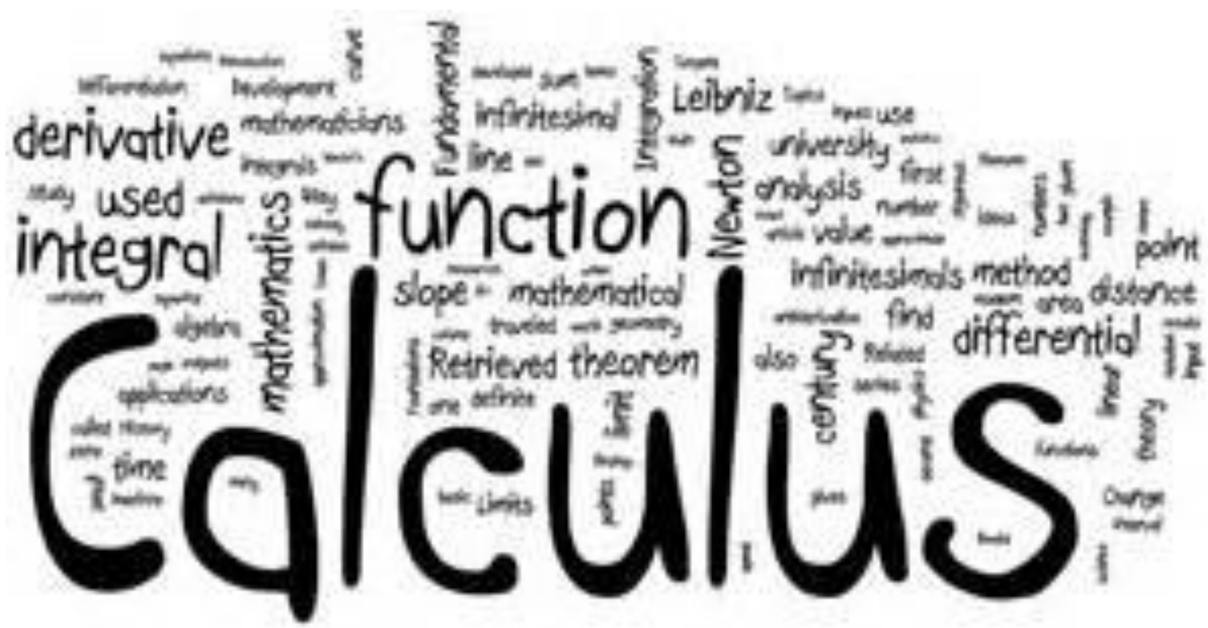
Regina Public Schools
Integral Calculus 30L
Spring 2016



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Course Description

Integral Calculus 30L is primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus 30 (AP) is a co-requisite for Integral Calculus 30L. Integral Calculus 30L is divided into 8 outcomes, as follows:

- Approximation methods
- Derivatives of inverse trigonometric functions
- Extended integration techniques
- Volumes of solids
- Non-geometric applications of integration
- Elementary single variable calculus theorems
- Differential equations and slope fields
- L'Hopital's rule

Rationale

Integral Calculus 30L is designed to challenge motivated and mathematically able students and to prepare them for the rigours of post-secondary mathematics. Students will explore more Integral Calculus topics than are studied in Calculus 30 and will thereby consolidate and strengthen their algebraic and calculus understandings and skills.

Schools now better recognize the diversity that exists amongst our students; interests and abilities vary tremendously and the most mathematically-inclined students are usually able to grasp concepts that are still two or more years away from them if they follow the traditional curriculum sequence in a lock-step manner. Many of our best students have been under-challenged even with Calculus 30. Every year many students complete their Calculus 30 in grade 9, 10 or 11 and are left with no further mathematics to study in high school. These same students often go to post-secondary institutions where they meet classmates who have studied not only Integral Calculus, but also statistics, linear algebra, and multivariate calculus while in high school. In a world that needs more people in STEM-related careers, it is important that we give our interested and able students the opportunities to develop the background necessary to reach their potential in math-dependent occupations.

Student Target Group

Integral Calculus 30L targets grade 11 and 12 students considering post-secondary programs or having an interest in STEM related professions and who are seeking opportunity for enriched and challenging mathematics content. Integral Calculus 30L incorporates opportunities for students to study a variety of concepts through authentic and relevant laboratory learning experiences. Student demonstrating an aptitude and interest in post-secondary mathematics study would find this course of benefit and a support to future study.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. Integral Calculus 30L contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to the following (Renewed Curricula: Understanding Outcomes, 2010):

- Lifelong Learners: Students who are engaged in constructing and applying their knowledge naturally build a positive disposition towards learning. Throughout their studies, students gain understandings, skills, and strategies to become more competent and confident learners.
- Sense of Self, Community, and Place: To learn, students need to interact with each other. Throughout their studies, students learn about themselves, others, and the world. The students use this knowledge to define who they are and to explore who they might become, to respond effectively with others, and to build community.
- Engaged Citizens: Throughout their studies, students are enabled to make a difference in their personal, peer, family, and community lives having developed a sense of agency and an ability to make a difference in their community and the world in which they live.

Cross Curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies are reflective of the Common Essential Learnings and are intended to be addressed in Integral Calculus 30L and include the following (Renewed Curricula: Understanding Outcomes, 2010):

- Developing Thinking:
  - Learners construct knowledge to make sense of the world around them. They develop understanding by building on what is already known. This key competency concerns the ability to make sense of information, experiences, and ideas through thinking contextually, critically, and creatively. The philosophy of learning across curricula is inquiry-based, and students are expected to use their thinking skills to explore a range of topics, issues, and themes.
  - In Integral Calculus 30 students will be engaged in the personal construction of mathematical knowledge. Inquiry and problem solving will challenge students to think critically and creatively. Moreover, students will experience mathematics in a variety of contexts, including a variety of occupations, everyday life, and research contexts. Students will consider questions such as: "How do you know

that . . . “, “Form a conclusion and justify it . . .”, “Is it reasonable to conclude that . . .”, “Based on the model you have developed, predict . . .”, “Explain your reasoning . . .” Students will work with the instructor, with each other, and independently to construct their Calculus understandings. Reflection, writing, and discussion are all helpful in assisting students in developing their thinking regarding Calculus.

