6.1 Assignment TEXTBOOK P274 #3, 5, 6 (Answer is 1200 g), 7, 11, 15, 16, 17 & Questions Below *(INTEGRAL CALCULUS 30L QUESTIONS)*

Be sure to show all work and the set up that leads to your final answer.

1. Let $f(x) = x^{2`}$
   a) Sketch *f(x)* on the interval from [0, 4]. Clearly label 5 coordinate points on the graph.
   b) Draw 4 rectangles of equal width using right endpoints for height. What is the width of each of these rectangles? Find the sum of the areas of each rectangle to approximate the area under *f(x)* on [0, 4].
   c) Shade the *exact* area "under the curve" on the interval [0, 4]. Is your right endpoint approximation from part (b) an over estimate for the exact area under the curve?
2. Let $f(x) = \sqrt{x}$
   a) Sketch *f(x)* on the interval from [0, 9] on both of the following xy-planes.
   b) On the left graph above, draw 3 rectangles of equal width using left endpoints for height and find the sum of the areas of each rectangle to approximate the area under *f(x)* in [1, 7]. (do not use a calculator. Leave your answer exact.) is this an over or under estimate for the exact area under the curve on [1, 7].
   c) On the right graph above, draw 3 rectangles of equal width using midpoints for height and find the sum of the areas of each rectangle to approximate the area under *f(x)* on [1, 7]. (do not use a calculator. Leave your answer exact.) can you determine whether this is an over or under estimation for the exacy area under the curve on [1, 7]? Explain.
3. Suppose *f* is a continuous, positive function defined on [-4, 4]. The following table represents selected values for *f*.

x	-4	-2	0	2	4
f(x)	1	3	5.5	6	7

   a. Plot the points in the table on the following xy-plane. Can you "connect the dots"? Explain.
   b. Sketch R_4 above and then compute $RRAM_4$.
   c. To conclude $RRAM_4$ is an over approximation for the exact area under the curve, we would need to guarantee *f(x)* is ___________________. Is it? Explain.
   d. Compute $MRAM_2$.
   e. Compute $LRAM_4$. Then find $\frac{RRAM_4+LRAM_4}{2}$. What is $\frac{RRAM_4+LRAM_4}{2}$?
4. A planted tree grows at a rate of $h(t) = 2t + 3$ ft/year. Sketch *h(t)* on [0, 3] and properly label the axes including units.
   a) Find $LRAM_3$.
   b) Shade the region that represents the area under the curve in [0, 3].
   c) Why is using rectangles on this particular problem not the best idea to approximate the exact area under the curve? Find the exact area using two other methods.
   d) What does the area represent in terms of the tree? Use units and geometry to explain your answer.

SOLUTIONS

1b) $RRAM_4 = 30$ c) Over approximation because the graph is concave up and increasing

2b) $LRAM_3 = 2(1+\sqrt{3}+\sqrt{5})$ c) $MRAM_3 = 2\sqrt{2}+4+2\sqrt{6}$

3 b) $RRAM_4=43$ c) increasing and concave up (not enough info to determine this fact) c) $MRAM_2=36$
d) $LRAM_4=31$ e) 37; this is the average of $RRAM_4$ and $LRAM_4$ and will be a better approximation than either4.
a) $LRAM_3=15$ c) Our curve is a straight line – we could use trapezoids instead of rectangles and find the exact area or we could use the large triangle and rectangle. A = 18 d) Area = $18\left(\frac{ft}{yr}\right)(yr)=18\,ft$ - our area represents the number of feet the tree grew over 3 years

6.5 Part 1 Assignment: Questions 1 – 8 Below

1. A car is traveling forward on a straight road and the velocity of measured at various times as indicated in the table below.

t (seconds)	0	2	5	9	13
v(t) (m/s)	0	15	21	30	34

Question: Use a Right Riemann Sum with the 4 subintervals given in the table to estimate the total distance travel by the car over the 13 seconds.

2. A car is traveling forward on a straight road and the velocity of measured at various times as indicated in the table below.

t (seconds)	0	2	5	9	13
v(t) (m/s)	0	15	21	30	34

Question: Use a Left Riemann Sum with the 4 subintervals given in the table to estimate the total distance travel by the car over the 13 seconds.

3. A car is traveling forward on a straight road and the velocity of measured at various times as indicated in the table below.

t (seconds)	0	2	5	9	13
v(t) (m/s)	0	15	21	30	34

Question: Use a Trapezoidal Sum with the 4 subintervals given in the table to estimate the total distance travel by the car over the 13 seconds.

4. A car is traveling forward on a straight road and the velocity of measured at various times as indicated in the table below.

t (seconds)	0	2	5	9	13
v(t) (m/s)	0	15	21	30	34

Question: Use a Trapezoidal Sum with the 4 subintervals given in the table to estimate the average velocity of the car over the 13 second interval.

5. Students are walking into school one morning starting 8:00AM. The rate, r(t) that students are entering the building is given in the table below.

t (min)	0	2	4	6	8	10	12
r(t) (students/min)	10	7	3	8	14	17	12

Question: Use a Left Riemann Sum with 3 subintervals of equal length given in the table to estimate the total students that entered the building over 12 minute interval.

6. Students are walking into school one morning starting 8:00AM. The rate, r(t) that students are entering the building is given in the table below.

t (min)	0	2	4	6	8	10	12
r(t) (students/min)	10	7	3	8	14	17	12

Question: Use a Trapezoidal Sum with 3 subintervals of equal length given in the table to estimate the total students that entered the building over 12 minute interval.

7. Students are walking into school one morning starting 8:00AM. The rate, r(t) that students are entering the building is given in the table below.

t (min)	0	2	4	6	8	10	12
r(t) (students/min)	10	7	3	8	14	17	12

Question: Use a Midpoint Riemann Sum with 3 subintervals of equal length given in the table to estimate the total students that entered the building over 12 minute interval.

8. Students are walking into school one morning starting 8:00AM. The rate, r(t) that students are entering the building is given in the table below.

t (min)	0	2	4	6	8	10	12
r(t) (students/min)	10	7	3	8	14	17	12

Question: Approximate the average rate that students are entering the building over the 12 minute interval using a Right Riemann Sum of three subintervals.

SOLUTIONS to 6.5 (Your answers will be the fractional equivalent to these)

1. 9.667 2. 349 3. 249 4. 299 5. 23 6. 108 7. 112 8. 128

6.2 Assignment P286 #1—29 odd, CALC ACTIVE: 33-36, 41-46

1. A bug is walking along a straight path for 0≤t≤16 seconds. The velocity of the bug can be modeled by the differentiable function v(t),in meters per second. Selected values of v(t) are given in the table:

t seconds	0	2	4	9	14	15	16
$v(t)$ m/s	3	-2	7	12	2	3	-5

A) Approximate $\int_0^{16} v(t)\,dt$ using a midpoint Riemann sum with the three subintervals indicated by the data above. Using correct units, interpret your answer in context of this problem.

