

MIDTERM REVIEW

A. Find each limit

1.  $\lim_{x \rightarrow 3} \frac{x-3}{x^2 - 4x + 3}$

2.  $\lim_{x \rightarrow 5} (x^2 + 2x - 3)$

10.  $\lim_{x \rightarrow 4} \frac{x+5}{x-4}$

11.  $\lim_{x \rightarrow 3} \frac{x+2}{(x-3)^2}$

3.  $\lim_{x \rightarrow 1} \frac{x^4 - 5x^2 + 1}{x + 2}$

4.  $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

12.  $\lim_{x \rightarrow 2} \frac{x^2 - 5x}{x^2 - 4}$

13.  $\lim_{x \rightarrow \infty} \frac{1}{x}$

5.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 3x + 2}$

6.  $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$

14.  $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x}{3x^2 + 2}$

15.  $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x - 3x^3}$

7.  $\lim_{x \rightarrow 0} \sqrt{x}$

8.  $\lim_{x \rightarrow 0^+} \sqrt{x}$

16.  $\lim_{x \rightarrow \infty} \frac{4x^4 + 5}{8 - 3x^3}$

17.  $\lim_{x \rightarrow -\infty} \frac{5x}{\sqrt{x^2 + 4}}$

9.  $f(x) = \begin{cases} -x-2 & \text{if } x \leq -1 \\ x & \text{if } -1 \leq x < 1 \\ x^2 - 2x & \text{if } x \geq 1 \end{cases}$

