing.
8. At what rate must air be forced into a spherical balloon in order for the radius to be growing at a rate of 2 cm/s when the radius of the balloon is 8 cm?
9. At what rate is water being poured into a cylindrical glass of diameter 10 cm if the depth of the water is increasing at a rate of 0.4 cm/s?
10. The surface area of a soap bubble is increasing at a rate of $128\pi\ \text{cm}^2/\text{s}$. At what rate is the volume of the soap bubble increasing when the radius is 4 cm?
11. Water is being poured into a conical flower vase at a rate of 50 cm^3/s. If the cone has a radius of 6 cm and a height of 30 cm, at what rate is the depth of the water increasing when the depth of the water above the vertex of the cone is 10 cm?
12. Wheat is being augured into a conical pile so that the radius of the pile is always three times its height. At what rate is the wheat being poured onto the pile if the radius of the pile is growing at a rate of 0.8 m/minute when the radius is 3 m? Give your answer in m^3/min.
13. Water is filling a conical storage tank (vertex down) at a rate of 20 L/s. The tank has a diameter of 200 cm and a height of 300 cm. At what rate is the depth of the water rising when the water is 150 cm above the vertex? Note that $1\ \text{L} = 1000\ \text{cm}^3$.
14. Gasoline is draining from a conical storage tank (vertex down) with radius 2 m and height 4 m. At the moment that the gasoline is 3 m deep, the depth is decreasing at a rate of 0.05 cm/s. At what rate is the gasoline being drained? Give your answer in cm^3/s.
15. A water trough of length 500 cm has a cross section in the shape of a triangle whose base is 120 cm and whose height is 40 cm. If water pours into the trough at a rate of 6500 cm^3/s, at what rate is the height of the water above the vertex of the triangle increasing when that height is 5 cm?

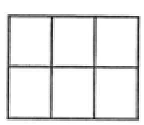


SOLUTIONS TO RELATED RATES REVIEW QUESTIONS: (May not be to 3 decimal AP Standard)

1. $10\,000\pi$ or $31\,415.93$ cm^2/s **2.** 1040 m^2/min **3.** 4π or 12.57 mm/day **4.** shrinking at a rate of 230 cm^2/s
5. 600 cm^2/s **6.** shrinking at a rate of 1/180 or $0.00\overline{5}$ mm/s **7.** (**a**) 480π or 1507.96 cm^2/s (**b**) 4800π or 15079.64 cm^2/s **8.** 512π or 1608.50 cm^3/s **9.** 10π or 31.42 cm^3/s **10.** 256π or 804.25 cm^3/s
11. $25/2\pi$ or 3.98 cm/s **12.** $12\pi/5$ or 7.54 m^3/min **13.** $8/\pi$ or 2.55 cm/s **14.** 1125π or 3534.29 cm^3/s
15. 13/15 or 0.87 cm/s **16.** 6/7 or 0.86 cm/min **17.** increasing at a rate of 1566π or 4919.73 cm^3/s
18. increasing at a rate of 524 cm^3/s **19.** $1/4\pi$ or 0.08 cm/s **20.** $5000/2401\pi$ or 0.66 cm/s

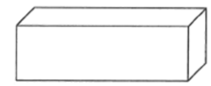
OPTIMIZATION & Related Rates Review Questions

4. The height of a ball above ground level, measured in metres, is given by the function $s(t) = -5t^2 + 60t + 8$, where t is time in seconds.
   (a) What was the initial height of the ball?
   (b) When did the ball reach its maximum height?
   (c) What was the maximum height reached by the ball?
   (d) When did the ball hit the ground? Round your answer to two decimal places.
   (e) With what velocity did the ball hit the ground? Round your answer to two decimal places.
5. A snowmobile operator, travelling along a narrow bush trail, came over a hill and saw another machine stalled 18 metres ahead. The operator immediately cut the throttle and travelled a distance of $s(t) = 12t - t^3$ metres thereafter, where t was the time in seconds. Was there a collision? Explain.
6. The height (in metres) of a bullet above ground level, t seconds after being fired from a gun, is given by the function $h(t) = 400t - 5t^2$.
   (a) Find the maximum height reached by the bullet.
   (b) If the fuselage of an airplane can withstand bullets travelling at a rate of 50m/s, what is the lowest height at which the plane can safely fly?
7. Two nonnegative numbers have a sum of 180. Find these numbers if the product of one of the numbers with the square of the other is to be a maximum. Find that maximum product.
8. Two nonnegative numbers have a product of 72. Find these numbers if the sum of one of them and twice the other is to be a minimum.
9. Two nonnegative numbers have a sum of 4. Find these numbers if the sum of the cube of one of them and the square of the other is to be a minimum. What is that minimum sum?
10. An open-topped box is to be made from a square piece of tin that is 54 cm by 54 cm by cutting out squares of equal size from each of the corners and folding up the flaps. Find the size of the square if the volume of the box is to be a maximum.
11. A dog kennel operator has 72 m of fencing and wants to enclose six congruent rectangular pens as shown below. What dimensions should be used for each pen in order to maximize the total area enclosed?

