Calculus 30

MIDTERM REVIEW

A. Find each limit

1. $\lim_{x\to 3} \frac{x-3}{x^2-4x+3}$

2. $\lim_{x\to 5}\left(x^2+2x-3\right)$

3. $\lim_{x\to 1} \frac{x^4-5x^2+1}{x+2}$

4. $\lim_{x\to 4} \frac{x^2-16}{x-4}$

5. $\lim_{x\to 2} \frac{x^3-8}{x^2-3x+2}$

6. $\lim_{h\to 0} \frac{(2+h)^2-4}{h}$

7. $\lim_{x\to 0}\sqrt{x}$

8. $\lim_{x\to 0^+}\sqrt{x}$

9. $$f(x)=\begin{cases} -x-2 & \text{if } x\le -1 \\ x & \text{if } -1\le x<1 \\ x^2-2x & \text{if } x\ge 1 \end{cases}$$

Find (a) $\lim_{x\to -1} f(x)$

(b) $\lim_{x\to 1} f(x)$

10. $\lim_{x\to 4} \frac{x+5}{x-4}$

11. $\lim_{x\to 3} \frac{x+2}{(x-3)^2}$

12. $\lim_{x\to 2^-} \frac{x^2-5x}{x^2-4}$

13. $\lim_{x\to\infty} \frac{1}{x}$

14. $\lim_{x\to\infty} \frac{2x^2-3x}{3x^2+2}$

15. $\lim_{x\to\infty} \frac{x^2+1}{2x-3x^3}$

16. $\lim_{x\to\infty} \frac{4x^4+5}{8-3x^3}$

17. $\lim_{x\to -\infty} \frac{5x}{\sqrt{x^2+4}}$

18. $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$

19. $\lim_{x\to 3} \frac{\sqrt{x^2-9}}{x-3}$

B. Find each derivative – simplify as much as possible.

1. $y=8x^3$

2. $f(x)=6x^{\frac{8}{3}}$

3. $y=2x^4+\sqrt{x}$

4. $y=3x\sqrt{x}$

5. $f(x)=\sqrt{x}\,(2-3x)$

6. $f(x)=\left(2x^3+5\right)\left(3x^2-x\right)$

7. $y=\frac{x^2+2x-3}{x^3+1}$

8. $y=\frac{\sqrt{x}}{1+2x}$

9. $y=\left(x^2-x+2\right)^8$

10. $y=\frac{1}{\sqrt[3]{1-x^4}}$

11. $y=\left(\frac{2x-1}{x+2}\right)^6$

12. $f(x)=\left(x^2+1\right)^3(2-3x)^4$

13. $x^2+y^2=25$

14. $2x^5+x^4y+y^5=36$

15. $x^2-x^3y^2-y^3=13$

C. Slope

1. Find the slope of the tangent line to $y = 2x - x^2$ at $(-1,-3)$

2. Find the slope of the tangent line at the point given
   (a) $y = (1-2x)(3x-4)$ at $x = 2$
   (b) $y = x^4\left(4x^3+2\right)$ at $x = -1$
   (c) $y^5 + x^2y^3 = 10$ at $(-3,1)$

D. Find the Equation of the Tangent Line

1. $y = \left(x^2-3\right)^8$ at $(2,1)$

2. $y = x + \dfrac{6}{x}$ at $(2,5)$

E. Point on a Curve

1. At what point on the curve $y = x^4 - 25x + 2$ is the tangent line parallel to the line $7x - y = 2$

2. Find the points on the curve $y = \dfrac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$

3. Find the points on the curve $y = 2 - \dfrac{1}{x}$ where the tangent is perpendicular to the line $y + 4x = 1$

4. Find the equation of the line through (2, 1) that is tangent to the curve $y = x^2 - 2$

F. Sketch the graphs by finding the relative extrema and inflection points

1. $y = x^4 - 8x^2$

2. $y = x^3 + 4x$

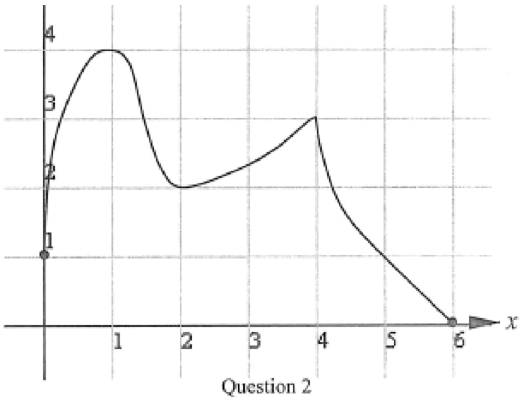
3. $y = 3x^{\frac{1}{3}} + 9x^{\frac{4}{3}}$

4. $y = \dfrac{4x}{x^2 - 4}$

5. $y = \dfrac{x^2 - 5x + 4}{x^2 + 4}$

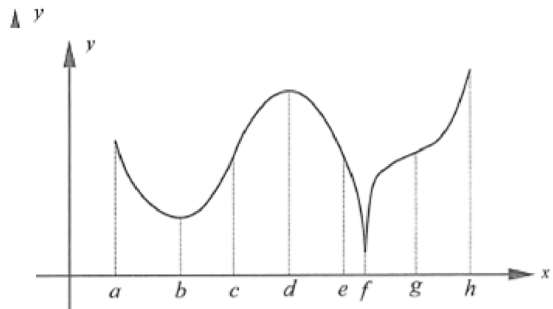
F. Curve Analysis

1. Consider the relation $x^2 + 4y^2 = 12$. Find $\frac{d^2 y}{dx^2}$.
2. Refer to the graph on the interval $[0, 6]$ at right to complete the table.
   (a) The absolute minimum value is _***_ .
   (b) The absolute maximum value is _***_ .
   (c) The relative maximum value(s) is/are _***_ .
   (d) The relative minimum value(s) is/are _***_ .