- Developing Identity and Interdependence:
    - The ability to act autonomously in an interdependent world requires an awareness of the natural environment, of social and cultural expectations, and of the possibilities for individual and group accomplishments. It assumes the possession of a positive self-concept and the ability to live in harmony with others and with the natural and constructed worlds. Achieving this competency requires understanding, valuing, and caring for oneself; understanding, valuing, and respecting human diversity and human rights and responsibilities; and understanding and valuing social and environmental interdependence and sustainability. In turn, students to explore ideas and issues of identity, social responsibility, diversity, sustainability, and personal agency.
    - Students will be expected to develop their self-confidence; one’s ability to progress in mathematical understanding is based in large part on one’s mathematical self-confidence. The value placed on individual student’s ideas, strategies, and thoughts will support the development of personal and mathematical confidence. Students will be expected to accept responsibility for the consequences of their choices and actions; for example, failure to invest time in developing understanding results in lower than optimal understanding and in lower marks. A positive learning environment in which every student is expected to develop strong understanding of Calculus concepts combined with strong pedagogical choices that engage students in learning will support students in behaving respectfully towards themselves and others
- Developing Literacies:
    - Literacies provide many ways, including the use of various language systems and media, to interpret the world and express understanding of it. Literacies involve the evolution of interrelated skills, strategies, and understandings that facilitate an individual’s ability to participate fully and equitably in a variety of roles and contexts – school, home, and local and global communities. To achieve this competency requires developing skills, strategies, and understandings related to various literacies in order to explore and interpret the world and communicate meaning. All curricula require students to use different literacies effectively and contextually to represent ideas and understanding in multiple, flexible ways.
    - Integral Calculus 30 students will develop their fluency in explaining and justifying their mathematical ideas in the English language as well as the symbolic language of mathematics. The relationships amongst various representations of ideas will be explored more fully than in pre-calculus courses; for many students, this will be the first mathematics course in which they use more words than numbers or symbols. Students will continually be required to

justify or explain their answers; effective and clear communication will be emphasized. Students will be required to explore and explain almost all of the concepts in Integral Calculus 30 through graphical and pictorial representations, symbolic/formula-based representations, tabular/numeric representations, and verbal/word-based representations. Students' technological literacy will grow through daily use of advanced calculator functions (much more than the arithmetical operations that are by far the most common use of a calculator in most high school mathematics courses). Students will become comfortable with using graphing calculator technology to help them explore and answer a variety of problems.

- Developing Social Responsibility:
    - Social responsibility is how people positively contribute to their physical, social, and cultural environments. It requires the ability to participate with others in accomplishing shared or common goals. This competency is achieved through using moral reasoning processes, engaging in communitarian thinking and dialogue, and taking action to contribute to learners' physical, social, and cultural environments. In all curricula, students explore their social responsibility and work toward common goals to improve the lives of others and the natural and constructed worlds.
    - Students will have opportunities to share and consider ideas, and resolve conflicts between themselves and others. The learning environment will support respectful, independent, and interdependent behaviours. Ideally every student will feel empowered to help others in developing their understanding, while finding ways to seek help from others. Students will be encouraged to explore mathematics in social contexts where they both ask and answer questions of each other. Mathematics is a subject dependent on social construction of ideas. Through the study of Integral Calculus, students will learn to become reflective and contributing members of their communities. Different perspectives and approaches will be considered, assessed for contextual validity and efficiency, and strengthened.

Aims and Goals

The aim of Integral Calculus 30L is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

The goals of Integral Calculus 30L are as follows:

1. Logical Thinking: In Integral Calculus 30, students will develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems. Integral Calculus 30's topics are rich with possibilities for logical thinking in new

situations and problems that students have not encountered before, particularly in the area of integrals as accumulators.

In particular, the development, statement, proof, and application of theorems in Integral Calculus 30 will help prepare students for post-secondary mathematics. At present, one of the difficulties that students have in transitioning from high school mathematics to university mathematics is the almost total absence of formal proofs and statements of theorems in high school math in contrast to the emphasis on formal mathematics at the university level. Integral Calculus 30 would include such theorems as the Fundamental Theorem Calculus (both parts), the Mean Value Theorem, Rolle's Theorem, and the Intermediate Value Theorem.

2. Number Sense: In Integral Calculus 30, students will develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including graphic, symbolic, tabular, and verbal) of numbers, and apply this understanding to new situations and problems. For example, numerical approximations to the integral and linear approximations to functions allow students to develop their number sense while avoiding the "symbolic quagmire" that calculus often involves. Another area in which students' number sense will be enhanced is the study of relative growth rates of functions; here students must develop strong understandings of the seemingly simple ideas of "big," "little," "fast," and "slow."

3. Spatial Sense: In Integral Calculus 30, students will develop an understanding of two-dimensional and three-dimensional objects, and the relationships between geometrical shapes and objects and numbers and symbolic representations, and apply this understanding to new situations and problems. While almost every topic in Integral Calculus 30 involves graphic/geometric representations of the ideas, the use of the integral to calculate volumes will specifically address the development of students' three-dimensional abilities: rotating two-dimensional objects and creating objects with known cross-sections challenge students to visualize and imagine objects unlike those that they have encountered in other mathematics courses.

4. Mathematics as a Human Endeavour: In Integral Calculus 30, students will develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs. This course is rich with opportunities for students to see mathematics as a human endeavour because: a) the students who enroll in such a course are capable and motivated to develop their own mathematical ideas and concepts; b) many of the calculus results that are studied have

names attached to them making it obvious that Calculus has been unfolding through human endeavour over a period of history (e.g. L'Hopital's Rule, Riemann sums, Euler's method); and c) the rich applications of the integral to a variety of human problems will also allow students to see the usefulness of mathematics to all humans.

Infusion of First Nations, Metis, and Inuit Ways of Knowing

First Nations, Metis, and Inuit content, perspectives, and ways of knowing are to be integrated into all curricula and embedded within the outcomes and indicators for each curriculum respectively. All students benefit from knowledge about the First Nations, Métis, and Inuit peoples and it is through such knowledge that misconceptions and bias can be eliminated. To that end, when completing various assignments, projects, portfolio components, etc., students are encouraged to address First Nations, Métis, and Inuit content and perspectives into their work. For further information, see Diverse Voices: Selecting Equitable Resources for Indian and Métis Education (Saskatchewan Education, 1992)

Content in Integral Calculus 30L and resources and materials will endeavor to present positive images of Aboriginal people and will complement the beliefs and values of First Nations, Métis and Inuit peoples. Students will recognize the connection between physical, mental and spiritual health and the importance of health, specifically, within the first nation's community (i.e. ways to combat the high incidents of diabetes within First Nations community). And, In providing information about First Nations games and sporting activities, a connection to aboriginal ways of knowing and perspectives will be highlighted. The integral role of activities within a holistic framework for life and spirituality will be examined.

Mathematics is cultural and all ways of teaching mathematics (and not just traditional ways) are culturally-based. These understandings will support First Nations, Metis, and Inuit students' development of personal mathematical understandings and mathematical self-confidence through holistic and constructivist approaches to learning. Factors that impact the mathematical success of all students, including First Nations, Metis and Inuit students, include cultural contexts and pedagogy.

Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current teaching practices. Mathematics instruction which focuses on the individual parts of the whole understanding may be challenging for students who rely on whole contexts to support understanding. In Integral Calculus 30, continual reference needs to be made to how the individual small parts relate to the two "big ideas" of rates of change and accumulation of change.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Direct translation of these mathematical ideas between cultural groups is often impossible or at least problematic. Teachers need to support students in uncovering differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Pedagogical practices influence the success of all students, including First Nations, Metis, and Inuit students, in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their way of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology. It is used to translate earlier forms of thinking into modern-day understandings. Individually, and as a class, teachers and students need to explore the big ideas that are foundational to Integral Calculus 30L and investigate how those ideas relate to them personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities supports learning by providing a holistic focus.