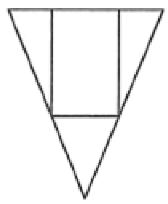


12. The sum of the lengths of two legs in a right triangle is 20 cm. Find the length of each leg if the length of the hypotenuse is to be minimized.
13. What is the shortest vertical distance between the graphs of the functions $f(x) = x^2 + 2$ and $g(x) = -(x-2)^2 - 1$?

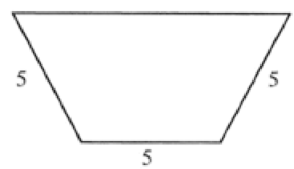
14. A storage box with square ends and no open sides is to be built to have a volume of 50 m^3. If the material for the square ends costs $\$80/\text{m}^2$ while the material for the rectangular sides costs $\$200/\text{m}^2$, find the dimensions of the box in order to minimize its cost.



15. Find the dimensions of the cylinder of greatest volume that can be inscribed in a cone of height 12 cm and radius 4 cm.



16. The shorter base and the legs of an isosceles trapezoid are each 5 cm in length. Find the length of the longer base if the area of the trapezoid is to be maximized.



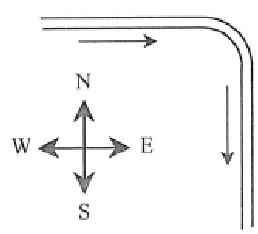
17. The base of a ladder 13 m long is pushed towards the wall at a rate of 10 cm/s. At what rate is the top of the ladder moving up the wall when the base is 5 m from the wall?
18. A right triangle has a hypotenuse of constant length 25 cm. One leg of the right triangle increases at a rate of 1.4 cm/s. When that leg is 24 cm long, find:
    (a) the rate at which the other leg is decreasing in length.
    (b) how the area is changing when the increasing leg is 24 cm.
19. A truck is parked 100 m directly south of an intersection. A car is travelling east at a rate of 20 m/s. At what rate is the distance between the car and the truck increasing 12 seconds after the car has passed through the intersection?
20. A bicyclist is approaching an intersection travelling south at a rate of 20 km/h. A motorcyclist is leaving the same intersection travelling west at a rate of 60 km/h. How is the distance between them changing when the cyclist is 42 m from the intersection and the motorcyclist is 40 m from the intersection?

21. A train of length 700 m maintains a constant speed of 100 km/h as it travels through a right-angled turn as shown. Let z denote the distance between the front and rear of the train at any time.

    (a) Why is $\frac{dz}{dt} = 0$ before the engine reaches the corner?

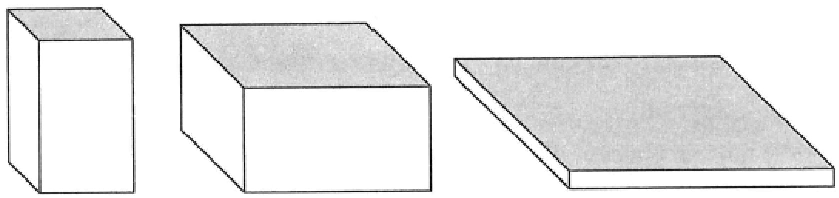
    (b) Find $\frac{dz}{dt}$ at the moment when the engine is 300 m past the corner and is travelling south.

    (c) When, if ever, is $\frac{dz}{dt}$ positive?