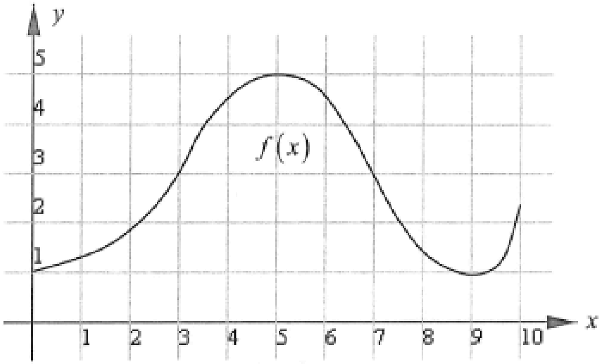
Question 2

3. Refer to the graph of the function shown below right. For each of the x-values a, b, c, d, e, f, g, and h, choose the words "absolute maximum", "absolute minimum", "relative maximum", "relative minimum", or "none of these". For some x-values, it may be necessary to use more than one set of words.



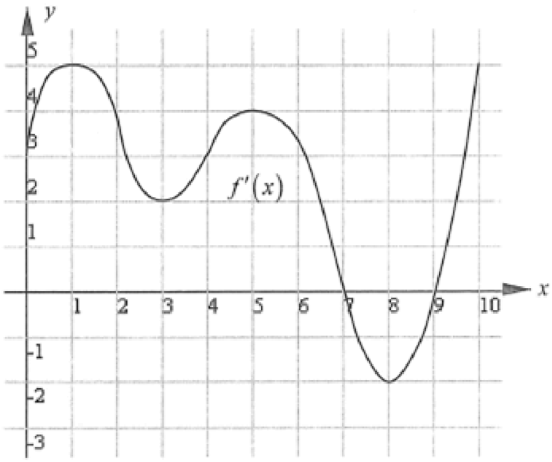
Question 3

4. Sketch the graph of a function that is continuous on $[2, 5]$ and has all of the following properties:
   - absolute maximum at $x = 2$
   - absolute minimum at $x = 5$
   - relative minimum at $x = 3$
   - relative maximum at $x = 4$
5. Sketch the graph of a function that is continuous on $[2, 5]$ and has all of the following properties:
   - absolute minimum at $x = 2$
   - absolute maximum at $x = 5$
   - relative minima at $x = 3.5$ and $x = 4.5$
   - relative maxima at $x = 3$ and $x = 4$
6. Sketch the graph of a continuous function that has a relative maximum at $x = 2$ and is differentiable at $x = 2$.
7. Sketch the graph of a continuous function that has a relative maximum at $x = 2$ but is not differentiable at $x = 2$.
8. Sketch the graph of a function on the interval $[2, 5]$ that has an absolute maximum at $x = 4$ but is not continuous at $x = 4$.
9. Use the graph of f at right to find:
   (a) the open interval(s) on which f is increasing.
   (b) the open interval(s) on which f is decreasing.
   (c) the open interval(s) on which f is concave up.
   (d) the open interval(s) on which f is concave down.
   (e) the coordinates of any relative extrema.
   (f) the coordinates of any inflection points.



Question 9

10. The graph of **the derivative of a function** ***f*** is shown below right. Use the graph to answer the following question about *f*.
   (a) On what open interval(s) is *f* increasing?
   (b) On what open interval(s) is *f* decreasing?
   (c) On what open interval(s) is *f* concave down?
   (d) On what open interval(s) is *f* concave up?
   (e) State the x-value(s) at which *f* has a point of inflection.
   (f) State the x-value(s) at which *f* has any relative extrema.
   (g) What are the critical numbers?



Question 10

11. Determine the y-intercept of the function $f(x)=\dfrac{x^4+6x^3-15x^2+24x-100}{2x^2-25}$.

12. Determine the x-intercepts of the function $f(x)=\dfrac{x^4-9x^2}{x^3+1}$.

13. Determine the equation(s) of any vertical asymptote line(s) for the function $f(x)=\dfrac{x^2-6x+9}{x^4+8x}$.

14. Use limits to determine the equation of the horizontal asymptote line to the function $f(x)=\dfrac{3x^2}{(x+1)^2}$.

15. Use limits to determine the equation of two horizontal asymptote lines to the function $f(x)=\dfrac{\sqrt{9x^2+3x+2}}{x-2}$.

16. Determine the absolute extrema for the function $f(x)=2x^3-3x^2-12x$ on the interval $[-2,4]$.

17. For the relation $x^2+xy=6$,

   (a) use implicit differentiation to show that $\dfrac{dy}{dx}=\dfrac{-2x-y}{x}$.

   (b) find the value of $\dfrac{dy}{dx}$ at the point $(-3,1)$. Based on this answer, does the point lie in an interval in which the relation is increasing or decreasing? Explain.