Find (a)  $\lim_{x \rightarrow -1} f(x)$

(b)  $\lim_{x \rightarrow 1} f(x)$

B. Find each derivative –  
simplify as much as possible.

1.  $y = 8x^3$

2.  $f(x) = 6x^{\frac{8}{3}}$

3.  $y = 2x^4 + \sqrt{x}$

4.  $y = 3x\sqrt{x}$

5.  $f(x) = \sqrt{x}(2 - 3x)$

6.  $f(x) = (2x^3 + 5)(3x^2 - x)$

7.  $y = \frac{x^2 + 2x - 3}{x^3 + 1}$

8.  $y = \frac{\sqrt{x}}{1 + 2x}$

9.  $y = (x^2 - x + 2)^8$

10.  $y = \frac{1}{\sqrt[3]{1 - x^4}}$

11.  $y = \left(\frac{2x-1}{x+2}\right)^6$

12.  $f(x) = (x^2 + 1)^3 (2 - 3x)^4$

13.  $x^2 + y^2 = 25$

14.  $2x^5 + x^4 y + y^5 = 36$

15.  $x^2 - x^3 y^2 - y^3 = 13$

### C. Slope

1. Find the slope of the tangent line to  $y = 2x - x^2$  at  $(-1, -3)$

2. Find the slope of the tangent line at the point given

(a)  $y = (1-2x)(3x-4)$  at  $x = 2$

(b)  $y = x^4(4x^3 + 2)$  at  $x = -1$

(c)  $y^5 + x^2y^3 = 10$  at  $(-3, 1)$

### D. Find the Equation of the Tangent Line

1.  $y = (x^2 - 3)^8$  at  $(2, 1)$

2.  $y = x + \frac{6}{x}$  at  $(2, 5)$

### E. Point on a Curve

1. At what point on the curve  $y = x^4 - 25x + 2$  is the tangent line parallel to the line  $7x - y = 2$

2. Find the points on the curve  $y = \frac{x}{x-1}$  where the tangent line is parallel to the line  $x + 4y = 1$

3. Find the points on the curve  $y = 2 - \frac{1}{x}$  where the tangent is perpendicular to the line  $y + 4x = 1$

4. Find the equation of the line through  $(2, 1)$  that is tangent to the curve  $y = x^2 - 2$

### F. Sketch the graphs

1.  $y = x^4 - 8x^2$

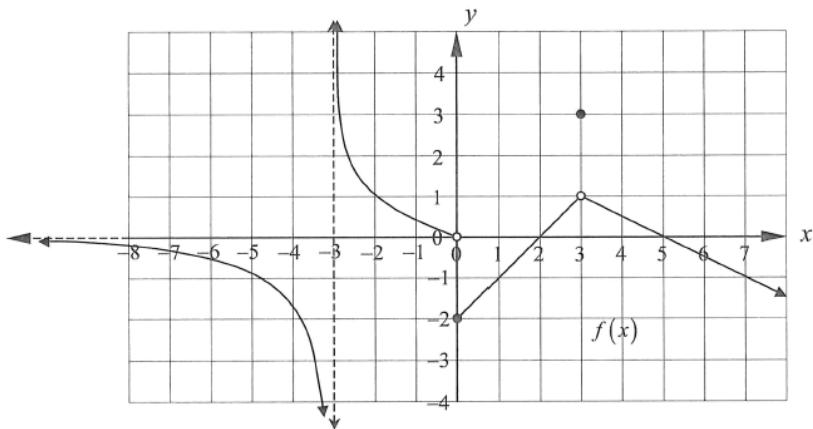
2.  $y = x^3 + 4x$

3.  $y = 3x^{\frac{1}{3}} + 9x^{\frac{4}{3}}$

4.  $y = \frac{4x}{x^2 - 4}$

5.  $y = \frac{x^2 - 5x + 4}{x^2 + 4}$

## G: Continuity



2. There are three conditions that a function must satisfy in order to be continuous at a point. In light of these conditions, explain why function  $f(x)$ , above, is not continuous at:
  - (a)  $x = -3$
  - (b)  $x = 0$
  - (c)  $x = 3$
3. Identify the type of discontinuity each function has at  $x = 1$ . Explain.
  - (a)  $f(x) = \frac{\sin(\pi x^2)}{x^2 - 1}$
  - (b)  $f(x) = \frac{x-1}{x^2 - 1}$
  - (c)  $f(x) = \frac{|x-1|}{x-1}$
4. Define  $f(3)$  so that  $f(x) = \frac{x^2 - 4x + 3}{x-3}$  will be continuous at  $x = 3$ .
5. Find the value(s) of  $x$ , if any, at which the function  $f(x) = \frac{x|x-4|}{x^3 - 2x^2 - 8x}$  is discontinuous. Explain your reasoning. Classify each discontinuity.

## Calculus 30

## Final Review Answers

### A. Limits

- |                   |                   |                    |                   |                    |               |
|-------------------|-------------------|--------------------|-------------------|--------------------|---------------|
| 1. $\frac{1}{2}$  | 2. 32             | 3. -1              | 4. 8              | 5. 12              | 6. 4          |
| 7. does not exist | 8. 0              | 9. a) -1           | b) does not exist | 10. does not exist |               |
| 11. $\infty$      | 12. $\infty$      | 13. 0              | 14. $\frac{2}{3}$ | 15. 0              | 16. $-\infty$ |
| 17. -5            | 18. $\frac{1}{4}$ | 19. does not exist |                   |                    |               |

### B. Derivatives

1.  $24x^2$
2.  $16x^{\frac{5}{3}}$
3.  $8x^3 + \frac{1}{2\sqrt{x}}$
4.  $\frac{9}{2}x^{\frac{1}{2}}$
5.  $x^{\frac{-1}{2}} - \frac{9}{2}x^{\frac{1}{2}}$
6.  $6x^2(3x^2 - x) + (6x - 1)(2x^3 + 5)$  or  $18x^4 - 6x^3 + (6x - 1)(2x^3 + 5)$
7.  $\frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$
8.  $\frac{1-2x}{2\sqrt{x}(1+2x)^2}$
9.  $(16x-8)(x^2-x+2)^7$
10.  $\frac{4}{3}x^3(1-x^4)^{-\frac{4}{3}}$
11.  $\frac{30(2x-1)^5}{(x+2)^7}$
12.  $-6(x^2+1)^2(2-3x)^3(5x^2-2x+2)$

$$13. \frac{-x}{y}$$

$$14. \frac{-10x^4 - 4x^3 y}{x^4 + 5y^4}$$

$$15. \frac{3x^2 y^2 - 2x}{2x^3 y - 3y^2}$$

**C. Slope**

1. 4

2. a) -13      b) 20

c)  $\frac{3}{16}$

**D. Find Equation of Tangent Line**

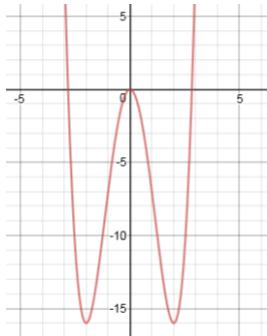
1.  $y = 32x - 63$       2.  $y = \frac{-1}{2}x + 6$