B) Approximate $\int_4^{16} |v(t)|\,dt$ using a right Riemann sum with four subintervals indicated by the data in the table. Using correct units, interpret your answer in context of this problem.

C) Using a trapezoidal approximation with three subintervals, approximate $\frac{1}{9}\int_0^9 v(t)\,dt$. Using correct units, interpret your answer in context of this problem.

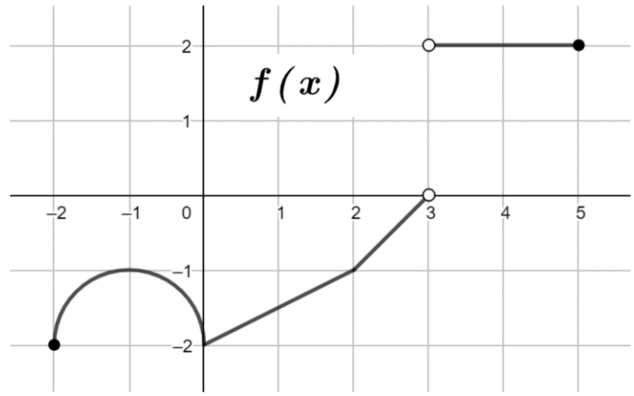
CONTINUED ON NEXT PAGE……

D) Approximate $a(3)$. Include units of measure with your answer.

E) Is there a time c, for $0 < c < 16$ such that $a(c) = -2$? Justify your answer.

2. The graph of $f(x)$ is shown above on the interval $[-2, 5]$ and consists of a semi circle and three line segments.

Use the graph of $f(x)$ to find the following.



A) $\int_{3}^{5} f(x)dx$ B) $\int_{-2}^{5} f(x)dx$ C) $\int_{2}^{5} [f(x)+5]dx$

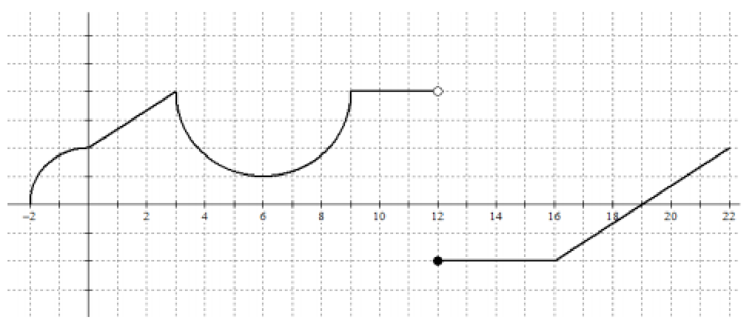
D) $\int_{-2}^{0} -6f(x)dx$ E) $\int_{4}^{-1} 2f(x)dx$ F) $\int_{3}^{0} [4-7f(x)]dx$

G) $\int_{0}^{2} f'(x)dx$ H) $\int_{5}^{3} [f'(x)-8]dx$ I) $\int_{4}^{5} [-3f''(x)+2]dx$

3. In the chart below, the limit of a Riemann sum has been provided for you. Write the corresponding definite integral. (Note: These questions use the variable *I* where our notes used the variable *k*). Be prepared to see either one on an exam!

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{2\left(\frac{6}{n}\right)+1}\right]\frac{6}{n}$ b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-2+\frac{5i}{n}\right)^2 - 3\right]\frac{5}{n}$ c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[3\left(1+\frac{5i}{n}\right)-4\right]\frac{5}{n}$

4. The graph to the right consists of a quarter circle, a half circle and four line segments. For each of the expressions below, fill in the missing definite integrals. Then determine the value of each definite integral using geometric formulas (without using a calculator).



Limit of Riemann Sum	Definite Integral	Value of Definite Integral
$\lim_{n\to\infty} \sum_{i=1}^{n} (-2)\left(\frac{4}{n}\right)$		
$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{3}\left(\frac{3i}{n}\right)+2\right)\left(\frac{3}{n}\right)$		
$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4-\left(-2+\frac{2i}{n}\right)^2}\right)\left(\frac{2}{n}\right)$		
$\lim_{n\to\infty} \sum_{i=1}^{n} (4)\left(\frac{3}{n}\right)$		

6.2 SOLUTIONS

1. a) $\int_0^{16} v(t)\,dt \approx 4(-2)+10(12)+2(3)$ = 118 , $\int_0^{16} v(t)\,dt$ is the displacement of the bug, in meters, from $t=0$ to $t=16$ seconds

b) $\int_4^{16} |v(t)|\,dt \approx 5(12)+5(2)+1(3)+1(5)$ =78, $\int_0^{16} |v(t)|\,dt$ is the total distance the bug travels, in meters, from $t=4$ to $t=16$ seconds

c) $a(3) \approx \dfrac{v(4)-v(2)}{4-2} = \dfrac{7+2}{4-2} = \dfrac{9}{2}\,m/s^2$

d) $v(9)=12\,, v(14)=2$: $\dfrac{2-12}{14-9} = -2$

Since $v(t)$ is continuous on [9, 14] and differentiable on (9, 14), the MVT guarantees that there is a time c, for $0<c<16$ such that $a(c)=-2$.

2. $A)\ 4$ $B)\ \dfrac{\pi-7}{2}$ $C)\ \dfrac{37}{2}$ $D)\ 24-3\pi$ $E)\ 7-\dfrac{\pi}{2}$ $F)\ \dfrac{-73}{2}$

$G)\ f(2)-f(0)=-1+2=1$ $H)\ 0+2(8)=16$ $I)\ -3[f'(4.5)-f'(4)]+0.5(2)=1$

3. a) $\int_0^6 \sqrt{2x+1}\,dx$ b) $\int_{-2}^3 x^2-3\,dx$ c) $\int_1^6 3x-4\,dx$

4.