   (c) use implicit differentiation to show that $\dfrac{d^2y}{dx^2}=\dfrac{2(x+y)}{x^2}$.

   (d) find the value of $\dfrac{d^2y}{dx^2}$ at the point $(-3,1)$. Based on this answer, does the point lie in an interval in which the relation is concave up or concave down? Explain.

18. What can you conclude about the point $(3,f(3))$ if:
   (a) $f'(3)=0$, $f''(3)=-4$?
   (b) $f'(3)=0$, $f''(3)=4$?
   (c) $f'(2.9)=1$, $f'(3)=0$, $f'(3.1)=-1$?
   (d) $f''(2.9)=-4$, $f''(3)=0$, $f''(3.1)=5$?

19. For a certain function, $f''(x)=\dfrac{(x-1)^3}{\sqrt{x^2+4}}$. Perform a sign analysis for $f''(x)$ and determine the open intervals on which $f(x)$ is concave up and concave down.

20. Find the equation of the oblique asymptote line to the function $f(x)=\dfrac{x^3-3x^2+5x-7}{x^2-x+1}$.

21. Explain why a cubic polynomial function cannot have more than two relative extrema.

22. Sketch the graph of a function that has the following sign analysis for $f'(x)$.

------------∞++++++++0------0---------- $f''(x)$

For each of the functions in questions 23 to 28, find:

(a) $f'(x)$, and perform a sign analysis.

(b) the open intervals on which $f(x)$ is increasing or decreasing.

(c) the coordinates of any relative extrema.

(d) $f''(x)$, and perform a sign analysis.

(e) the open intervals on which $f(x)$ is concave up or concave down.

(f) the coordinates of any inflection points.

(g) the x and y intercepts.

(h) the equations of any asymptote lines.

(i) a sketch of the function showing all of the above features.

23. $f(x)=\dfrac{2x^2}{x^2+12}$

24. $f(x)=3x^5-10x^3$

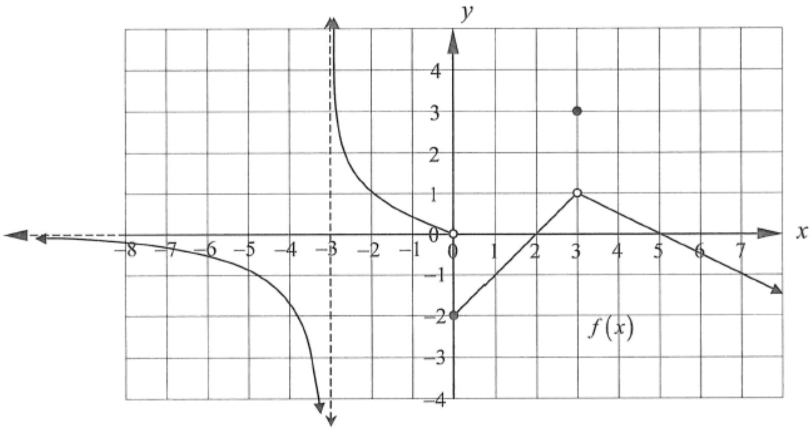
25. $f(x)=\dfrac{3x}{(x+1)^2}$

26. $f(x)=\dfrac{x^2-7x+10}{x-1}$

27. $f(x)=3x^{2/3}-x^2$

28. $f(x)=\dfrac{x+2}{\sqrt{x^2+2}}$

G: Continuity



2. There are three conditions that a function must satisfy in order to be continuous at a point. In light of these conditions, explain why function $f(x)$, above, is not continuous at:

   (a) $x=-3$ (b) $x=0$ (c) $x=3$

3. Identify the type of discontinuity each function has at $x=1$. Explain.

   (a) $f(x)=\dfrac{\sin\left(\dfrac{\pi x^2}{2}\right)}{x^2-1}$ (b) $f(x)=\dfrac{x-1}{x^2-1}$ (c) $f(x)=\dfrac{|x-1|}{x-1}$

4. Define $f(3)$ so that $f(x)=\dfrac{x^2-4x+3}{x-3}$ will be continuous at $x=3$.

5. Find the value(s) of x, if any, at which the function $f(x)=\dfrac{x|x-4|}{x^3-2x^2-8x}$ is discontinuous. Explain your reasoning. Classify each discontinuity.

Calculus 30

Midterm Review Answers

A. Limits

1. $\frac{1}{2}$ 2. 32   3. -1   4. 8   5. 12   6. 4

7. does not exist   8. 0   9. a) -1   b) does not exist   10. does not exist

11. ∞ 12. ∞ 13. 0   14. $\frac{2}{3}$ 15. 0   16. - ∞

17. -5   18. $\frac{1}{4}$ 19. does not exist

B. Derivatives

1. $24x^2$ 2. $16x^{\frac{5}{3}}$ 3. $8x^3 + \frac{1}{2\sqrt{x}}$ 4. $\frac{9}{2}x^{\frac{1}{2}}$ 5. $x^{\frac{-1}{2}} - \frac{9}{2}x^{\frac{1}{2}}$

6. $6x^2(3x^2 - x) + (6x-1)(2x^3+5)$ or $18x^4 - 6x^3 + (6x-1)(2x^3+5)$

7. $\frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$ 8. $\frac{1-2x}{2\sqrt{x}(1+2x)^2}$ 9. $(16x-8)(x^2-x+2)^7$