**E. Points on a Curve**

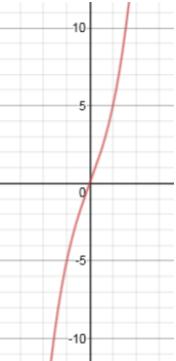
1. (2, -32)      2.  $\left(3, \frac{3}{2}\right)$  and  $\left(-1, \frac{1}{2}\right)$       3.  $\left(2, \frac{3}{2}\right)$  and  $\left(-2, \frac{5}{2}\right)$       4.  $y = 6x - 11$  and  $y = 2x - 3$

**F. Graphs**

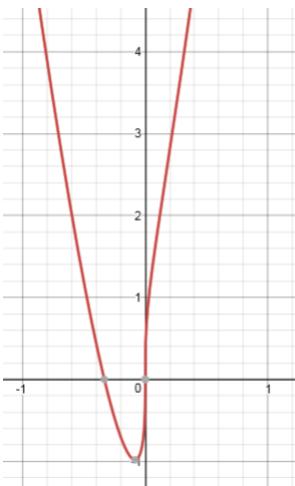
1. Max (0, 0), Min (2, 16), (-2, -16), Inflection Points (-1.155, -8.89), Intercepts: (0,0), (2.828, 0), (-2.828, 0)



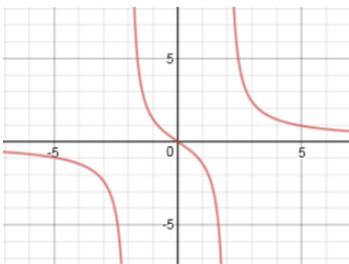
2. No Max/Min points, Inflection Point (0, 0), intercepts: (0,0)



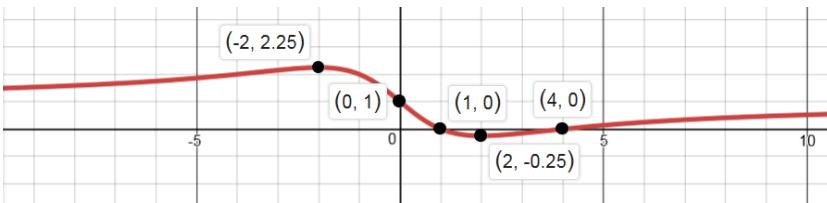
3. Min : (-1/12, -0.98), IP : (0,0), (1/6, 2.48), y-int : (0,0), x-int : (0,0), (-1/3, 0)



4. No extrema, IP : (0,0), intercepts : (0,0), VA:  $x = 2, x = -2$ , HA:  $y=0$



5. Max:  $(-2, 2.25)$ , Min :  $(2, -0.25)$ , IP :  $(0,1), (2\sqrt{3}, -0.08), (-2\sqrt{3}, 2.08)$  Intercepts :  $(1,0), (4,0), (0,1)$ , HA:  $y=1$



## G: Continuity

~~(k) 1 (l) 0 (m) 0 (n)  $\infty$~~  2. (a) The first condition is not satisfied, namely  $f(-3)$  does not exist—it is

undefined. (b) The second condition is not satisfied, namely  $\lim_{x \rightarrow 0} f(x)$  does not exist. (c) The third

condition is not satisfied, namely  $\lim_{x \rightarrow 3} f(x) \neq f(3)$ . 3. (a)  $f(1)$  yields  $\frac{1}{0}$ . A nonzero numerator and a zero denominator is indicative of a vertical asymptote line. Thus there is an infinite discontinuity.

(b)  $f(1)$  yields  $\frac{0}{0}$ , which is indeterminate, but  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1} = 1/2$ . Thus there is a removable discontinuity.

By defining  $f(1) = 1/2$ , the discontinuity can be removed. (c)  $\lim_{x \rightarrow 1^+} f(x) = 1$  while  $\lim_{x \rightarrow 1^-} f(x) = -1$ .

Thus there is a jump discontinuity. 4. Define  $f(3)$  as 2. 5. There is a removable discontinuity at  $x = 0$

since  $f(0)$  yields  $\frac{0}{0}$ , but  $\lim_{x \rightarrow 0} f(x) = -\frac{1}{2}$ . There is an infinite discontinuity at  $x = -2$  since  $f(-2)$

yields  $-\frac{12}{0}$ . There is a jump discontinuity at  $x = 4$  since  $\lim_{x \rightarrow 4^+} f(x) = \frac{1}{6}$  but  $\lim_{x \rightarrow 4^-} f(x) = -\frac{1}{6}$ . ~~6. (a)  $\frac{4}{5}$~~