	Definite Integral	Value of Definite Integral
a)	Answer: $\int_{12}^{16}(-2)\,dx$	Answer: -8
b)	Answer: $\int_0^3\left(\frac{2}{3}x+2\right)dx$	Answer: 9

	Definite Integral	Value of Definite Integral
c)	Answer: $\int_{-2}^0\sqrt{4-x^2}\,dx$	Answer: π
d)	Answer: $\int_9^{12}4\,dx$	Answer: 12

6.3 Day 1 Assignment P294 #1-7, 11-18

1. A bug is walking along a straight path for 0≤t≤16 seconds. The velocity of the bug can be modeled by the differentiable function v(t),in meters per second. Selected values of v(t) are given in the table:

t seconds	0	2	4	9	14	15	16
$v(t)$ m/s	3	-2	7	12	2	3	-5

Using a trapezoidal approximation with three subintervals, approximate $\frac{1}{9}\int_0^9 v(t)\,dt$. Using correct units, interpret your answer in context of this problem.

t hours	1	2	5	7
$s'(t)$ in/hr	1.9	1.6	0.9	0.5

2. Overnight, snow begins falling heavily in Plugville. The rate that snow is falling is modeled by the continuous and decreasing function $s'(t)$, measured in inches per hour, where t represents hours since 12AM. Selected values of $s'(t)$ are given in the table above.

A) Using a left Riemann sum with the intervals indicated in the table above, approximate $\int_1^7 s'(t)\,dt$

Using correct units, interpret your answer in context of this problem.

B) Is your answer from part A) an over or under approximation? Give a reason for your answer.

C) Can you guarantee a time r, where $1 < r < 7$ such that $s'(r) = 1$? Justify your answer.

D) It is known that $\frac{1}{6}\int_1^7 s'(t)dt \approx 0.9121$. Using correct units, interpret the meaning of this in context of this problem.

6.3 Day 1 SOLUTIONS

1. $\frac{1}{9}\int_0^9 v(t)\,dt \approx \frac{1}{9} * \frac{1}{2}[2(3-2) + 2(-2+7) + 5(7+12)]$ = $\frac{107}{18}$

$\frac{1}{9}\int_0^9 v(t)\,dt$ is the average velocity of the bug, in meters per second, from $t = 0$ to $t = 9$ seconds

2. a) $\int_1^7 s'(t)\,dt \approx 1(1.9) + 3(1.6) + 2(0.9)$ = 8.3

$\int_1^7 s'(t)\,dt$ is the total amount of snow, in inches, that fell in Plugville from 1AM to 7AM.

b) Overapproximation because $s'(t)$ is decreasing.

c) $s'(1) = 1.9,\quad s'(7) = 0.5$

Since $s'(t)$ is continuous on $[1,7]$, the IVT guarantees a time r, where $1 < r < 7$ such that $s'(r) = 1$.

d) 0.9121 is the average rate that snow is falling, in inches per hour, from 1AM to 7AM

6.3 Day 2 Assignment (1-45 Odd Below)

For each of the following, find a function $f(x)$ that has the given derivative or differential.

1. $f'(x)=6x^2+12x-7$
2. $f'(x)=x+1+x^{-1}+x^{-2}$
3. $f'(x)=\cos x+\sin x+e^x$
4. $dy=(20x+3)dx$
5. $dy=\cos 6x dx$
6. $dy=\left(e^{2x}+e^{-2x}\right)dx$

For each of the following, find the function $f(x)$ that passes through the given point and has the given derivative.

7. $(3,4)$; $f'(x)=2x-4$
8. $(-2,-3)$; $f'(x)=4x^3-3x^2$
9. $\left(2,4+e^2\right)$; $f'(x)=x+e^x$

Determine each of the following integrals by sight. Some may require simplification before you can integrate.

10. $\int 14dx$
11. $\int -\frac{3}{5}dx$
12. $\int 12xdx$
13. $\int 24x^5 dx$
14. $\int r^{3/5}dr$
15. $\int h^{1/2}dh$
16. $\int 14w^{4/3}dw$
17. $\int x^{-7/8}dx$
18. $\int g^{-1/4}dg$
19. $\int 2x^{-2/3}dx$
20. $\int \frac{1}{m}dm$
21. $\int \sqrt[6]{x^5}\,dx$
22. $\int x^4\left(x^2-1\right)dx$
23. $\int (x-3)(x+2)dx$
24. $\int (4-3t)(2t-1)\,dt$
25. $\int (3a+2)^2 da$
26. $\int \sqrt{b}(b+6)\,db$
27. $\int \frac{6}{x^3}dx$
28. $\int \frac{2}{\sqrt{x}}dx$
29. $\int \sqrt[3]{x}(x-1)^2\,dx$
30. $\int \frac{10}{x}dx$
31. $\int \frac{2x^2-4}{x^3}dx$
32. $\int \sin 3xdx$
33. $\int \cos 8udu$
34. $\int -\sin\frac{1}{6}udu$
35. $\int 12\cos\frac{1}{4}xdx$
36. $\int e^{6x}dx$
37. $\int 2e^{\frac{1}{6}x}dx$
38. $\int \frac{1+x-x^2+x^3}{x^2}dx$
39. $\int dx$
40. $\int \frac{\pi}{x}dx$
41. $\int \frac{x}{e}dx$
42. $\int 3^x dx$
43. $\int x^3 dx$
44. $\int \sqrt{3x}dx$
45. $\int \frac{3}{x}dx$

SOLUTIONS to 6.3 Day 2

1. $2x^3+6x^2-7x+C$ **2.** $\frac{1}{2}x^2+x+\ln|x|-x^{-1}+C$ **3.** $\sin x-\cos x+e^x+C$ **4.** $10x^2+3x+C$

5. $\frac{1}{6}\sin 6x+C$ **6.** $\frac{1}{2}e^{2x}-\frac{1}{2}e^{-2x}+C$ **7.** $f(x)=x^2-4x+7$ **8.** $f(x)=x^4-x^3-27$

9. $f(x)=\frac{1}{2}x^2+e^x+2$ **10.** $14x+C$ **11.** $-\frac{3}{5}x+C$ **12.** $6x^2+C$ **13.** $4x^6+C$ **14.** $\frac{5}{8}r^{8/5}+C$

15. $\frac{2}{3}h^{3/2}+C$ **16.** $6w^{7/3}+C$ **17.** $8x^{1/8}+C$ **18.** $\frac{4}{3}g^{3/4}+C$ **19.** $6x^{1/3}+C$ **20.** $\ln|m|+C$

21. $\frac{6}{11}x^{11/6}+C$ **22.** $\frac{1}{7}x^7-\frac{1}{5}x^5+C$ **23.** $\frac{1}{3}x^3-\frac{1}{2}x^2-6x+C$ **24.** $-2t^3+\frac{11}{2}t^2-4t+C$

25. $3a^3+6a^2+4a+C$ **26.** $\frac{2}{5}b^{5/2}+4b^{3/2}+C$ **27.** $-3x^{-2}+C$ **28.** $4x^{1/2}+C$

29. $\frac{3}{10}x^{10/3}-\frac{6}{7}x^{7/3}+\frac{3}{4}x^{4/3}+C$ **30.** $10\ln|x|+C$ **31.** $2\ln|x|+2x^{-2}+C$ **32.** $-\frac{1}{3}\cos 3x+C$