10. $\frac{4}{3}x^3(1-x^4)^{-\frac{4}{3}}$ 11. $\frac{30(2x-1)^5}{(x+2)^7}$ 12. $-6(x^2+1)^2(2-3x)^3(5x^2-2x+2)$

13. $\frac{-x}{y}$ 14. $\frac{-10x^4 - 4x^3y}{x^4 + 5y^4}$ 15. $\frac{3x^2y^2 - 2x}{2x^3y - 3y^2}$

C. Slope

1. 4   2. a) -13 b) 20   c) $\frac{3}{16}$

D. Find Equation of Tangent Line

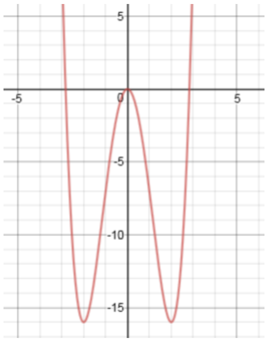
1. $y = 32x - 63$ 2. $y = \frac{-1}{2}x + 6$

E. Points on a Curve

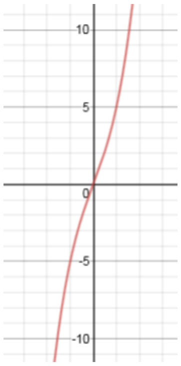
1. (2, -32)   2. $\left(3, \frac{3}{2}\right)$ and $\left(-1, \frac{1}{2}\right)$ 3. $\left(2, \frac{3}{2}\right)$ and $\left(-2, \frac{5}{2}\right)$ 4. $y = 6x - 11$ and $y = 2x - 3$

F. Graphs

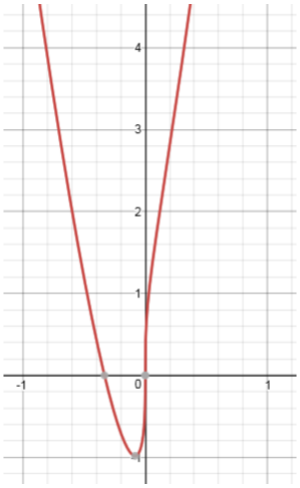
1. Max (0, 0), Min (2, 16), (-2, -16), Inflection Points (-1.155, -8.89), Intercepts: (0,0), (2.828, 0), (-2.828, 0)



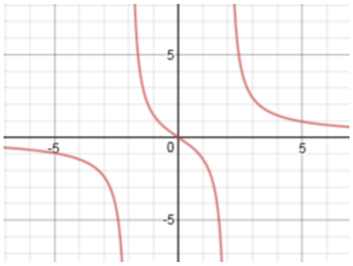
2. No Max/Min points, Inflection Point (0, 0), intercepts: (0,0)



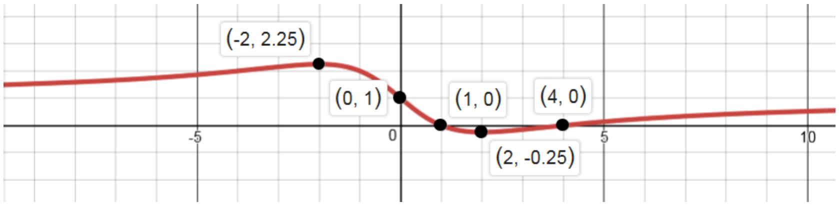
3. Min : (-1/12, -0.98), IP : (0,0), (1/6, 2.48), y-int : (0,0), x-int : (0,0), (-1/3, 0)



4. No extrema, IP : (0,0), intercepts : (0,0), VA: x = 2, x = -2, HA: y=0



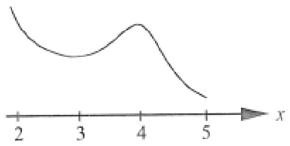
5. Max: (-2, 2.25), Min : (2, -0.25), IP : (0,1), $(2\sqrt{3}, -0.08)$, $(-2\sqrt{3}, 2.08)$ Intercepts : (1,0), (4,0), (0,1), HA: y=1



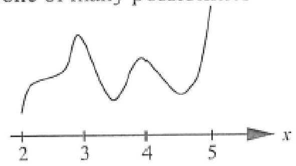
G:

1. $\frac{-4y^2-x^2}{16y^3}=\frac{-3}{4y^3}$ 2. **(a)** 0 **(b)** 4 **(c)** 4 and 3 **(d)** 2 3. *a*—none of these; *b*—relative minimum; *c*—inflection point; *d*—relative maximum; *e*—none of these; *f*—relative and absolute minimum; *g*—inflection point; *h*—absolute maximum