Integral Calculus 30L, much more than traditional Calculus courses, will use real-world data sets and real-world contexts. For example, exercises on the integral as net change could include questions involving motion of a car (displacement, velocity, and acceleration), oil consumption, home electricity use, population density, oil flow, bagel sales, filling milk cartons, stretching springs, inflation rates, and centres of mass; some of the problems will present students with graphed data, some with tabular data, some with a formula, and yet others with only a verbal description. All ideas will be contextualized in Integral Calculus 30L, thus helping, not only First Nations, Metis, and Inuit students, but all students.

Big Ideas and Questions for Deeper Understanding

It is important that teachers and students learn within meaningful contexts that relate to their lives, communities, and world. Teachers and students need to identify big ideas and questions for deeper understanding central to the area of study.

Big ideas are at the core of the subject; they need to be uncovered. The big ideas at the core of a subject are arrived at, sometimes surprisingly slowly, via teacher-led inquiries and reflective work by students. Big ideas encompass concepts, broad or overarching themes, skills, attitudes, and habits of mind which help students make sense of and apply what they learn. A big idea can be thought of as providing a focusing conceptual 'lens' for study; breadth of meaning by connecting and organizing many facts, skills, and experiences; serving as the linchpin of understanding; ideas at the heart of expert understanding; great transfer value and applying to many other inquiries and issues across subject areas and over time and both in the curriculum and out of school (Renewed Curricula: Understanding Outcomes, 2010).

Questions for deeper understanding are used to initiate and guide the inquiry and give students direction for developing deep understandings about a topic or issue under study. It is essential to develop questions that are evoked by student interests, have potential for rich and deep learning, are compelling and able to assist students to grasp important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school (Renewed Curricula: Understanding Outcomes, 2010).

*Refer to Course Overview for big ideas and questions for deeper understanding specific to Integral Calculus 30L.

Outcomes and Indicators

The learning expected of students in Saskatchewan is defined by curriculum outcomes for each grade. As Saskatchewan students achieve the grade-specific outcomes identified in curricula, they will deepen their understanding of each area of study as a living field of knowledge. Outcomes define what a student is expected to know and be able to do at the end of the grade or secondary level course. Outcomes require that students develop a combination of factual, conceptual, procedural, and metacognitive knowledge and are developed based on current research to ensure coherence and rigor. Therefore, all curriculum outcomes are required. Indicators clarify the breadth and depth of each outcome and are examples of ways that students might be asked to demonstrate achievement of an outcome. They serve as examples of the type of evidence that teachers would accept to determine the extent to which students have achieved the desired learning results. When teachers are planning for instruction, they must be aware of the set of indicators to understand fully the breadth and depth of the outcome. Based on this understanding of the outcome, teachers may develop their own indicators that are responsive to their students' interests, lives, and prior learning. These teacher-developed indicators must maintain the intent of the outcome.

*Refer to Course Overviews for outcomes and indicators specific to Integral Calculus 30L.

Incorporation and Explanation of Various Core Curriculum Components and Initiatives

Common Essential Learnings

The Common Essential Learnings can be integrated into all aspects of planning and instruction. It is through using these learnings that students can translate thoughts into actions. Refer to Understanding the Common Essential Learnings: A Handbook for teachers (Saskatchewan Education, 1988). Some suggestions within this curriculum are:

<u>Communication</u>

- Use formal procedures required within the subject area
- Use language, vocabulary, and structure appropriate to audience and purpose

- Use a variety of resources to cover the depth and breadth of topic

Numeracy
- Read and interpret graphs, tables, and algebraic representations of functions
- Discuss how to estimate effectively to gain new insight
- Choose the most appropriate method of solution for a particular problem and/or task

Critical and Creative Thinking
- Recognize the difference between recognition/familiarity and comprehension/deep understanding
- Question an idea and/or solution for its accuracy and completeness
- Understanding the intellectual virtues needed for critical thinking and good scholarship such as perseverance and open mindedness

Technological Literacy
- Develop technological expertise using a variety of tools including a graphic calculator
- Understand the impact of technology on the study of mathematics
- Search for, generate, collate, and judge the reliability and usefulness of information

Personal and Social Development
- Demonstrate commitment to and responsibility for own learning
- Participate effectively in a variety of cooperative groups
- Develop understanding of concepts such as learned dependence and empowerment in order to go above and beyond expectation

Independent Learning
- Derive (ha ha) enjoyment from learning
- Connect background understanding to new knowledge
- Determine own learning needs and design a plan to meet them

Adaptive Dimension

It is a teachers' responsibility to make adjustments in approved educational programs to accommodate diversity in student learning needs. Meeting the needs of all students includes those practices the teacher undertakes to ensure curriculum, instruction and the learning environment are meaningful and appropriate for each student. (The Adaptive Dimension in Core Curriculum, Saskatchewan Education, Training and Employment, 1992.)The teacher should consult individual student PPP (Personal Program Plans) for specific direction on accommodation for individuals.

The resources included in Integral Calculus 30L will allow teachers the opportunity to do the following accommodations, when appropriate, to address the needs of students.
- Provide a visual outline of lessons on the board, overhead, or handout.
- Provide key visuals or graphic organizers for assignments as an alternative.

- Partner students to work with appropriate people or resources.
- Provide key vocabulary or reference notes.
- Assist students to recall prior knowledge before introducing new information.
- Use appropriate visual materials rather than print material to convey information.

When addressing the adaptive dimension to meet student learning and behavioural needs, teachers are encouraged to refer to Regina Public Schools Intervention First documentation and processes with a focus on the provision and documentation of Tier 1 Universal /School-wide/Classroom-wide Support.

Multicultural Education

Multicultural education, as integrated into Integral Calculus 30L is an interdisciplinary educational process which fosters a broad and comprehensive understanding and acceptance of one's own and others' culture and ethnicity in addition to fostering empathy, and constructive and harmonious relations among peoples of diverse cultures. It encourages learners of all ages to view different cultures as a source of learning and enrichment and stresses the acquisition of skills in analysis, communication and inter-group relations, which enables one to function effectively in varying cultural environments. Multiculturalism recognizes the diversity of the cultural differences which exist in society. It endorses a society in which individuals of all cultures are accepted and accorded respect. It encourages a positive acceptance of races, religions and cultures, and recognizes such diversity as healthy. For further information, see Multicultural Education (Saskatchewan Education, 1994).

The classroom environment must be structured so that students from all cultures feel empowered and expected to learn and so that all these students develop a respect for all cultures. The teacher should model a respect for diversity of cultures and ideas and should require that students similarly show respect for all others no matter what their background is. Encouraging students to work together in solving problems and developing understandings can facilitate mutual respect, especially when the teacher helps form the groups that will work together.

Integral Calculus 30 L is designed to be much more of a reformed Calculus course rather than a traditional Calculus course, and as such takes a broader less Eurocentric approach to mathematics. While traditional Calculus courses, such as Calculus 30, focus primarily on the Eurocentric abstract, symbolic, algebraic approach to calculus with its manipulation of variables, reform Calculus courses take a more holistic approach which values context and the concrete more highly. In reform Calculus, every concept is approached not just algebraically, but also numerically, visually, and verbally. This integration and variety of approaches allows students from a wider variety of cultural backgrounds to relate to the mathematics.