33. $\frac{1}{8}\sin 8u+C$ **34.** $6\cos\frac{1}{6}u+C$ **35.** $48\sin\frac{1}{4}x+C$ **36.** $\frac{1}{6}e^{6x}+C$ **37.** $12e^{\frac{1}{6}x}+C$

38. $-x^{-1}+\ln|x|-x+\frac{1}{2}x^2+C$ **39.** $x+C$ **40.** $\pi\ln|x|+C$ **41.** $\frac{1}{2e}x^2+C$ **42.** $\frac{1}{\ln 3}3^x+C$ **43.** $\frac{1}{4}x^4+C$

44. $\frac{2}{3}\sqrt{3}x^{3/2}+C$ **45.** $3\ln|x|+C$ **46.** $\frac{1}{2}\sec 2x+C$ **47.** $3\sin^{-1}x+C$ **48.** $6\tan\frac{1}{3}x+C$

6.4 Day 1 Assignment: 5-23 odd, 27, 31, 45 & 46

EVALUATE THE FOLLOWING (No Calculator)

5. $\int_0^2 6x\,dx$ **7.** $\int_{-1}^0 (2x-1)\,dx$ **9.** $\int_{-1}^1 (t^2-2)\,dt$

11. $\int_0^1 (2t-1)^2\,dt$ **13.** $\int_1^2 \left(\frac{3}{x^2}-1\right)dx$ **15.** $\int_1^4 \frac{u-2}{\sqrt{u}}\,du$

17. $\int_{-1}^1 \left(\sqrt[3]{t}-2\right)dt$ **19.** $\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$ **21.** $\int_{-1}^0 \left(t^{1/3}-t^{2/3}\right)dt$

23. $\int_0^5 |2x-5|\,dx$ **25.** $\int_0^4 |x^2-9|\,dx$

27. $\int_0^{\pi} (1+\sin x)\,dx$ **31.** $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$

For question #45, find the value(s) of *c* guaranteed by the Mean Value Theorem for Integrals for f(x)

45. $f(x) = x^3, \quad [0, 3]$

46. The function G and its derivatives are continuous. Use the table below and properties of limits, to find the following:"

x	1	3	4	7
$G(x)$	2	-3	1	4
$g(x)$	-1	5	3	7
$g'(x)$	8	-4	-2	9
$g''(x)$	11	0	-6	2

A) $\int_3^7 g(x)dx$ B) $\int_4^1 5g(x)dx$ C) $\int_1^3 [g(x)-2]dx$

D) $\int_1^7 g'(x)dx$ E) $\int_4^3 -2g'(x)dx$ F) $\int_7^1 [3-2g'(x)]dx$

SOLUTIONS TO 6.4 Day 1

5. 12 **7.** -2 **9.** $-\frac{10}{3}$ **11.** $\frac{1}{3}$ **13.** $\frac{1}{2}$ **15.** $\frac{2}{3}$ **17.** -4 **19.** $-\frac{1}{18}$ **21.** $-\frac{27}{20}$ **23.** $\frac{25}{2}$ **25.** $\frac{64}{3}$

27. $\pi + 2$ **31.** $2\sqrt{3}/3$ **45.** $3\sqrt[3]{2}/2 \approx 1.8899$

A) $G(7)-G(3)$
$= 4+3 = 7$

B) $5[G(1)-G(4)]$
$= 5(2-1) = 5$

C) $G(3)-G(1)-2(2)$
$= -3-2-4 = -9$

D) $g(7)-g(1)$
$= 7+1 = 8$

E) $-2[g(3)-g(4)]$
$= -2(5-3) = -4$

F) $-6(3)-2[g(1)-g(7)$
$= -18-2(-1-7) = -34$

6.4 Day 2 Assignment TEXTBOOK P294 11& 13 (CALC ACTIVE), 31, 35, 45-50
TEXTBOOK Page 306 #1-39 Odd
DUO TANG Page 85 #1

1. Given to the right is the graph of $f(t)$ which consists of three line segments and one semicircle.

   Additionally, let the function $g(x)$ be defined to be $g(x)=\int_{-1}^{x} f(t)dt$.

   a) Find $g(-6)$.

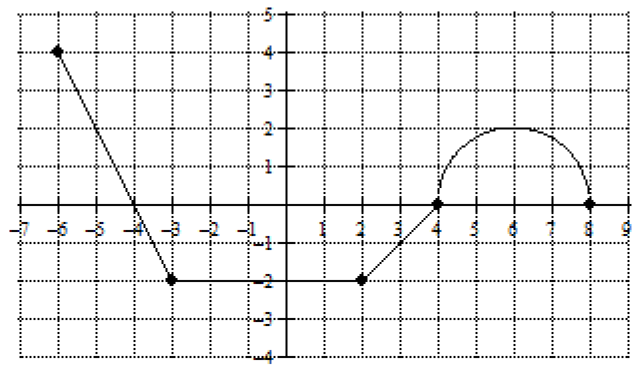
   b) Find $g(6)$.

   c) Find $g'(6)$.

   d) Find $g'(2)$.

   e) Find $g''(2)$. Give a reason for your answer.

   f) Find $g''(-4)$. Give a reason for your answer.



2004 AP® CALCULUS AB
Question 5

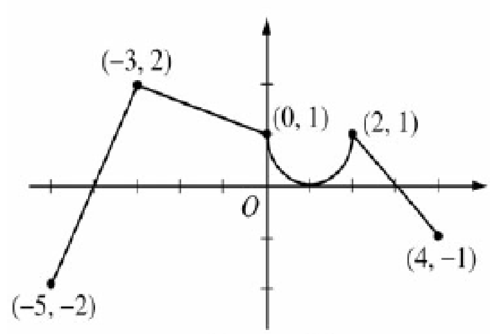
The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x)=\int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



Graph of f

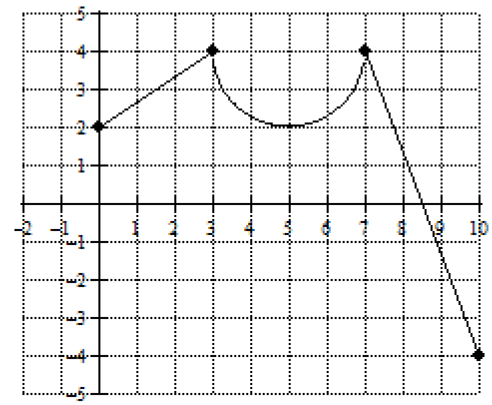
SOLUTIONS TO 6.4 Day 2

1 a) 1 b) $-8+\pi$ c) 2 d) -2 e) g'(2)=f'(2) which is undefined, f'(2) is undefined because the graph of f(t) has a cust at x=2 f) g"(4)=f'(-4)=-2, f'(4) represents the slope of the tangent line drawn to f(t) at x=-4. The graph of f(t) has a slope of -2 when x=-4.