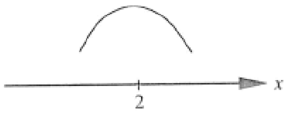
4. one of many possibilities



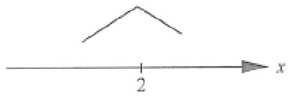
5. one of many possibilities



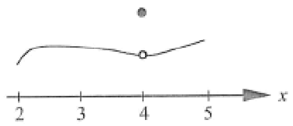
6. one of many possibilities



7. one of many possibilities



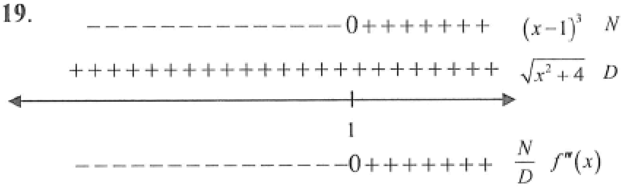
8. one of many possibilities



9. **(a)** $(0,5)\cup(9,10)$ **(b)** $(5,9)$
(c) $(0,3)\cup(7,10)$ **(d)** $(3,7)$
(e) $(5,5)$ is a relative maximum
and $(9,1)$ is a relative minimum.
(f) $(3,3)$ and $(7,3)$

10. **(a)** $(0,7)\cup(9,10)$ **(b)** $(7,9)$ **(c)** $(1,3)\cup(5,8)$ **(d)** $(0,1)\cup(3,5)\cup(8,10)$ **(e)** $x\in\{1,3,5,8\}$ **(f)** relative maximum at $x=7$; relative minimum at $x=9$ **(g)** $x\in\{7,9\}$ **11.** 4 **12.** $x\in\{0,\pm3\}$ **13.** $x=0$, $x=-2$ **14.** $y=3$ **15.** $y=3$, $y=-3$ **16.** absolute maximum, $f(4)=32$; absolute minimum, $f(2)=-20$ **17.** **(b)** $\left.\frac{dy}{dx}\right|_{(-3,1)}=-\frac{5}{3}$; the relation is decreasing because $\frac{dy}{dx}<0$ **(d)** $\left.\frac{d^2y}{dx^2}\right|_{(-3,1)}=-\frac{4}{9}$; the point is in a concave down interval since $\frac{d^2y}{dx^2}<0$ **18.** **(a)** it is a relative maximum **(b)** it is a relative minimum **(c)** it is a relative maximum **(d)** it is a point of inflection

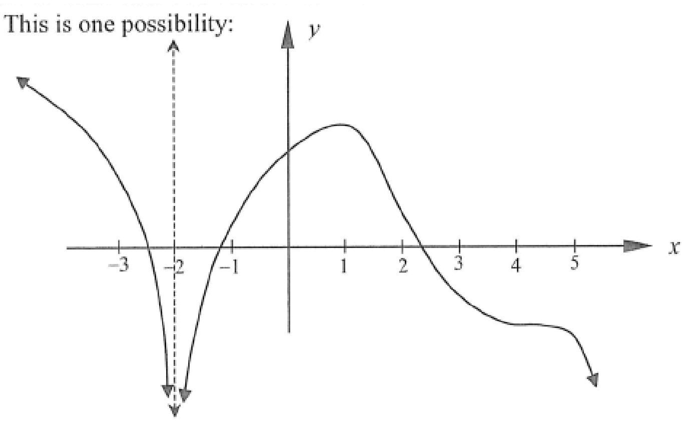
19.



$f(x)$ is concave up for $x\in(1,\infty)$ because $f''(x)>0$.
$f(x)$ is concave down for $x\in(-\infty,1)$ because $f''(x)<0$.

20. $y=x-2$ **21.** If $f(x)=ax^3+bx^2+cx+d$, then $f'(x)=3ax^2+2bx+c$. Recall that relative extrema may occur only at critical numbers. The derivative function is quadratic and has at most two zeros. At no value of *x* could the derivative function be undefined. Thus there can be at most two critical numbers and hence no more than two relative extrema.

22. This is one possibility:



23. (a) $f'(x)=\dfrac{48x}{\left(x^2+12\right)^2}$

−−−−−−−−−−−0++++++++++	$48x$	N
++++++++++++++++++++++	$\left(x^2+12\right)^2$	D
0		
−−−−−−−−−−−0+++++++++++	$\frac{N}{D}$	$f'(x)$

(b) $f(x)$ increases for $x\in(0,\infty)$ because $f'(x)>0$. $f(x)$ decreases for $x\in(-\infty,0)$ because $f'(x)<0$.

(c) $(0,0)$ is a relative minimum point because $f'(x)$ goes $-\ 0\ +$.

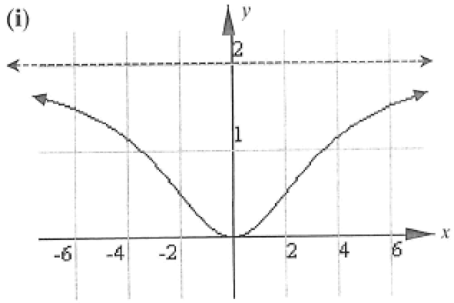
(d) $f''(x)=\dfrac{-144(x-2)(x+2)}{\left(x^2+12\right)^3}$

(e) $f(x)$ is concave up for $x\in(-2,2)$ because $f''(x)>0$. $f(x)$ is concave up down $x\in(-\infty,-2)\cup(2,\infty)$ because $f''(x)<0$.

++++++++++++++++0−−−−	$-144(x-2)$	N
−−−−−0++++++++++++++++	$x+2$	
++++++++++++++++++++++	$\left(x^2+12\right)^3$	D
−2 2		
−−−−−0++++++++++++0−−−−	$\frac{N}{D}$	$f''(x)$

23. (f) $(\pm2,1/2)$ are inflection points because $f''(x)$ changes signs at each.
(g) The x-intercept is 0; the y-intercept is 0.
(h) $y=2$ is a horizontal asymptote line.