While the mathematical concepts in modern Calculus were largely developed by Europeans, these Europeans encompassed a variety of Eastern and Western cultures. Students should understand that no single cultural or ethnic group has had a monopoly on the development of mathematics.

Treaty Education

The Saskatchewan Ministry of Education is committed to providing the appropriate supports and programs that reflect and affirm the unique status of First Nations and Métis people – Treaty Education. Four Treaty Education goals have been identified as the basis for building understanding and nurturing appreciation. These goals are based upon the Treaty Essential Learnings and are intended to be addressed through various subject areas, including Integral Calculus 30L as able and appropriate, and include:

- Treaty Relationships: By the end of grade 12, students will understand that Treaty relationships are based on a deep understanding of peoples' identity which encompasses: languages, ceremonies, worldviews, and relationship to place and the land.
- Spirit and Intent of Treaties; By the end of grade 12, students will recognize that there is interconnectedness between thoughts and actions which is based on the implied and explicit intention of those actions. The spirit and intent of Treaties serve as guiding principles for all that we do, say, think, and feel.
- Historical Context of Treaties: By the end of grade 12, students will acknowledge that the social, cultural, economic, and political conditions of the past played and continue to play a significant role in both the Treaty reality of the present and the reality they have yet to shape.
- Treat Promises and Provisions: By the end of grade 12, students will appreciate that Treaties are sacred covenants between sovereign nations and are the foundational basis for meaningful relationships that perpetually foster the well-being of all people

While each of four Treaty Education goals are presented separately, these goals can only be understood when considered as parts of a whole. The outcomes and indicators at each grade level are designed to engage learners on a journey of inquiry and discovery. When meaningfully and thoughtfully incorporated into subject areas, Treaty Education moves beyond an idea to become actualized as a belief that benefits all learners. For further information, see Treaty Education Outcomes and Indicators (Saskatchewan Ministry of Education, 2013).

Saskatchewan and Canadian Content and Perspectives

Integral Calculus 30L encourages students to explore identity in this province and in Canada. It is important that students become familiar with their own heritage and surroundings. If they study Saskatchewan and Canadian culture students will recognize themselves, their environment, their concerns and their feelings expressed in many different ways. They will learn that both similarities and differences between various identities in Saskatchewan and Canada are cause for celebration.

Gender Equity

All course material for Integral Calculus 30L has been created with the concept of gender equity as a guiding principle. In all units efforts have been made to balance content and acknowledge the role of both genders as both participants as well as leaders. As well occupations related to the field of Exercise Science reach out to both men and woman. (see Incorporating Career Development Competencies)

Students of both genders must believe that success in mathematics is not related to one's gender. All students must be challenged and all students must be encouraged to develop the work habits which will ensure success. While the traditional North American mentality is that mathematical aptitude is primarily genetically determined, the teacher can do much to help students to understand that their success in understanding mathematics is strongly correlated with their work ethic. A focus on what a student does rather than on who a student is helps students of both genders achieve success in the mathematics classroom.

Not only must the instructor of Integral Calculus 30L send strong messages about the gender-neutrality of the course, but the selected textbook and other resources must employ gender-balanced examples, illustrations, and questions.

Resource-Based Learning

Resource-based instruction is an approach to learning in which students use a variety of types of resources to achieve foundational and related learning objectives and reflects a student-centered approach to instruction. Teachers are encouraged to assess their current resource collection, identifying those that continue to be useful, and to acquire new resources in order to provide students with a broad range of perspectives and information. For further information, see Resource-Based Learning Policy, Guidelines and Responsibilities for Saskatchewan Learning Resource Centers (Saskatchewan Education, 1987), and Selecting Fair and Equitable Learning Materials (Saskatchewan Education, 1991).

Since no single textbook is ever ideal for a particular class of unique students, the teacher ought to use a variety of resources, not restricted to textbooks. Several innovative textbooks are listed and briefly discussed in the annotated list of instructional materials at the end of this document. Teachers ought to consider the explanations and question types provided in a variety of textbooks when teaching any topic.

Since almost all Calculus topics can be explored graphically, the graphing calculator is a resource that every student should have ready access to. In the annotated list of instructional materials, most of the textbooks provide suggestions on the use of the graphing calculator and several calculator-specific resources are listed under technology.

Many interactive applets illustrating calculus concepts are now available on the internet. In particular, the applets which illustrate the construction of solids of revolution and solids of known cross-section can be particularly helpful for the many students that initially encounter difficulties in visualizing these three-dimensional objects. Even if teachers are unable to reserve computer lab time for their students during class, students can be encouraged or required to spend time exploring these applets on their own time either at home or at other non-class times when school computers are available.

The internet now provides a variety of resources which students can readily access. Teachers should provide students with a list of these. Websites such as Khan Academy provide instructional videos on almost all Integral Calculus 30 L topics. Other websites provide text explanations of calculus topics and a variety of assessment tools that students can use to check and develop their understanding.

*Refer to Instructional Materials for full list of resources for Integral Calculus 30L.

Career Development and Exploration

The integration of career development competencies across curricula Integral Calculus 30L and to connecting learning to life/work is part of a broad career development strategy designed to equip students with the skills required to achieve fulfillment in personal, social, and work roles through exposure to a career building process. The career development framework, as outlined by Blueprint for Life/Work Designs includes the continuous development of the following competencies. For further information, see Blueprint for Life/Work Designs.

Personal Management:

- Building and maintaining a positive self-image
- Interacting positively and effectively with others
- Changing and growing throughout one's life

Learning and Work Exploration:

- Participating in lifelong learning supportive of life/work goals
- Locating and effectively using life/work information
- Understanding the relationship between work and society/economy

Life/Work Building:

- Securing, creating, and maintaining work

- Making life/work enhancing decisions
- Maintaining balanced life and work goals
- Understanding the changing nature of life/work roles
- Understanding, engaging in, and managing one's own life/work building processes.

The classroom environment and activities must be structured so as to facilitate the acquisition and actualization of the career development competencies. A positive climate in which all people in the classroom demonstrate respect for each other and for their own learning will help build and maintain positive self-image in students. Large and small group discussions can help build understandings; these discussions aid students in learning to interact positively and effectively with others. Since Calculus is the study of change, data sets that examine the changes that occur in humans throughout their lives will help students understand their own potential for dynamic growth for the rest of their lives.

Students should be encouraged to set goals for themselves not only in terms of their achievement within the course, but also in terms of their work and study habits. Students should reflect and record their reflections on what their career goals are and how Integral Calculus 30 L can help them achieve those. Through exposure to a broad range of calculus applications, students will come to realize how calculus is used in a large variety of careers. Specific data sets dealing with careers, work, and society will help them understand the relationships among these aspects of their lives.