SOLUTION TO AP QUESTION 2004 #5 can be found online on AP Central

6.4 Day 3 Assignment:

1. The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the x – axis on the time interval $0 \leq t \leq 10$. The initial position of the particle at time $t = 0$ is 12.



   a) On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

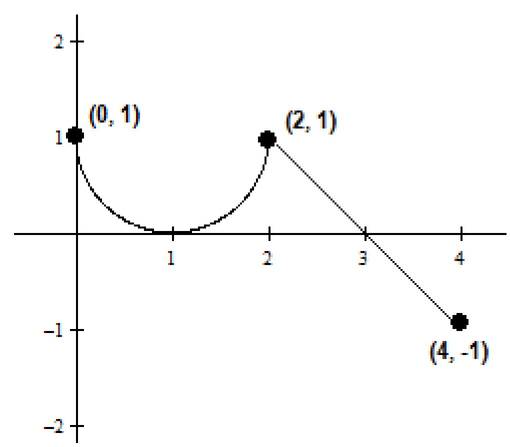
   b) What is the total distance that the particle has traveled on the time interval $0 \leq t \leq 7$. Leave your answer in terms of π. Indicate units of measure.

   c) What is the net distance that the particle travels on the interval $5 \leq t \leq 10$? Round your answer to the nearest thousandth. Indicate units of measure.

   d) What is the acceleration of the particle at time $t = 2$? Indicate units of measure.

   e) What is the position of the particle at time $t = 5$? Indicate units of measure.

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the questions 2-7.



2. For what values is the particle moving to the right? Justify your answer.

3. For what values is the particle moving to the left? Justify your answer.

4. For what values is the speed of the particle increasing? Justify your answer.

5. For what values is the speed of the particle decreasing? Justify your answer.

6. What is the net distance that the particle travels on the interval [0, 4]?

7. What is the total distance that the particle travels on the interval [0, 4]?

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous and differentiable functions on $0 \leq t \leq 60$. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

8. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)| dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

9. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t) dt$ in terms of the car's motion. Find the exact value of the integral.

10. Is there a value of t such that $a'(t) = 0$? If so, identify an interval on which such a value of t exists? Justify your reasoning.

11. Using appropriate units, approximate the value of $a(31)$. What does this value say about the velocity of the car at $t = 31$? Give a reason for your answer.

12. Using appropriate units, find the value and explain the meaning of $\frac{1}{35}\int_{25}^{60} a(t) dt$.

2002 AP® CALCULUS AB (Form B)

Problem #3

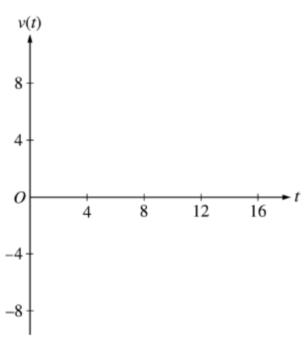
A particle moves along the x-axis so that its velocity v at any time t, for $0 \leq t \leq 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.



SOLUTIONS TO 6.4 Day 3

1. a) The particle is moving to the right for 0<t<8.5 because v(t)>0 on this interval. The particle is moving to the left for 8.5<t<10 because v(t) <0 on this interval

b) Total Dist. $= \int_0^7 |v(t)|\,dt$

$= \frac{1}{2}(3)(2+4) + \left[4(4) - \frac{1}{2}\pi(2)^2\right]$

$= 9 + 16 - 2\pi = \boxed{25 - 2\pi \text{ meters}}$

c) Net Dis. $= \int_5^{10} v(t)\,dt$

$= 2(4) - \frac{1}{4}\pi(2)^2 + \frac{1}{2}(1.5)(4) + \frac{1}{2}(1.5)(-4)$

$= 8 - \pi = \boxed{4.858 \text{ meters}}$

d) $v'(2) = a(2) =$ slope of $v(t)$ at $t = 2$

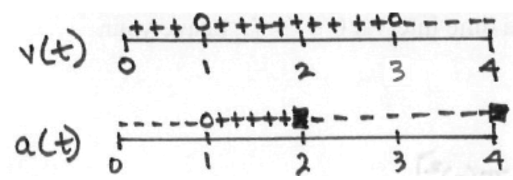
$= \boxed{\frac{2}{3} \text{ meters}/\text{sec}^2}$

e) $\int_0^5 v(t)\,dt = p(5) - p(0)$

$\frac{1}{2}(3)(2+4) + \left[2(4) - \frac{1}{4}\pi(2)^2\right] = p(5) - 12$

$9 + 8 - \pi + 12 = p(5) \rightarrow \boxed{p(5) = 29 - \pi \text{ meters}}$

2. The particle is moving to the right on 0<t<1 and 1<t<3 because v(t)>0 on those intervals
3. The particle is moving to the left on 3<t<4 because v(t)<0 on that interval
4.



Speed is increasing when v(t) and a(t) have the same sign which occurs for $1 < t < 2$ and $3 < t < 4$.

5. speed is decreasing when v(t) and a(t) have different signs which occurs for 0<t<1 and 2<t<3

6. Net Distance $= \int_0^4 v(t)\,dt$

$= \left[2(1) - \frac{1}{2}\pi(1)^2\right] + \frac{1}{2}(1)(1) + \frac{1}{2}(1)(-1)$

$= \boxed{2 - \frac{1}{2}\pi}$

7. Total Distance $= \int_0^4 |v(t)|\,dt$

$= \left[2(1) - \frac{1}{2}\pi(1)^2\right] + \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1)$

$= 2 - \frac{\pi}{2} + \frac{1}{2} + \frac{1}{2}$

$= \boxed{3 - \frac{\pi}{2}}$

8. $\int_0^{60} |v(t)|\,dt \approx 25(30) + 10(14) + 25(0) \approx 890$ feet

$\int_0^{60} |v(t)|\,dt$ represents the total distance the car travels during the first 60 seconds of movement.

9. $\int_{15}^{50} a(t)\,dt = v(50) - v(15) = 0 - (-30) = 30$ feet per second

$\int_{15}^{50} a(t)\,dt$ represents the change in velocity between $t = 15$ and $t = 50$ seconds.

10. a(t) is continuous on [25,35] and differentiable on (25,35) and $\frac{f(35)-f(25)}{35-25} = 0$, the Mean Value Theorem guarantees a value of c, 25<c<35 such that f'(c)=0

SOLUTIONS TO 6.4 Day 3 CONT.