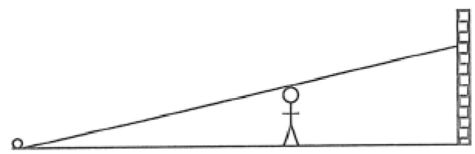


22. A fictitious piece of pie is in the shape of a sector with radius r and arc length s. If the radius is increasing at a rate of 4 cm/s while the arc length is decreasing at a rate of 3 cm/s, what is happening to the area of the piece of pie when the radius is 8 cm and the arc length is 7 cm? Recall that the area of such a sector is $\frac{1}{2}rs$.
23. A metal ball bearing is heating which causes the radius to grow at a rate of 2 mm/min. At what rate is the volume of the ball increasing when the radius of the ball is 1.5 cm? Give your answer in mm^3/min.

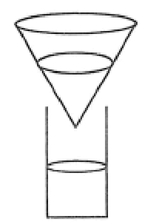
24. A spherical balloon is being inflated at a rate of 20π m^3/h. At what rate is the radius increasing when the radius is 5 m?
25. A conical paper cup has a radius of 4 cm and a height of 12 cm. If water is leaking from the cup at a rate of 6 cm^3/s, at what rate is the water level falling when the height of the water is 8 cm?
26. Sand leaves a conveyor belt at a rate of 0.72 m^3/min, forming a conical pile whose radius is the same as its height. At what rate is the height of the pile rising when the height of the pile is 6 m?
27. A conical wading pool has a diameter of 12 m and a depth of 1 m. Water is draining from the pool at a rate of 2 m^3/minute. At what rate is the height of the water decreasing when it is 25 cm deep above the vertex of the cone? Give your answer in cm/min.
28. Each side of the square top of a box is growing at a rate of 8 cm/s while the height of the box is shrinking at a rate of 5 cm/s. What is happening to the volume of the box when it measures 60 cm by 60 cm by 10 cm?



29. A spotlight on the ground shines towards a wall 12 m away. A man 2 m tall walks directly towards the wall at a rate of 1.6 m/s. At what rate is his shadow length against the wall decreasing when he is 4 m from the wall?



30. Water is leaking from a cone with radius 6 cm and height 12 cm into a cylindrical glass jar with radius 5 cm. It is observed that the height of the water in the jar is increasing at a rate of 3 cm/min. At what rate is the height of the water in the cone decreasing when the height of the water in the cone is 5 cm?



SOLUTIONS TO OPTIMIZATION & RELATED RATES REVIEW QUESTIONS: (May not be to 3 decimal AP

1. **(a)** 600 **(b)** 2100 **(c)** 732 bacteria/h **(d)** 700 bacteria/h **2.** **(a)** \$30 000 **(b)** \$17 142.86 **(c)** \$4285.71/yr **(d)** \$3918.37/yr **3.** **(a)** $v(t)=6t^2-42t+72$; $a(t)=12t-42$ **(b)** $s(8)=256$ u; $v(8)=120$ u/s; $a(8)=54$ u/s^2 **(c)** $t\in(0,3)\cup(4,\infty)$ **(d)** $t\in(3,4)$ **(e)** 87 u **(f)** 128 u/s **4.** **(a)** 8 m **(b)** 6 s **(c)** 188 m **(d)** 12.13 s **(e)** -61.32 m/s **5.** No. The operator's velocity was 0 in 2 seconds. During that time the snowmobile travelled 16 m, thus stopping 2 m before the stalled machine. **6.** **(a)** 8000 m **(b)** 7875 m **7.** 120 (the number to square) and 60; 864 000 **8.** 6 (the number to double) and 12 **9.** 4/3 (the number to cube) and 8/3 (the number to square); the minimum sum is 256/27 **10.** 9 cm by 9 cm **11.** 4m by 4.5 m **12.** each leg is 10 cm **13.** 5 u **14.** 5 m by 5 m by 2 m **15.** $r=8/3$ cm, $h=4$ cm **16.** 10 cm **17.** 25/6 cm/s **18.** **(a)** 4.8 cm/s **(b)** decreasing at 52.7 cm^2/s **19.** 240/13 or 18.46 m/s **20.** increasing at 780/29 or 26.90 km/h **21.** **(a)** the distance between the front of the train and the back of the train is constant at that time **(b)** decreasing at 20 km/h **(c)** when the engine is more than 350 m south of the corner **22.** increasing at 2 cm^2/s **23.** 1800π or 5654.87 mm^3/min **24.** 1/5 m/h **25.** $27/32\pi$ or 0.27 cm/s **26.** $1/50\pi$ or 0.0064 m/min **27.** $800/9\pi$ cm/min **28.** decreasing at 8400 cm^3/s **29.** 3/5 m/s **30.** 12 cm/min