(i)



24. (a) $f'(x)=15x^4-30x^2=15x^2\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$

+++++++++++0++++++++++	$15x^2$
−−−−−−−−−−−−−−−−0++++	$x-\sqrt{2}$
−−−−−0+++++++++++++++++	$x+\sqrt{2}$
$-\sqrt{2}$ 0 $\sqrt{2}$	
+++++0−−−−−−0−−−−−−0++++	$f'(x)$

(b) $f(x)$ increases for $x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$ because $f'(x)>0$. $f(x)$ decreases for $x\in\left(-\sqrt{2},\sqrt{2}\right)$ because $f'(x)<0$. Note that $x=0$ is included in the interval because it is an isolated point at which $f'(x)=0$.

(c) $\left(\sqrt{2},-8\sqrt{2}\right)$ or $(1.41,-11.31)$ is a relative minimum point because $f'(x)$ goes $-\ 0\ +$. $\left(-\sqrt{2},8\sqrt{2}\right)$ or $(-1.41,11.31)$ is a relative maximum point because $f'(x)$ goes $+\ 0\ -$.

(d) $f''(x)=60x^3-60x=60x(x-1)(x+1)$

−−−−−−−−−−−0++++++++++	$60x$
−−−−−−−−−−−−−−−−0++++	$x-1$
−−−−−0+++++++++++++++++	$x+1$
−1 0 1	
−−−−−0++++++0−−−−−0++++	$f''(x)$

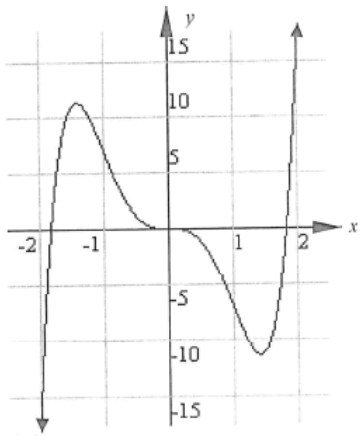
(e) $f(x)$ is concave up for $x \in (-1,0) \cup (1,\infty)$ because $f''(x) > 0$. $f(x)$ is concave down for $x \in (-\infty,-1) \cup (0,1)$ because $f''(x) < 0$.

(f) $(-1,7)$, $(0,0)$, and $(1,-7)$ are inflection points because $f''(x)$ switches signs at each.

(g) The x-intercepts are $x = 0$ and $x = \frac{\pm\sqrt{30}}{3}$; the y-intercept is 0.

(h) There are no asymptotes

(i)



25. **(a)** $f'(x) - \frac{-3(x-1)}{(x+1)^3}$

$+++++++++++++++++0----\qquad -3(x-1)\quad N$

$-----0++++++++++++++++\qquad (x+1)^3\quad D$

$-----\infty+++++++++++0----\qquad \frac{N}{D}\quad f'(x)$

(b) $f(x)$ decreases for $x \in (-\infty,-1) \cup (1,\infty)$ because $f'(x) < 0$. $f(x)$ increases for $x \in (-1,1)$ because $f'(x) > 0$.

(c) $(1,3/4)$ is a relative maximum because $f'(x)$ goes $+$ 0 $-$.

25. **(d)** $f''(x) = \frac{6(x-2)}{(x+1)^4}$

$----------------0++++\qquad 6(x-2)\quad N$

$+++++0++++++++++++++++\qquad (x+1)^4\quad D$

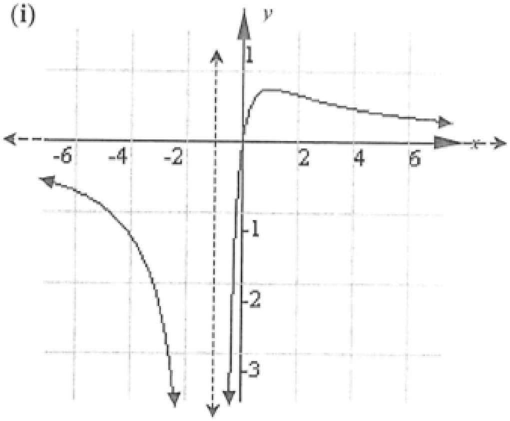
$-----\infty-----------0++++\qquad \frac{N}{D}\quad f''(x)$

(e) $f(x)$ is concave down for $x \in (-\infty,-1) \cup (-1,2)$ because $f''(x) < 0$. $f(x)$ is concave up for $x \in (2,\infty)$ because $f''(x) > 0$. **(f)** $(2,2/3)$ is an inflection point because $f''(x)$ changes signs at $x = 2$.

(g) The x-intercept is 0; the y-intercept is 0.

(h) $x = -1$ is a vertical asymptote; $y = 0$ is a horizontal asymptote.