11. $v'(31) \approx \dfrac{v(30)-v(35)}{30-35} \approx \dfrac{-14-(-10)}{-5} \approx \boxed{\frac{4}{5} \text{ feet per second}^2}$

Since $v'(31) > 0$, then the velocity is increasing at $t = 31$ seconds.

12. $\frac{1}{35}\int_{25}^{60} a(t)\,dt = \frac{1}{35}\left[v(60)-v(25)\right] = \frac{1}{35}\left[10-(-20)\right]$

$= \boxed{\frac{6}{7} \text{ feet per second}^2}$

$\frac{1}{35}\int_{25}^{60} a(t)\,dt$ represents the average acceleration of the car from $t = 25$ to $t = 60$ seconds.

6.4 Day 4 Assignment: Duo – Tang Page 89&90 #1-7 & AP Question

At time *t* = 0, there are 120 pounds of sand in a conical tank. Sand is being added to the tank at the rate of $S(t) = 2e^{\sin^2 t} + 2$ **pounds per hour. Sand from the tank is used at a rate of** $R(t) = 5\sin^2 t + 3\sqrt{t}$ **pounds per hour. The tank can hold a maximum of 200 pounds of sand. (Calculator Active)**

1. Find the value of $\int_0^4 S(t)dt$. Using correct units, what does this value represent?
2. Find the value of $\int_1^3 R(t)dt$. Using correct units, what does this value represent?
3. Find the value of $\frac{1}{4}\int_0^4 S(t)dt$. Using correct units, what does this value represent?
4. Write a function, *A*(*t*), containing an integral expression that represents the amount of sand in the tank at any given time, *t*.
5. How many pounds of sand are in the tank at time *t* = 7?
6. After time *t* = 7, sand is not used any more. Sand is, however, added until the tank is full. If *k* represents the value of *t* at which the tank is at maximum capacity, write, but do not solve, an equation using an integral expression to find how many hours it will take before the tank is completely full of sand

7. A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t\sin^2\left(\frac{t}{14}\right)$**. The rate at which** the snow melts is modeled by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. Both $S(t)$ and $M(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 24$. At time $t = 0$, the slope holds 50 cubic yards of snow. (Calculator Active)

a. Compute the total volume of snow added to the mountain over the first 6-hour period.

b. Find the value of $\int_0^6 M(t)dt$ and $\frac{1}{6}\int_0^6 M(t)dt$. Using correct units of measure, explain what each represents in the context of this problem.

c. Is the volume of snow increasing or decreasing at time $t = 4$? Justify your answer.

d. How much snow is on the slope after 5 hours? Show your work.

e. Suppose the snow machine is turned off at time $t = 10$. Write, but do not solve, an equation that could be solved to find the time $t = K$ when the snow would all be melted.

2005 AP® CALCULUS AB

Problem #2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

SOLUTIONS TO 6.4 Day 4

1. $\int_0^4 S(t)dt = 21.173$ pounds of sand were added to the tank in the first 4 hours

2. $\int_1^3 R(t)dt = 14.878$ pounds of sand were used during the hours from $t=1$ to $t=3$.

SOLUTIONS TO 6.4 Day 4 CONT:

3. $\frac{1}{4}\int_0^4 s(t)\,dt = 5.293$ pounds per hour.
This value represents the average # of pounds of sand added to the tank per hour from $t=1$ to $t=4$.

4. $A(t) = 120 + \int_0^t S(t)\,dt - \int_0^t R(t)\,dt$

w many pounds of sand are in the tank at time $t = 7$?

5. $A(7) = 120 + \int_0^7 S(t)\,dt - \int_0^7 R(t)\,dt$
$= \boxed{104.420 \text{ pounds}}$

6. $104.420 + \int_7^K S(t)\,dt = 200$

7. a) $\int_0^6 S(t)\,dt = \boxed{142.413 \text{ cubic yards}}$

b) $\int_0^6 M(t)\,dt = 81.823\ yd^3$ $\qquad$ $\int_0^6 M(t)\,dt$ represents the total volume of snow that has melted from $t=0$ to $t=6$ hours.

$\frac{1}{6}\int_0^6 M(t)\,dt = 13.637\ yd^3/\text{hour}$ $\qquad$ $\frac{1}{6}\int_0^6 M(t)\,dt$ represents the avg value of the rate at which the snow is melting from $t=0$ to $t=6$ hours.

4? Justify your answer.

c) $A(t) = 50 + \int_0^t S(t)\,dt - \int_0^t M(t)\,dt$

$A'(t) = S(t) - M(t)$

$A'(4) = S(4) - M(4) = 11.800$ $\qquad$ Since $A'(4) > 0$, the volume of snow is increasing at $t=4$ hours

d) $A(5) = 50 + \int_0^5 S(t)\,dt - \int_0^5 M(t)\,dt$
$= \boxed{95.335\ yd^3}$

e) $A(10) = 183.193$

$183.193 - \int_{10}^K M(t)\,dt = 0$

AP Calculus

Ch 6 REVIEW ASSIGNMENT

TEXTBOOK: P 319 #9, 13, 15, 17, 19, 20-25, 27, 30, 31, 36-39, 41, 42, 45-47, 49-50, 54

Riemann Sums – You might be given a function's equation, data in graphs, or data in charts.

1. If $f(x) = -x^2 + 2$, estimate the value of $\int_0^2 f(x)dx$ using 4 subintervals of equal length and the approximation method indicated.

(a) A left Riemann sum
(b) A right Riemann sum
(c) A midpoint approximation (Scientific Calc Allowed)
(d) A trapezoidal approximation

2. Use your calculator to evaluate the integral from question 1. Then for each approximation draw a sketch of the strips used and justify why the value is an overestimate or underestimate.

x	0	0.8	2	3.6	4
$f(x)$	9.58	3.5	-0.82	2.38	4.78

3. Selected values for the function f are given above. Approximate the value of $\int_0^4 f(x)dx$ using:

(a) A left Riemann sum using 4 subintervals.
(b) A right Riemann sum using 4 subintervals.

x	0	0.4	0.8	1.2	1.6	2	2.4
$f(x)$	1	1.16	1.64	2.44	3.56	5	6.76

4. Selected values for the function f are given above. Approximate the value of $\int_0^{2.4} f(x)dx$ using:

(a) A midpoint approximation using 3 subintervals.
(b) A trapezoidal approximation using 3 equal subintervals.

Be able to describe what you are finding.

5. The function $v(t)$ gives a particle's velocity at a time t in meters per second.

(a) Using correct units, explain what the integral $\int_1^3 v(t)dt$ would find.
(b) Using correct units, explain what $v'(4)$ would find.