Examples of Instructional Approaches

Students learn best when they are active, exploring, questioning/searching for meaning, investigating/ experimenting, looking for connections/relationships/patterns, sharing/discussing with others and reflecting. Considering how students learn, the optimal conditions for learning and the learning skills needed to develop an effective program that focuses on improved student learning. In addition, by beginning with a topic of deep interest to students, they are more willing to engage in activities which reinforce and build their skills.

1. Improved student learning is enhanced when we understand how students learn. Take into consideration developmental stages, learning preferences, learning styles, and learning environment.

2. Create conditions for optimal learning by considering the full range of teaching and learning strategies. Teaching and learning strategies are instructional practices that:
    - involve a sequence of steps or a number of related concepts;
    - determine the approach a teacher may take to achieve learning objectives and meet diverse learner needs;
    - should be selected based on an understanding of how students learn.

Direct Instruction

Lecture:	an oral presentation of facts or principles during which the learner is responsible for taking appropriate notes
Demonstrations/modelling:	performing a skill or activity in order to show how to do it
Didactic Questions:	guiding students to predetermined learning through the use of lower order questions
Drill and Practice:	repetition of fundamental skills to enhance speed and accuracy of performance
Guides for Reading, Listening, and Viewing:	Structured formats intended to direct students to appropriate learning expectations in reading, listening, or viewing

Indirect Instruction

Problem Solving:	an organized process for solving a problem
Research:	gathering and interpreting data on a specific topic
Case Studies:	investigation of a specific event, situation, or person to develop an understanding of factors that can be generalized to other situations
Concept Formation:	an inductive thinking strategy in which students sort, classify, and/or group items, ideas, opinions, into categories to draw inferences, make generalizations, and develop concepts

Concept Attainment:	Clarifying a concept by providing positive and negative examples of that concept
Reflection:	process of thinking about and connecting ideas, experiences, and learning
Debate:	the presentation of opposing sides of an issue by two teams/individuals before an audience or judge

<u>Interactive Instruction</u>

Cooperative Learning:	a variety of interdependent learning structures where students learn in small heterogeneous groups
Jigsaw:	Students are divided into “home” groups. Each student in the group moves into a different expert group to gather information (provided by the teacher or through research) and then goes back to the home group to share that information
Think/Pair/Share:	Students begin thinking about a concept on their own, then work with a partner to share and discuss ideas
Snowballing:	pairs of students begin sharing ideas. After a few minutes, the pairs join with another pair to form a group of four to share ideas. The groups continue to combine to form groups of eight, then 16. New ideas are added and discussed
Numbered Heads:	Numbered heads is a structure whereby students number off, e.g., four in a group, and the teacher poses a problem and sets a time limit for each group to investigate. The teacher calls a number and the student with that number in each group responds.
Learning Circles:	small groups of students who discuss a common test, topic, or problem in order to deepen understanding
Brainstorming:	a group activity in which participants are encouraged to think uncritically about all possible ideas, approaches, or solutions
Role Playing:	assuming the role of another and acting out a situation to develop understanding and insights
Peer Coaching:	a structured situation where students teach and learn from each other
Experiential Learning:	A situation requiring a high level of active involvement in his/her own learning that is inductive, learner centred and activity oriented. These activities may include field trips, simulations, model building, analysing, drawing inferences or conclusions, providing reasons and evidence for conclusions, or reflecting on experiences in analysing, inferring, decision-making, and conclusions.

Independent Instruction

Independent Project:	A formal assignment on a topic related to the curriculum
Learning Centres:	A specially organized space containing specific resources and/or equipment
Learning Contracts:	A plan of instruction allowing students to proceed at their own rate in learning specified material.

Inquiry Instruction

Mini Inquiry:	Spontaneous inquiry for which students are provided the opportunity to ask questions, search for and find information relatively quickly, and satisfy curiosity
Curricular Inquiry:	Inquiry for which content and concepts are determined by provincial or locally developed outcomes.
Open Inquiry:	Inquiry for which students are provided the opportunity to select a topic of inquiry with teacher guidance

Project/Problem/Design-Based Learning (PBL/PBL/DBL):

Inquiry that results in the completion of an product, event, or presentation to an audience (project-based learning); define a problem and identify solutions (problem-based learning); or design and create an artifact that requires application and understanding (design-based learning)

Inquiry Continuum:

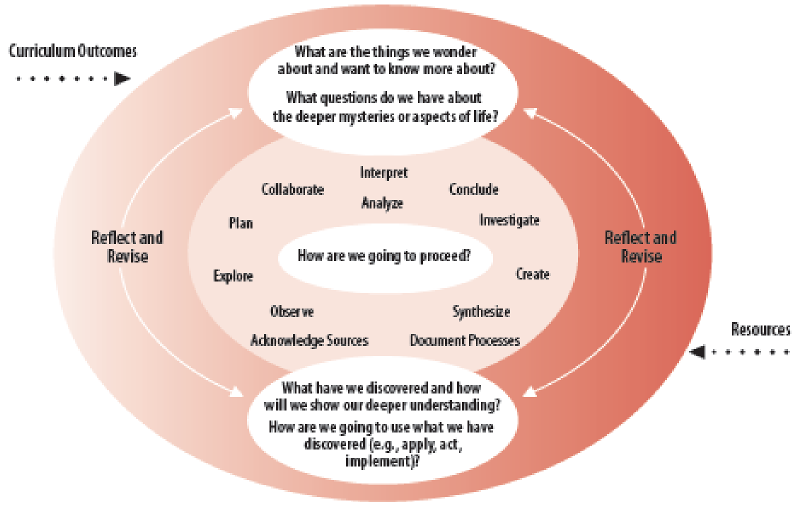
← Teacher Directed --- Collaborative --- Student Directed →

← Large Group --- Small Group --- Individual →

← Intra-disciplinary --- Inter-disciplinary →

← Mini --- Curricular --- Open (PBL/PBL/DBL)→

Constructing Understanding Through Inquiry



3. Help students develop effective learning skills.
   Consider:
   - The skills and knowledge required to participate in learning, e.g., working independently, self-assessment, setting goals and monitoring progress, adapting to change, inquiry skills;
   - The self-knowledge, personal and interpersonal skills to interact positively with others, e.g., self-management, getting along with others, social responsibility;
   - The skills and knowledge required to plan their present and future lives and to determine the learning required to implement the plan, e.g., self-assessment, exploring and obtaining information, awareness of opportunities.

Examples of Assessment and Evaluation Techniques

Assessment and evaluation are ongoing and serve different purposes at different times.