(i)



26. **(a)** $f'(x)=\dfrac{(x-3)(x+1)}{(x-1)^2}$

$- - - - - - - - - - - - - - - - - - 0 + + +$ $\quad x-3$ $\Big\}$ N

$- - - - - 0 + + + + + + + + + + + + + + + +$ $\quad x+1$

$+ + + + + + + + + + + 0 + + + + + + + + + +$ $\quad (x-1)^2$ $\quad D$

$-1 \qquad 1 \qquad 3$

$+ + + + + 0 - - - - - - \infty - - - - - - 0 + + + +$ $\quad \frac{N}{D}$ $\quad f'(x)$

(d) $f''(x)=\dfrac{8}{(x-1)^3}$

$+ +$ $\quad 8$ $\quad N$

$- - - - - - - - - - - - - - - 0 + + + + + + + +$ $\quad (x-1)^3$ $\quad D$

1

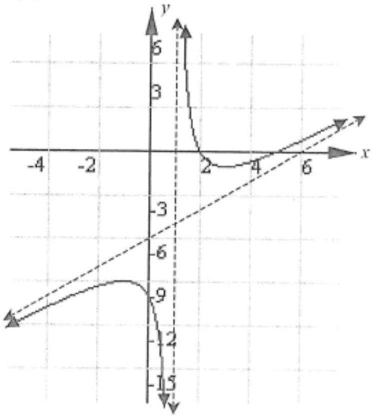
$- - - - - - - - - - - - - - - 0 + + + + + + + +$ $\quad \frac{N}{D}$ $\quad f''(x)$

(e) $f(x)$ is concave down for $x\in(-\infty,1)$ because $f''(x)<0$. $f(x)$ is concave up for $x\in(1,\infty)$ because $f''(x)>0$. **(f)** There are no inflection points. **(g)** The x-intercepts are $x=2$ and $x=5$; the y-intercept is -10. **(h)** $x=1$ is a vertical asymptote; $y=x-6$ is an oblique asymptote.

(b) $f(x)$ increases for $x\in(-\infty,-1)\cup(3,\infty)$ because $f'(x)>0$. $f(x)$ decreases for $x\in(-1,1)\cup(1,3)$ because $f'(x)<0$.

(c) $(-1,-9)$ is a relative maximum point because $f'(x)$ goes $+\ 0\ -$. $(3,-1)$ is a relative minimum point because $f'(x)$ goes $-\ 0\ +$.

(i)



27. (a) $f'(x)=2x^{-1/3}-2x=2x^{-1/3}\left(1-x^{4/3}\right)=\dfrac{2\left(1-x^{4/3}\right)}{x^{1/3}}$

$- - - - - 0 + + + + + + + + + + + + 0 - - - -$ $\quad 2(1-x)^{4/3}$ $\quad N$

$- - - - - - - - - - - 0 + + + + + + + + + + +$ $\quad x^{1/3}$ $\quad D$

$-1 \qquad 0 \qquad 1$

$+ + + + + 0 - - - - - \text{DNE} + + + + 0 - - - -$ $\quad \frac{N}{D}$ $\quad f'(x)$

(b) $f(x)$ increases for $x\in(-\infty,-1)\cup(0,1)$ because $f'(x)>0$. $f(x)$ decreases for $x\in(-1,0)\cup(1,\infty)$ because $f'(x)<0$.

(c) $(-1,2)$ and $(1,2)$ are relative maxima because $f'(x)$ goes $+\ 0\ -$ at each. $(0,0)$ is a relative minimum because $f'(x)$ goes $-$ DNE $+$.

27. **(d)**

$$f''(x) = -\frac{2}{3}x^{-4/3} - 2 = -\frac{2}{3}x^{-4/3}\left(1+3x^{4/3}\right) = \frac{-2\left(3x^{4/3}+1\right)}{3x^{4/3}}$$

– $-2\left(3x^{4/3}+1\right)$ N

+++++++++++0++++++++++ $3x^{4/3}$ D

0

– – – – – – – – – – – ∞ – – – – – – – – – – $\frac{N}{D}$ $f''(x)$

(e) $f(x)$ is concave down for $x \in (-\infty,0)\cup(0,\infty)$ because $f''(x)<0$. **(f)** There are no inflection points. **(g)** The *x*-intercepts are 0 and $\pm\sqrt[4]{27} \approx \pm 2.28$; the *y*-intercept is 0

(h) There are no asymptotes.

28. **(a)** $f'(x) = \frac{-2(x-1)}{\left(x^2+2\right)^{3/2}}$

+++++++++++++++++0– – – – $-2(x-1)$ N

+++++++++++++++++++++++ $\left(x^2+2\right)^{3/2}$ D

1

+++++++++++++++++0– – – – $\frac{N}{D}$ $f'(x)$

(d) $f''(x) = \frac{2(x-2)(2x+1)}{\left(x^2+2\right)^{5/2}}$

– – – – – – – – – – – – – – – – – 0++++ $2(x-2)$

– – – – 0++++++++++++++++++ $2x+1$ $\Big\}N$

++++++++++++++++++++++ $\left(x^2+2\right)^{5/2}$ D

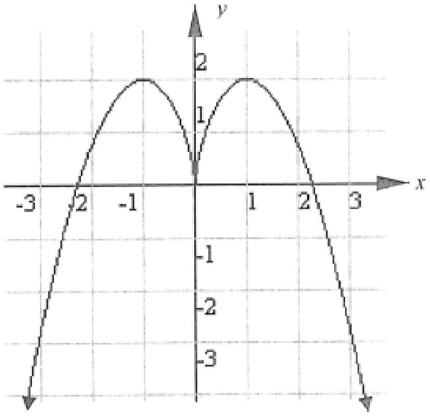
$-1/2$ 2

++++0– – – – – – – – – – – – 0++++ $\frac{N}{D}$ $f''(x)$

(e) $f(x)$ is concave up for $x \in (-\infty,-1/2)\cup(2,\infty)$ because $f''(x)>0$. $f(x)$ is concave down for $x \in (-1/2,2)$ because $f''(x)<0$.