Integrals can be expressed using summation notation.

6. Write an integral that is equal to the limit given below:

$$\lim_{n\to\infty}\sum_{k=1}^{n} 4\left(1+\frac{2k}{n}\right)^2 \frac{2}{n}$$

b) $$\lim_{n\to\infty}\sum_{i=1}^{n}\left[5\left(2+\frac{4i}{n}\right)+7\right]\frac{4}{n}$$

7. Write the following definite integrals as limits.

(a) $\int_1^2 x^2 dx$ (b) $\int_1^2 4x^2 dx$ (c) $\int_0^1 (1+x)^2 dx$ (d) $\int_0^1 4(1+x)^2 dx$

Know how to manipulate integrals to solve problems:

8. Evaluate $\int_1^5 f(x)dx$ if $\int_1^7 2f(x)dx = 8$ and $\int_5^7 f(x)dx = 3$.

Know how to apply the Fundamental Theorem of Calculus, parts I and II.

9. Find $\frac{dy}{dx}$ when $y = \int_0^{x^2} \tan 5t\, dt$

10. If the antiderivative of $f(x)$ is $\sin(x^2 - 3)$, evaluate $\int_1^2 f(x)dx$ (Calc Active)

11. If $g(x) = \int_1^x f(x)dx$, where the function f is continuous and $f(3) = 5$, $f(1) = 2$, and $f(0) = 9$. Evaluate $g'(3)$.

12. Due to warmer temperatures, the falling snow is melting once it lands on the ground. The rate that the snow is melting at time t, where t is hours since 12AM, is given by m(t), measured in inches per hour.

Selected values of m(t) and M(t), the antiderivative of m(t) are given in the table above.

t	1	3	6	7
$m(t)$	0.2	0.8	0.25	0.1
$M(t)$	0.1	0.7	1.5	2.2

A) How many total inches of snow melted between 1AM and 7AM?

B) By 6AM, there has been 4 inches of snow that has fallen. How many inches of snow are on the ground at time t=6?

C) Approximate m' (5). Include units of measure

D) Write an equation of the tangent line to M(t) at t=3. Use this tangent line to approximate M(4)

t (minutes)	0	1	5	7	10
$C'(t)$ (cars per minute)	5	12	32	39	45

13. After a crash on the interstate, cars begin to back up on the road. The rate that cars are backing up, in cars per minute, can be modeled by the continuous and differentiable function $C'(t)$, where t is measured in minutes. $C'(t)$ is increasing and concave down on $0 < t < 10$

(a) Using a trapezoidal approximation with the four subintervals indicated in the table above, approximate $\int_0^{10} C'(t)dt$. Using correct units, interpret the meaning of this answer in context of the problem.

(b) Is your answer from part (a) an over or under approximation? Give a reason for your answer.

(c) Estimate $C''(6)$. Include units of measure. Interpret this result.

(d) Can you guarantee a time r, $0 < r < 10$ when $C'(r) = 40$? Give a reason for your answer.

(e) Explain why there must be at least one time t, for $0 < t < 10$, such that $C''(t) = 5$.

t	0	3	6	7	8	10	12
$v(t)$	5	2	-3	-4	1	5	2

14. A particle is moving along the x axis for $0 \le t \le 12$ seconds. The velocity of the particle, $v(t)$, is differentiable and measured in meters per second. Selected values of $v(t)$ are given in the table above.

   (a) Use a midpoint Riemann sum with three subintervals to approximate $\int_0^{12} v(t)\,dt$. Using correct units, interpret the meaning of your answer in context of the problem.

   (b) Use a right Riemann sum with three subintervals to approximate $\int_0^{12} |v(t)|\,dt$. Using correct units, interpret the meaning of your answer in context of the problem.

   (c) Use a left Riemann sum with the four subintervals indicated in the table to approximate $\frac{1}{7}\int_3^{10} v(t)\,dt$. Using correct units, interpret the meaning of your answer in context of the problem.

   (d) Is there a time $c, 0 < c < 12$, such that $a(c) = -\frac{1}{4}$? Explain your reasoning.

15. A Fed Ex driver is delivering packages ordered from Amazon. The function $P'(t)$ represents the rate, in packages per hour, that the driver is delivering packages, where t is measured in hours. Using correct units, interpret the meaning of the following in context of this situation: $\int_0^6 P'(t)\,dt = 76$

16. A hot-air balloon is floating in the air at an altitude of $H(t)$, where H is measured in feet and t is measured in minutes since take off. Using correct units, interpret the following in context of this situation:

   $a.\ \int_{10}^{11} H'(t)dt = -23$ $\qquad$ $b.\ \frac{1}{20}\int_0^{20} H(t)dt = 115$

17. At time t hours, the amount of money in a certain cash register at the grocery store is changing at a rate of $D(t)$, measured in dollars per hour. Using correct units, interpret the following in context of this situation:

   $a.\ \int_4^7 D(t)dt = 5{,}210$ $\qquad$ $b.\ \frac{1}{2}\int_3^5 D(t)dt = -74$

18. A local mail carrier is delivering mail along her route. For $0 \le t \le 7$ hours, the rate that she is delivering packages is given by $M(t)$, measured in packages per hour. How many total packages does she deliver during the seven hours?

19. As a bowl of soup is removed from a microwave, the temperature of the soup cools down at a rate modeled by S' for $0 \le t \le 5$, where t is measured in minutes and $S'(t)$ is measured in degrees Fahrenheit per minute.
   a. What is the total change in temperature, in degrees Fahrenheit, of the soup during the five-minute interval?
   b. What is the average rate, in degree Fahrenheit per minute, that the temperature is changing over the first three minutes it begins to cool?

20. A bug moves along a straight path with velocity v, measured in inches per second over the interval $[0,8]$ seconds.
   a. What is the total distance the bug travels during the first five seconds?
   b. What is the displacement of the bug after the first 3 seconds?

21. The number of books inside a library changes as patrons check out and return books. At a local library, the rate that books are checked out is modeled by $L(t)$ while books are returned at the rate $E(t)$. $E(t)$ and $L(t)$ are both measured in books per hour and t is measured in hours since the library opens. The library is open for ten hours and has 24,547 books inside when it first opens.
   a. How many total books are checked out during the first 3 hours the library is open?
   b. How many books are in the library when it closes at the end of the day?