	Diagnostic Assessment	Formative		Summative Evaluation
		Assessment	Evaluation	
What?	- assessing what students know and are able to demonstrate prior to instruction	- assessing what students know and are able to do as they progress through the learning and practice opportunities	- evaluating what students know and are able to do at certain points during the process of learning and practising	- evaluating students' demonstration of what they know and are able to do at the end of the instruction
When?	- occurs before instruction begins	- is ongoing as students learn and practise	- occurs at one or more checkpoints throughout the process of learning and practising	- occurs at the end of the instructional unit, e.g., unit, course, and will not be judged again in the course
Why?	- helps determine starting points and helps the teacher program appropriately for individual students	- provides ongoing meaningful feedback to help students improve as the learning/ practice builds, becomes more complex and connects with other learning	- provides a snapshot of students' achievement, e.g., mark, level at specific points in the course before the final demonstration (summative evaluation)	- provides students with the opportunity to synthesize knowledge and skills and demonstrate their achievement

	Diagnostic	Formative		Summative Evaluation
How?	- assessment strategies to provide a holistic picture of the learning students have acquired in the past	- assessment strategies to provide opportunities for students to learn and practise	- strategies that are relevant to: a) the expected learning; b) the point students have progressed to in the learning process; c) The summative evaluation (demonstration) planned for the end of the instructional unit.	- strategies that: a) require students to synthesize and apply the key learnings; b) require students to demonstrate learning in new or unfamiliar context (but not new learning); c) present students with engaging, challenging problems; d) allow for individual student accountability.
Note	- information from diagnostic assessments must not count towards the final grade	- formative assessment may be taken into consideration in determining students' final grades	- formative evaluation may count towards students' final grades	- summative evaluation will always count towards students' final grades

Under each of these categories, there are various types of assessment strategies. Some examples have been listed above beside each assessment method.

One of the critical professional judgments teachers must make is to appropriately match the assessment strategy (ies) to the type(s) of learning being assessed. There are a wide variety of assessment strategies available to teachers. Assessment strategies are what the teacher will have the students doing to demonstrate their learning.

Specific Examples of Instructional Approaches and Corresponding Formative and Summative Assessment and Evaluation Techniques

Instruction

Integral Calculus 30 L should be taught using a variety of instructional approaches: direct instruction, indirect instruction, interactive instruction, independent instruction, and experiential instruction.

Direct instruction involving explicit teaching should frequently be used when introducing new vocabulary or words which are used in new or specific ways in the context of calculus (e.g. convergence, absolute, speed, distance, series, arc length, bounded, disk, washer, shell, initial value). Ultimately the way that students fully understand new calculus terminology is by hearing and reading new terms used and then by using these terms in their own speaking and writing, but initially these terms need to be clearly defined. Direct instruction is just a small initial step which makes it possible for students to become active participants in the development of calculus understandings.

While the emphasis of Integral Calculus 30 L is on calculus ideas rather than on calculus computations, there will be exercises requiring the instructional strategy of drill and practice. For example, while often students will be given the values of a derivative function for a variety of domain values or the values of integrals, often they will have to calculate these. The practice of calculating the values of derivatives and definite integrals by hand without the aid of any technology will help students understand how these are measures of rates of change or the accumulation of change.

An example of an indirect instructional strategy that is useful in understanding calculus is concept mapping. Because Integral Calculus 30 L emphasizes ideas rather than calculations, concept mapping will be particularly useful in helping students visualize and understand the relationships amongst disparate ideas. Memorizing of definitions, techniques, or theorems is either insufficient or near impossible without the understanding of the relationships amongst the terms, techniques, and theorems; concept mapping will help students understand those relationships.

Students should learn indirectly through the problems they solve. By analyzing, representing, and describing a given problem in a variety of ways as they seek solutions, students will come to understand the limitations, strengths and weaknesses of the different representations, analyses, and verbal descriptions.

Another indirect instructional strategy which will be used throughout the course is concept attainment. For example, when presented with a mathematical problem, students will need to determine if the question can answered using differentiation or integration, whether the question is asking for an instantaneous rate of change, an average rate of change, a rate of a rate of change, or an accumulation of change.

Integral Calculus 30 L offers opportunity to incorporate pair discussions and larger group discussions. The rich ideas of calculus can be expressed in a variety of ways and students benefit from hearing descriptions by a variety of people. Only from examining a variety of ways of expressing calculus ideas can students develop an appreciation for the necessity of precision when expressing mathematical ideas.

Independent study instructional strategies that will be employed in the class include short reading assignments and individual projects.

Assessment and Evaluation

The majority of the assessment that occurs in a mathematics classroom is formative rather than summative. Students need to be reminded that whenever the instructor or a student is working through a problem with the class, that each student must each be thinking ahead to what should happen next and then, when it happens, they should assess their predictions. Each time a student works through an assignment, the student must reflect on and assess how well they understand the concepts and techniques with which they are working. Students need to be encouraged to write notes to themselves assessing their understanding for future reference as they review material later prior to summative assessments.

All assessment should enhance learning and thus is “assessment as learning.” Student skills and understanding are enhanced even on a summative assessment such as a final examination, since students have opportunities to practice, apply and enhance their understandings as they work through calculus problems posed to them.

In Integral Calculus 30L, the various types of assessments should flow from the learning tasks and provide direct and indirect feedback to the students regarding their progress in attaining the desired learning as well as opportunities for the students to set and assess personal learning goals related to the content of Integral Calculus 30L.

Types of Assessment and Evaluation

1. Any question posed to the class whether orally on in writing: Students need to learn that even if their responses to such questions are not written (whether these responses are their unexpressed thoughts or spoken answers), they can still assess their responses. Students must understand that their thoughts are more important than their pencil scratchings; the pencil scratchings are merely evidence of their thoughts. Students that

respond to orally posed questions and assess their own responses develop understandings much more quickly than students than merely wait to hear answers from the teacher or other students.

2. Regular individual assignments: which are strictly for formative assessment allow students to explore and make errors without social, psychological, or grade penalty. Students must be encouraged to seek assistance from each other or the teacher when they encounter difficulty in understanding a problem or its potential solutions. Questions in these assignments should be carefully sequenced so as to gradually develop student understandings and skills. The questions should, as much as possible, include a variety of representations of the mathematical ideas involves: verbal, graphical, numeric, and symbolic. As often as possible, a few questions of types which are new to students should be included, so that students are able to learn to take risks in a consequence-free environment.

3. Open-book and closed-book assignments, quizzes and tests: Students should be given opportunities to demonstrate what they can do both with the assistance of a textbook, notes, and formula sheet and without such assistance. In their lives, they will encounter mathematical problems in situation where they will and will not have ready access to such aids.

4. Calculator-active and calculator-inactive assignments, quizzes and tests: Students should be given opportunities to demonstrate their mathematical understandings both with and without the aid of a graphing calculator. Again, in their lives, they will need to respond to mathematical situations both when they have and do not have ready access to a calculator. While a calculator is a valuable tool and should often be used for such mundane tasks as evaluating a derivative at a point, or evaluating a definite integral on an interval, students need to be able to demonstrate their ability to perform these tasks without a calculator.

5. Individual and partner assessments, quizzes, and tests: allow students to develop both independent and collaborative learning skills.

Course Overview

Big Ideas and Questions for Deep Understanding

Integral Calculus 30L	
Big Ideas	Essential Questions
- Approximation methods - Derivatives of inverse trigonometric functions - Extended integration techniques - Volumes of solids - Non-geometric applications of integration - Elementary single variable calculus theorems - Differential equations and slope fields - L'Hopital's rule	- How can you calculate the areas under a curve? - What is rate of change? How is it used? - What is an inverse? How does it apply to trigonometric functions? - How can you determine the integral of a function without substitute of by sight? - What happens when you make a graph 3-dimentional? - What is deductive reasoning? When studying a theorem - what is true and how do you know it to be true? - What is a differential equation? What can it tell you? - What is an indeterminate form?