(f) $(-1/2,1)$ and $\left(2,2\sqrt{6}/3\right) \approx (2,1.63)$ are inflection points since $f''(x)$ changes signs at $x=-1/2$ and $x=2$.

(i)



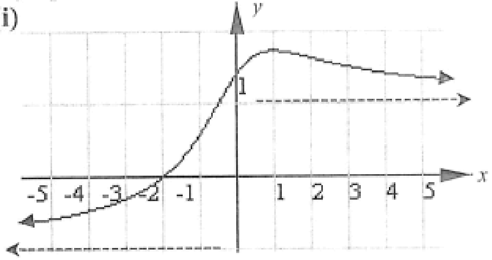
(b) $f(x)$ increases for $x \in (-\infty,1)$ because $f'(x)>0$. $f(x)$ decreases for $x \in (1,\infty)$ because $f'(x)<0$.

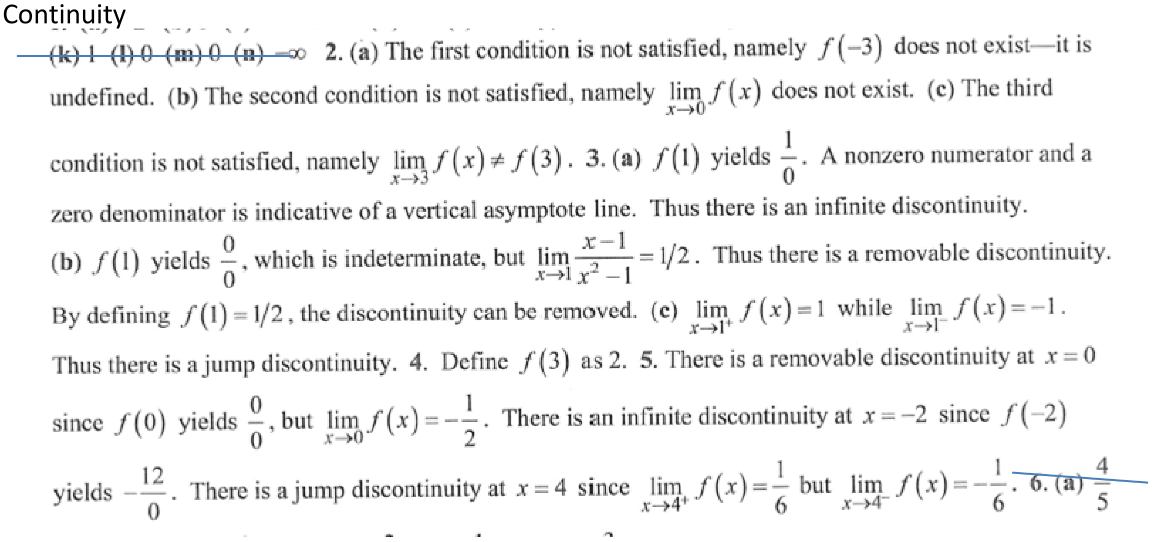
(c) $\left(1,\sqrt{3}\right)$ is a relative maximum point because $f'(x)$ goes $+\ 0\ -$.

(g) The *x*-intercept is $x=-2$; the *y*-intercept is $\sqrt{2} \approx 1.41$.

(h) $y=1$ is a horizontal asymptote for $x>0$; $y=-1$ is a horizontal asymptote for $x<0$. There are no vertical asymptotes.

(i)





~~(k) 1 (l) 0 (m) 0 (n) $-\infty$~~ **2.** (**a**) The first condition is not satisfied, namely $f(-3)$ does not exist—it is undefined. (**b**) The second condition is not satisfied, namely $\lim_{x\to 0} f(x)$ does not exist. (**c**) The third condition is not satisfied, namely $\lim_{x\to 3} f(x) \neq f(3)$. **3.** (**a**) $f(1)$ yields $\frac{1}{0}$. A nonzero numerator and a zero denominator is indicative of a vertical asymptote line. Thus there is an infinite discontinuity. (**b**) $f(1)$ yields $\frac{0}{0}$, which is indeterminate, but $\lim_{x\to 1}\frac{x-1}{x^2-1} = 1/2$. Thus there is a removable discontinuity. By defining $f(1) = 1/2$, the discontinuity can be removed. (**c**) $\lim_{x\to 1^+} f(x) = 1$ while $\lim_{x\to 1^-} f(x) = -1$. Thus there is a jump discontinuity. **4.** Define $f(3)$ as 2. **5.** There is a removable discontinuity at $x = 0$ since $f(0)$ yields $\frac{0}{0}$, but $\lim_{x\to 0} f(x) = -\frac{1}{2}$. There is an infinite discontinuity at $x = -2$ since $f(-2)$ yields $-\frac{12}{0}$. There is a jump discontinuity at $x = 4$ since $\lim_{x\to 4^+} f(x) = \frac{1}{6}$ but $\lim_{x\to 4^-} f(x) = -\frac{1}{6}$. ~~**6.** (**a**) $\frac{4}{5}$~~