SOLUTIONS TO CH 6 REVIEW ASSIGNMENT

1. a) 9/4 b) ¼ c) 1.375 d) 9/4

2. 1.3; LRAM is an overestimate on this decreasing interval; RRAM is an underestimate on a decreasing interval; TRAP is an overestimate an a decreasing interval

3. a) 11.504 b) 7.536 4. a) 6.880 b) 7.264

5. a) The displacement of the particle in meters from t=1 to t=3 seconds

b) The acceleration of the particle in m/s^2 at t=4 seconds 6. $\int_1^3 4x^2\,dx$ or $\int_0^2 4(x^2+1)dx$

7. a) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\left(\frac{k}{n}\right)\right)^2\left(\frac{1}{n}\right)$ b) $\lim_{n\to\infty}\sum_{k=1}^{n}4\left(1+\left(k\frac{1}{n}\right)\right)^2\left(\frac{1}{n}\right)$ c) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\left(\frac{k}{n}\right)\right)^2\left(\frac{1}{n}\right)$ d) $\lim_{n\to\infty}\sum_{k=1}^{n}4\left(1+\left(k\frac{1}{n}\right)\right)^2\left(\frac{1}{n}\right)$

8. 1 9. $2x\tan(5x^2)$ 10. 1.751 11. 5

12. a) $\int_1^7 m(t)dt = M(7)-M(1) = 2.2-0.1 = 2.1$ inches b) Snow at $t=7$: $4+\int_1^7 s'(t)dt-\int_1^7 m(t)dt \approx 4+8.5-2.1 = 10.4$ inches

c) $m'(5)\approx\frac{m'(6)-m'(3)}{6-3}=\frac{0.25-0.8}{6-3}$ inches per hour per hour d) $y(t)=0.7+0.8(t-3)$. $M(4)\approx y(4)=0.7+0.8(4-3)$ = 1.5

13. a) $\int_0^{10} C'(t)dt \approx \frac{1}{2}(1)(5+12)+\frac{1}{2}(4)(12+32)+\frac{1}{2}(2)(32+39)+\frac{1}{2}(3)(39+45)\approx 293.5$ cars. 293.5 cars represents the total number of cars that were added to the backup of cars from t=0 to t=10 minutes

b) The answer from part (a) is an under approximation because it is a trapezoidal approximation on an increasing interval c) $C''(6)\approx\frac{C'(7)-C'(5)}{7-5}\approx 3.5\,\frac{\text{cars}/\text{min}}{\text{min}}$. At 6 minutes, the rate of cars backing up is INCREASING at a rate of 3.5 cars/min per minute. d) C'(r) is continuous on [0,10], C'(0)=5, C'(10)=45, by the Intermediate Value Theorem there must exist a value of r on (0,10) such that C'(r)=40 because 5<40<45 (or because C'(0)<C'(r)<C'(10))

e) Within (0, 10), C'(r) is continuous on [1, 5] and differentiable on (1, 5), C'(1)=12 and C'(5)=32, $\frac{32-12}{5-1}=5$, therefore by the Mean Value Theorem there is guaranteed to be a value, r, within (1, 5) (which is part of the larger interval (0, 10)) where C"(r) = 5.

14. a) $\int_0^{12} v(t)dt \approx 6(2)+(-4)(2)+(4)(5)\approx 24$ meters. The particle moves approximately 24 meters from t=0 to t=12 seconds. b) $\int_0^{12}|v(t)|\,dt \approx 6(3)+(1)(2)+(4)(2)\approx 26$. The total distance travelled by the particle on t=0 to t=12 seconds is approximately 26 meters. C) $\frac{1}{7}\int_3^{10} v(t)dt \approx \frac{1}{7}[3(2)+(1)(-3)+(1)(4)+(2)(1)]\approx\frac{1}{7}$. The average velocity of the particle from t=3 to t=10 seconds is $\frac{1}{7}$ meters per second. d) v(8)=1, v(12)=2, $\frac{v(12)-v(8)}{12-8}=\frac{1}{4}$. v(t) is differentiable on (0, 12) therefore it is differentiable on (8, 12) and continuous on [8, 12]. By the Mean Value Theorem there must exist a value of t=c on 8<c<12 where v'(c)=$\frac{1}{4}$.

15. The driver delivers 76 packages from time t=0 to t=6 hours.

16. a) The balloon experienced loss of altitude of 23 feet during time t=10 to t=11 minutes since take off.

b) The average altitude of the balloon from time t=1 to time t=20 minutes since take off is 115 feet.

17. a) \$5210 is the net change in the amount in the cash register from t=4 to t=7 hours b) The average rate that money is changing in the cash register from t=3 to t= 5 hours was -\$74 per hour

18. $\int_0^7 M(t)dt$ 19. a) $\int_0^5 S'(t)dt$ b) $\frac{1}{3}\int_0^3 S'(t)dt$ 20. a) $\int_0^5 |v(t)|dt$ b) $\int_0^3 v(t)dt$

21. a) $\int_0^3 L(t)dt$ b) $24{,}547+\int_0^{10} E(t)dt-\int_0^{10} L(t)d$

CHAPTER 6 FORMULA'S TO KNOW

AP CALC: DERIVATIVES	AP CALC: INTEGRALS
$\frac{d}{dx}(x^n) = nx^{n-1}dx$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\ln u) = \frac{1}{u} * \frac{du}{dx}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(a^u) = (\ln a)a^u \frac{du}{dx}$	$\int a^x dx = \frac{a^x}{\ln a} + C$
$\frac{d}{dx}(\log_a u) = \frac{1}{(\ln a)u} * \frac{du}{dx}$	$\int \frac{1}{x \ln a} dx = \log_a x + C$
$\frac{d}{dx}(\sin u) = (\cos u)\frac{du}{dx}$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\cos u) = -(\sin u)\frac{du}{dx}$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\tan u) = (\sec^2 u)\frac{du}{dx}$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\sec u) = (\sec u \tan u)\frac{du}{dx}$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}(\cot u) = -(\csc^2 u)\frac{du}{dx}$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}(\csc u) = -(\csc u \cot u)\frac{du}{dx}$	$\int \csc x \cot x dx = -\csc x + C$
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} * \frac{du}{dx}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} * \frac{du}{dx}$	
$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} * \frac{du}{dx}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2} * \frac{du}{dx}$	
$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{\lvert u\rvert\sqrt{u^2-1}} * \frac{du}{dx}$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}} dx = \sec^{-1} x + C$
$\frac{d}{dx}(\csc^{-1} u) = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}} * \frac{du}{dx}$	

- **Definition of the Derivative (both versions)**
- **Intermediate Value Theorem**
- **Mean Value Theorem for Derivatives**
- **Extreme Value Theorem**
- **Mean Value Theorem for Integrals**
- **Average Value of a Function**
- **First Fundamental Theorem of Calculus**
- **Second Fundamental Theorem of Calculus**
- **Total Area Vs Net Area**