Ch 5 INTEGRAL CALCULUS 30L PRACTICE QUESTIONS (5.1 – 5.6)

The following questions are additional practice questions that will help you prepare for the 30L Chapter 5 test. The questions that allow for graphing calculator will be indicated within the instructions and/or by the image of a calculator beside the question. Please note that many questions given within the calculator section do not actually require the use of the calculator to answer the question most efficiently – it is quite possible that the calculator will be of no use at all. A blank version with extra space will be available on my website. SOLUTIONS for 26 – 34 TO BE SHARED LATER.

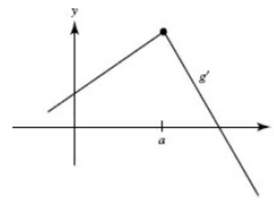
1. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined
(B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist
(D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

2. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?

I. g is continuous.
II. g is differentiable.
III. g is increasing.

(A) I only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

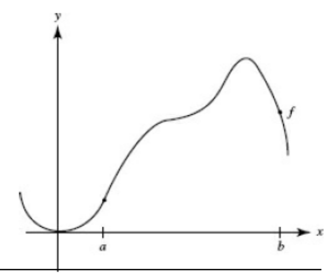


3. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

4. At how many points on the interval $[a, b]$ does the function graphed satisfy the Mean Value Theorem?

(A) none
(B) 1
(C) 2
(D) 3
(E) 4



For Questions 5 and 6 $f'(x) = x \sin x - \cos x$ for $0 < x < 4$.

5. f has a local maximum when x is approximately

(A) 0.9
(B) 1.2
(C) 2.3
(D) 3.4
(E) 3.7

6. The graph of f has a point of inflection when x is approximately

(A) 0.9
(B) 1.2
(C) 2.3
(D) 3.4
(E) 3.7

7. The graph of $f\,'$ is shown below. If we know that $f(2) = 10$, then the local linearization of f at $x = 2$ is $f(x) \simeq$

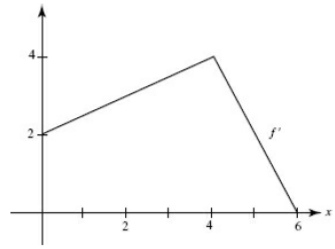
(A) $\frac{x}{2}+2$

(B) $\frac{x}{2}+9$

(C) $3x-3$

(D) $3x+4$

(E) $10x-17$



8. Suppose $f\,'(x) = x^2(x-1)$. Then $f\,''(x) = x(3x-2)$. Over which interval(s) is the graph of f both increasing and concave up?

I. $x < 0$ **III.** $\frac{2}{3} < x < 1$

II. $0 < x < \frac{2}{3}$ **IV.** $x > 1$

(A) I only **(D)** I and III

(B) II only **(E)** IV only

(C) II and IV

9. Which of the following statements is true about the graph of $f(x)$ in Question 8.

(A) The graph has no relative extrema.

(B) The graph has one relative extremum and one inflection point.

(C) The graph has two relative extrema and one inflection point.

(D) The graph has two relative extrema and two inflection points.

(E) None of the preceding statements is true.

10. At what point in the interval $[1, 1.5]$ is the rate of change of $f(x) = \sin x$ equal to its average rate of change on the interval?

(A) 0.995 **(D)** 1.253

(B) 1.058 **(E)** 1.399

(C) 1.239

11. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

(A) $\lim_{x\to a} f(x)$ exists.

(B) $\lim_{x\to a} f(x) = f(a)$.

(C) $f\,'(a)$ exists.

(D) $f(a)$ is defined.

(E) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

12. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

A. The change in temperature during the first day

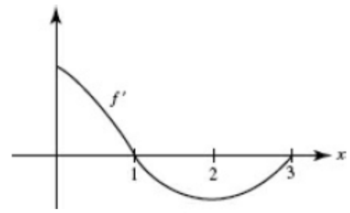
B. The change in the temperature during the 24^{th} hour.