Outcomes	Indicators
1 Demonstrate understanding of approximation methods in calculus including: • **tangent line approximations** • **estimations of rates of change** • **Riemann sums** • **trapezoidal approximations**	a. Graph and compare any function and its tangent line over large and small intervals of the domain of the function. b. Determine an approximation for a function’s value using a line tangent to the function’s curve when given a domain value and the function. c. Compare the approximation found using a tangent line with the actual value of the function. d. Analyze situations in which a tangent line approximation is useful and is reasonably accurate. e. Approximate or estimate the rate of change (i.e. derivative) at a particular domain value of a quantity when given a table of values or a graph (and no algebraic rule). f. Compare various approximations of the rate of change with the actual instantaneous rate of change computed using the derivative of a function. g. Estimate the area under a curve using right, left, and midpoint Riemann sums with both equal and unequal subintervals when given the curve’s equation, a graph of a curve, or a list of values on the curve. h. Estimate the area under a curve using a trapezoidal approximation with both equal and unequal subintervals when given the curve’s equation, a graph of a curve, or a list of values on the curve . i. Develop, generalize and explain rules for determining when right sums, left sums, midpoint sums, and trapezoidal sums will under-approximate or over-approximate the area under a curve based on whether the curve is increasing, decreasing, concave-up, or concave-down.

2 Demonstrate an understanding of the derivatives of the tangent, cotangent, cosecant, secant, inverse sine, inverse cosine, inverse tangent, and general inverse functions.	a. Develop, generalize, explain, and apply rules for finding the derivatives of the tangent, cotangent, cosecant, and secant functions using previously developed rules for the sine and cosine functions. b. Graph and compare the tangent, cotangent, cosecant, and secant functions and their derivative functions with the aid of technology. c. Determine the derivatives of functions which include tangent, cotangent, cosecant, and secant expressions. d. Apply the derivatives of the tangent, cotangent, cosecant, and secant functions to solve a variety of problems including, but not limited to, optimization, related rates, and motion problems. e. Graph, analyze, and describe the inverse sine, inverse cosine, and inverse tangent functions, including, but not limited to, identification of the functions' domains, ranges, monotonicity, and concavity. f. Determine, with and without technology, values of the primary inverse trigonometric functions and expressions involving the primary inverse trigonometric functions. g. Develop, generalize, explain, and apply rules for finding the derivatives of the inverse sine, inverse cosine, and inverse tangent functions using implicit differentiation. h. Graph and compare inverse sine, inverse cosine, and inverse tangent functions with their derivative functions with the aid of technology. i. Determine the derivatives of functions which include inverse trigonometric expressions. j. Apply the derivatives of the inverse sine, inverse cosine, and inverse tangent functions to solve a variety of problems including, but not limited to, optimization, related rates, and motion problems. k. Develop, explain, and apply strategies for finding the derivative of the inverse of a function when given an equation for the original function. l. Determine the value of the derivative of the inverse of a function when given the equation and/or values of the original function or its derivative.

3 Extend understanding of integral properties and integration techniques including:

- **Integration by parts**
- **Integration by partial fractions**
- **Integration by trigonometric substitution**

a. Apply and graphically illustrate the following integral properties to simplify, evaluate and compare integrals:

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^a f(x)dx = 0$
- $\int_a^b kf(x)dx = k\int_a^b f(x)dx =$
- $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
- $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$
- Minimum of f·(b-a) $\leq \int_a^b f(x)dx$ $\leq$ maximum of f•(b-a)
- If f(x) $\geq$g(x) on [a,b], then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$

b. Develop, generalize, verify, explain and apply strategies for integrating using parts, partial fractions and trigonometric substitution.

c. Identify and correct errors in the integration of an expression using integration by parts, partial fractions and trigonometric substitution.

d. Apply integration by parts, integration by partial fractions, and integration by trigonometric substitution to solve problems involving areas under a function and above the horizontal axis, areas trapped between a curve and the horizontal axis, areas between two curves on a given interval, and areas trapped by two intersecting curves.

e. Develop, generalize, verify, explain and apply strategies for integrating expressions involving the tangent, cotangent, cosecant, secant, and primary inverse trigonometric functions.

f. Critique statements such as: “All expressions can be integrated.”

g. Express an integral as the limit of a sum and vice versa.

4 Demonstrate understanding of finding volumes of solids of revolution and solids of known cross-section.

a. Develop, generalize, verify, explain, and apply strategies (including, but not limited to, the disc and washer methods) for determining (both with and without technology) the volume of a solid formed by revolving a two-dimensional region around:
   - the horizontal axis
   - any horizontal line which does not intersect the interior of the region
   - the vertical axis
   - any vertical line which does not intersect the interior of the region.

b. Explain when it is appropriate to use the disc method and when it is appropriate to use the washer method.

c. Develop, generalize, verify, explain, and apply strategies for determining the volume of a solid with a given base and known cross-sections, including, but not limited to cross-sections which are:
   - semi-circular
   - square
   - rectangular with a given ratio between the length and width
   - right triangles with a given ratio between the base and height of the triangles
   - equilateral triangles

d. Estimate the volumes of solids of revolution by comparing them to simpler solids and by using a finite number of discs or washers.

e. Critique statements such as: "The volume of any solid can be found by integrating."

f. Determine the volume of a real-world object (such as a pond when given parameters for the surface area and depth of the pond).

g. Solve contextual problems that require the calculation of volume using integration techniques.

h. Verify or develop standard volume formulas for spheres, cones, and cylinders using integration techniques

5 Demonstrate understanding of non-geometric applications of integration.	a. Explain the relationships amongst the following quantities and how one can determine one from any of the others: distance, displacement, speed, velocity, and acceleration. b. Solve situational questions involving one-dimensional motion with both scalar and vector quantities where integration (and possibly differentiation) is required, including but not limited to questions involving: • whether speed, velocity, and/or acceleration is/are increasing or decreasing at particular moments in time • maximum and/or minimum distances, displacements, speeds, velocities, and acceleration • initial conditions c. Solve situational questions involving the integral of a function which is a rate of change. (e.g. Find the net change in population over a time interval when given a function representing the rate of change of the population; or find the oil consumption of a country over a time period when given a function representing the rate of oil consumption over that time period.) d. Solve non-situational and situational questions concerning the average value of a function over an interval when given an equation for the function. e. Estimate the average value of a function using integration approximation methods. f. Explain, using concrete examples, illustrated verbally, symbolically, and graphically, the difference between the average rate of change of a function and the average value of a function. g. Critique statements such as: “If we can define a function symbolically, then we can calculate its average value.”

6 Demonstrate an understanding of the basic theorems of elementary single-variable calculus including the: • **Extreme Value Theorem** • **Intermediate Value Theorem** • **Mean Value Theorem** • **Rolle's Theorem** • **Fundamental Theorem of Calculus (both parts)**	a. Identify the conditions necessary for the application of each of the five theorems. b. Prove each of the five theorems. c. Identify what each of the first four theorems guarantees the existence of. d. Illustrate each of the five theorems graphically and numerically. e. Identify the implications of each of the five theorems for a particular function on a particular interval. f. Prove the existence of a phenomenon in situational questions using one of the first four theorems. (e.g. Prove that a drive must have exceeded a speed limit at least once given the times and places that they drive began and ended.) g. Apply the Fundamental Theorem of Calculus, Part 1 to determine the derivative of an integral where one or both limits of integration are variable expressions. h. Analyze functions of the form $g(x) = \int_a^x f(t)dt$ when given either a graph of f(t) or an equation for f(t), by determining features of g(x), including but not limited to: • function values g(x) at given values of the domain • values of the derivative of g(x) at given values of the domain • values of the second derivative of g(x) at given values of the domain • intervals of increase and decrease of g(x) • local or global maximum and minimum values of g(x) • concavity of the graph of g(x) • points of inflection of g(x) i. Determine the net change in the antiderivative of a function using the Fundamental Theorem of Calculus, Part 2. j. Critique statements such as: "Each of the five theorems is a precise statement of common sense."

7 Demonstrate understanding of differential equations and their graphic representations in slope fields.	a. Create a slope field for a differential equation in which dy/dx is isolated. b. Interpret a slope field of a differential equation. c. Explain the relationship between the graph of a function and the graph of its slope field. d. Solve simple differential equations in which the derivative is isolated. e. Solve simple differential equations subject to a given initial condition. f. Solve separable differential equations by separating the variables and integrating. g. Solve separable differential equations subject to a given initial condition. h. Determine and justify the domain and range of a differential equation. i. Model exponential growth using equations of the form $y' = ky$. j. Apply separable differential equations to solve growth problems.
8 Demonstrate understanding of Relative Growth Rates and L'Hopital's Rule.	a. Compare the growth rates of two functions (including polynomial functions, exponential functions, logarithmic functions, and trigonometric functions) using the graphs of the functions, tables of values of the functions, or L'Hopital's Rule. b. Identify the conditions necessary for the application of L'Hopital's Rule in evaluating the limit of a rational function. c. Apply L'Hopital's Rule to evaluate the limits of appropriate functions.

Instructional Materials

Key Resources

Finney, Ross L., Franklin D. Demana, Bert K. Waits and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*. 3rd ed. Boston: Pearson/Prentice Hall, 2007. [This textbook is used in many high schools as well as in many colleges. It incorporates graphing calculator technology and is much more progressive in its questions than traditional calculus textbooks. There are a wide variety of exercises and examples that approach concepts not only through the traditional algebraic lens, but also through graphical and numerical perspectives. The text covers all of the topics in Integral Calculus 30 in a fairly traditional order. Students may often be left wanting more examples than this text provides, though.]

Hughes-Hallett, Deborah, et al. *Calculus—Single Variable.* 4th ed. New York: Wiley & Sons, 2005. [This text is used primarily with college students in the better colleges in the US. It was developed at Harvard and is the original "reform calculus" text. Its approach is very conceptual and it is probably not appropriate for most Saskatchewan high school Calculus classes, but it has excellent thought-provoking questions and would serve as a good teachers' resource.]

Ostebee, Arnold, and Paul Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed. Boston: Houghton Mifflin, 2002. [This reform text has a wonderful informal writing style with wit that is appreciated by bright high school students. The problems are thought-provoking and innovative. The distributor of this text changes often. Try Key College Press.]

Stewart, James. Single Variable Calculus: Early Transcendentals. 5th ed. Belmont, CA: Thomson-Brooks/Cole, 2003. [Stewart is the Canadian author of the world's best-selling Calculus textbook, often called the "violin book" after the cover art. This text is traditional but thoughtful, well-organized, and a source of excellent traditional exercises. Universities that use the text usually prefer that students not use this text in high school. Stewart has authored other Calculus texts including a reform text and a text designed for high school students.]

Additional Resources

Foerster, Paul A. *Calculus Explorations.* Emeryville, Calif.: Key Curriculum Press, 1997. [Foerster has also written a good textbook which is ideally suited to high school students. He has taught high school calculus for longer than almost anyone in the world, so he knows what works with students.]

Kamischke, Ellen. *A Watched Cup Never Cools: Lab Activities for Calculus and Precalculus.* Emeryville, Calif.: Key Curriculum Press, 1999. [This is the classic book of lab activities for Calculus.]

Online/Technology Resources

AP Central (apcentral.collegeboard.com) [This is the College Board site, and it has a plethora of helpful free resources for high school Calculus teachers, including Mark Howell's *Teachers' Guide,* worksheets, an electronic discussion group for high school Calculus teachers, a huge list of reviews for Calculus resources, articles on a variety of Calculus topics, and very rich free response questions from old exams.]

Gough, Sam, et al. *Work Smarter Not Harder—Calculus Labs for the TI-82 and TI-83* (also available for the TI-83 Plus) Andover, Mass.: Venture Publishing, n.d. www.vent-pub.com [Each book contains a disk with wonderful calculator programs.]

Husch, Lawrence S. Visual Calculus Web site. Mathematics Department, University of Tennessee, Knoxville. archives.math.utk.edu/visual.calculus/ [Visit this site for a collection of excellent modules that can be used by teachers and students.]

Parris, Richard. *Winplot*. Phillips Exeter Academy. math.exeter.edu/rparris/winplot.html [This is a free general-purpose plotting utility to draw and animate curves and surfaces. It is effective for enabling students to see and manipulate solids of revolution.]

Texas Instruments Inc. education.ti.com [Any of the links on this site will lead you to a variety of calculus activities and resources.]

Texas Instruments Inc. *TI InterActive!* CD-ROM. [The program contains a word processor with an integrated mathematics system, TI graphing calculator functionality, and an integrated Web browser.]

Weeks, Audrey. *Calculus in Motion.* CD-ROM. Burbank, Calif.: Calculus in Motion, 2005. [This CD has great calculus animations for *Geometer's Sketchpad* v4. www.calculusinmotion.com]

Evaluation of the Locally Developed Course of Study

Following the completion of this Locally Developed Course, instructors will complete and submit the following questionnaire within two weeks of completing the course. Completed questionnaires can be faxed to:

Supervisor of Instruction
Regina Public Schools
Phone (306) 523-3136
Fax (306) 523-3031

1. Enrolment
   a) How many students enrolled in this course?
   b) How many students successfully completed this course?
   c) Which semester did you offer this course in?
2. Reflection
   a) What successes were experienced in the teaching and learning of this course? Explain.
   b) What challenges were experienced in the teaching and learning of this course? Explain.
   Be sure to reference
   - Learning outcomes
   - Core curricular components and initiatives
   - Career development competencies
   - Instructional approaches
   - Assessment and evaluation techniques
   - Instructional materials

3) Interpretation
   a) How might successes identified be enhanced? What supports might be required? Explain.
   b) How might challenges identified be overcome? What supports might be required? Explain.
   c) What revisions, additions, deletions, would you recommend be made to this course as currently developed? Explain