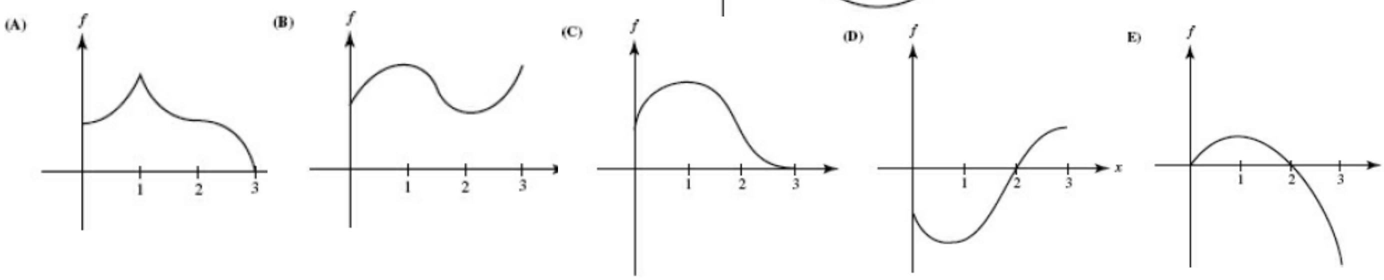
C. The average rate at which the temperature changed during the 24^{th} hour

D. The average rate at which the temperature is changing during the first day

E. The rate at which the temperature is changing at the end of the 24^{th} hour.

13. Given f' as graphed, which could be the graph of f?





14. On the interval $0 < x < 10$, how many relative minimums does the graph of $g(x)$ have if $g'(x) = \dfrac{\sin x}{x+2}$?

A. 0
B. 1
C. 2
D. 3
E. 4

15. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

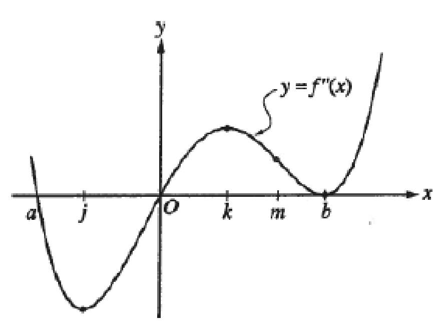
x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

16. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

A. $-2<x<2$ only
B. $-2\le x\le 2$ only
C. $-1\le x\le 1$ only
D. $x\ge -2$
E. $x\le -2$ or $x\ge 2$
F. $x\ge 2$ only

17. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of $f''(x)$ is shown to the right. For what values of x does the graph of $f'(x)$ have a relative maximum?

A. j and k only
B. a and b only
C. a only
D. 0 only
E. a and 0 only



18. A table of function values for a twice differentiable function, $f(x)$, is pictured to the right. Which of the following statements is/are true if $f(x)$ has only one zero on the $-3 \le x \le 3$? Use only the given points to answer!

I. $f'(x) < 0$ on the interval $-3 < x < 3$.
II. $f(x)$ has a zero between $x = 1$ and $x = 3$.
III. $f''(x) > 0$ on the interval $-3 < x < 3$.

x	$f(x)$
-3	10
-1	8
1	2
3	-13

A. I only B. I and II only C. III only

D. II and III only E. I, II and III

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19. The function f has a first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1+x+x^3}$. What is the x–coordinate of the point of inflection of the graph of f? (CALCULATOR PROBLEM)

A. 1.008 D. -0.278
B. 0.473 E. The graph has no points of inflection.
C. 0

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20. If $h(x)$ is a twice differentiable function such that $h(x) < 0$ for all values of x, then at what value(s) does the graph of $g(x)$ have a relative maximum if $g'(x) = (9 - x^2)\cdot h(x)$?

A. $x = 3$ and $x = -3$ B. $x = 3$ only
C. $x = 9$ only D. $x = -3$ only
E. $g(x)$ does not have a relative maximum

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21. For $t \ge 0$, the velocity of a particle moving along the x – axis is given by $v(t) = e^{\tan t} + t^2 - 5$. Which of the following statements is/are true?

I. The particle first changes directions at $t = 1$.

II. On the interval $0 < t < 1$, the mean value theorem guarantees a time t at which the instantaneous acceleration is equal to the average acceleration of the particle.

III. At $t = 2$, the speed of the particle is decreasing.

A. II and III only B. I and III only
C. III only D. I, II and III

---

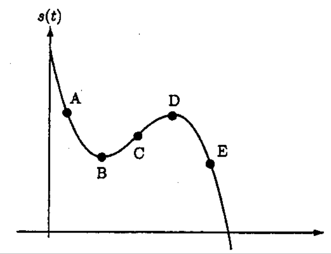
22. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \dfrac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

A. 2.55 B. 0.74 C. 1.32 D. -5.19

23. The position of a particle moving along the x – axis is given by the function $p(t) = (t-1)\cos(2t)$. At what value of t does the particle change directions the second time on the interval $0 < t < 3$?

A. 0.543 B. 1.386 C. 0.892 D. 1.839

24. The graph below shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time, t. Which of the following points marked is closest to the point where the acceleration first becomes negative?



A. A B. B
C. C D. D

25. A particle moves along the x – axis so that at any time $t \geq 0$ its velocity is given by the function $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

A. 1.500 B. 29.453 C. 20.453 D. 74.860

THE REMAINDER OF THE ASSIGNMENT REQUIRES WORK AND *JUSTIFICATION* TO BE SHOWN

26. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c on the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following values could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

27. For $t \geq 0$, the temperature of a cup of coffee in degrees Fahrenheit t minutes after it is poured is modeled by the function $F(t) = 68 + 93(0.91)^t$. Find the value of $F'(4)$. Using correct units of measure, explain what this value means in the context of the problem. **[CALC]**

28. A particle moves along the x-axis so that any time $t > 0$, its velocity is given by $v(t) = 2t\ln t - t$.

a. Write an expression for the acceleration of the particle. NO CALCULATOR!

b. What are the values of t for which the particle is moving to the right? Justify your answer.

c. Is the particle speeding up or slowing down at $t = 1$? Show the analysis that leads to your conclusion.

d. Find the absolute minimum velocity of the particle. Show the analysis that leads to your conclusion.

29. The function $f'(x) = \cos(\ln x)$ is the first derivative of a twice differentiable function, $f(x)$.

a. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a relative maximum. Justify your answer.

b. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a relative minimum. Justify your answer.

c. On the interval $0 < x < 10$, find the x – value(s) where $f(x)$ has a point of inflection. Justify your answer.

30. For the functions in exercises 1 and 2, determine if the Mean Value Theorem holds true for $0 < c < 5$? Give a reason for your answer. If it does hold true, find the guaranteed value(s) of c. **[CALC]**

a) $f(x) = -2 + \frac{1}{2}|x-3|$

b) $g(x) = -2x + \sin^2 x$

31. The price of a share of stock in dollars over a week is given by the function $P(t) = \sqrt{2t+1} + 2\cos t + 20$ where t is measured in days and $0 \le t \le 5$.

a. Find the average rate of change of the price of the stock over [0, 5]. Use correct units.

b. Apply the Mean-Value Theorem to P on [0, 5] and explain the result in the context of the problem situation.

c. On what value of t over the 5-day period is the price of the stock increasing the fastest?

32. Administrators at a hospital believe that the number of beds in use is given by the function

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50,$$

where t is measured in days. **[CALC]**

a. Find the value of $B'(7)$. Using correct units of measure, explain what this value means in the context of the problem.

b. For $12 \le t \le 20$, what is the maximum number of beds in use?

33. Determine whether $g(x) = \sin 2x + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, \pi]$ If so, find all numbers c in (a, b) such that $f(b) - f(a) = f'(c)(b-a)$. **NO CALCULATOR.**

34. For questions 5 – 8, use the table given below which represents values of a differentiable function g on the interval $0 \leq x \leq 6$. Be sure to completely justify your reasoning when asked, citing appropriate theorems, when necessary.

NO CALCULATOR!

x	0	2	3	4	6
$g(x)$	-3	1	5	2	1

a) Estimate the value of $g'(2.5)$.

b) If one exists, on what interval is there guaranteed to be a value of c such that $g(c) = -1$? Justify your reasoning.

c) If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 0$? Justify your reasoning.

d) . If one exists, on what interval is there guaranteed to be a value of c such that $g'(c) = 4$? Justify your reasoning.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS

1. E	2. E	3. B	4. D	5. D	6. C	7. D	8. E	9. E	10. D
11. C	12. E	13. C	14. B	15. D	16. B	17. C	18. B	19. B	20. D
21. A	22. A	23. D	24. C	25. B					

Questions 26 – 34 Solutions will be shared in full detail in class. The "numerical" part of these solutions are worth very little – the work, justifications and explanations are the most important part of the solution.