

Final Exam Review

Chapter 1 Outcome 7

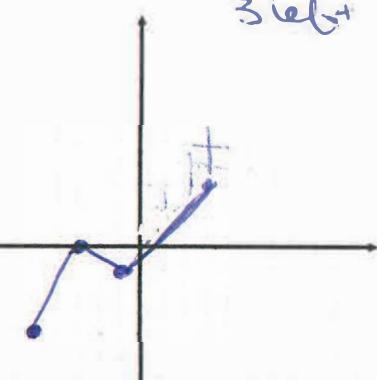
Level 2

1. Consider the graph of $y = f(x)$.

a) Sketch each of the transformed function:

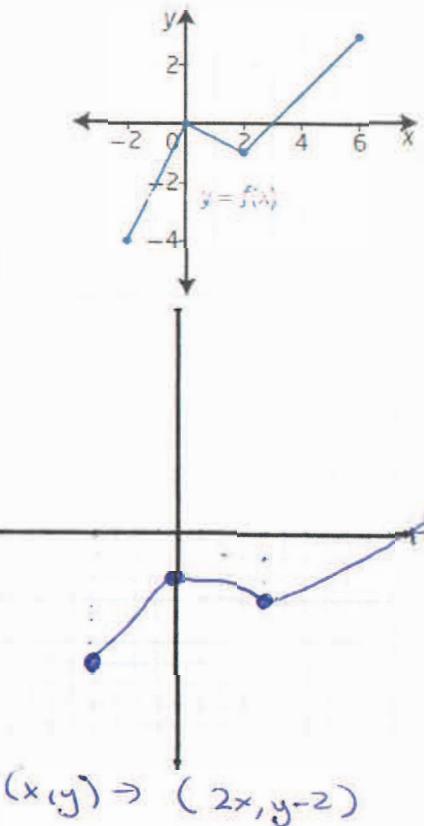
i) $y = f(x+3)$

3 left +



$$(x, y) \rightarrow (x-3, y)$$

$$(x, y) \rightarrow (x, -2y)$$



$$(x, y) \rightarrow (2x, y-2)$$

ii) $y = -2f(x)$

V. reflection about x-axis

iii) $y = f(\frac{1}{2}x) - 2$

H.S x 2

Down 2

x-axis

y-axis

2. For each equation, describe how the graph was translated, reflected or stretched.

a) $y = -2f(3(x-4))$

V. stretch factor of 2

V. reflection about x-axis

H. stretch factor of $\frac{1}{3}$

H. translation 4 right

b) $y = f(x-5)-3$

H. translation 5 right

V. translation 3 down

c) $y = f(-2x)+5$

H. stretch factor $\frac{1}{2}$

H. reflection about y-axis

3. Consider the graph of $y = f(x)$ and $y = g(x)$.

Determine the equation of the translated function in the form

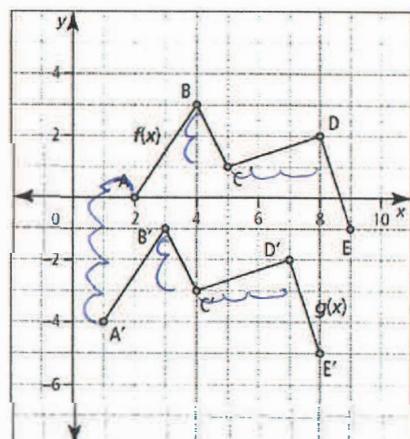
$$y = af(b(x-h))+k.$$

$$\boxed{y = f(x+1) - 4}$$

4 down $\Rightarrow k = -4$

1 left $\Rightarrow h = -1$

no V. or H
stretch.



5. Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$

$$\begin{aligned} x &= 3y - 6 \\ x + 6 &= 3y \\ y &= \frac{x+6}{3} \\ y &= \frac{1}{3}x + 2 \end{aligned}$$

b) $f(x) = x^2 - 7$

$$\begin{aligned} x &= y^2 - 7 \\ x + 7 &= y^2 \\ y &= \pm\sqrt{x+7} \end{aligned}$$

c) $y = (x - 5)^2 - 9$

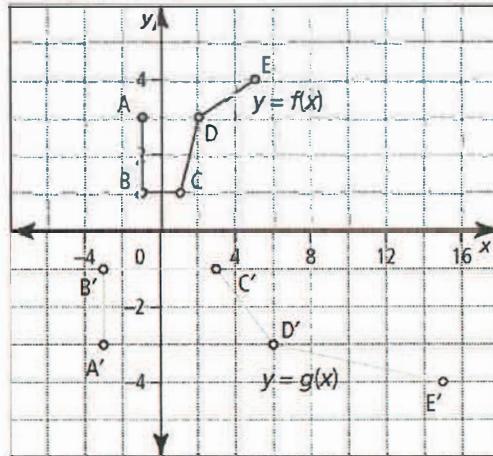
$$\begin{aligned} x &= (y - 5)^2 - 9 \\ \pm\sqrt{x+9} &= \sqrt{(y - 5)^2} \\ y - 5 &= \pm\sqrt{x+9} \\ y &= \pm\sqrt{x+9} + 5 \end{aligned}$$

Level 3

6. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine an equation for $g(x)$.

$$y = -f(\frac{1}{3}x)$$

V. reflection
 $a \Rightarrow -ve$
 H. stretch $\times 3$
 $b = \frac{1}{3}$



$$(x, y) \rightarrow (3x, -y)$$

7. Write the equation for each transformation of $y = x^2$ in the form $y = af(b(x - h)) + k$.

- a) a vertical stretch by a factor of 3, reflected in the y -axis, and translated 3 units left and 2 units down

$$y = 3f(-(x+3))-2$$

- b) a horizontal stretch by a factor of 2, reflected in the x -axis, and translated 7 units up

$$y = -f(\frac{1}{2}x) + 7$$

9. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

$$y = -2f(x-3)-2$$

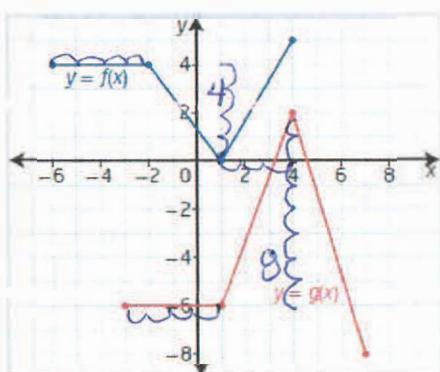
V. reflection

$a \Rightarrow -ve$

no H. stretch

V. stretch $\times 2$

3 right, 2 up.



10. The key point $(-18, 12)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $-3f(x + 5) + 4$

$$(x, y) \rightarrow (x + 5, -3y + 4)$$

$$(-18, 12) \rightarrow (-18 + 5, -3(12) + 4)$$

$$\rightarrow (-23, -32)$$

b) $y = 2f(6x)$

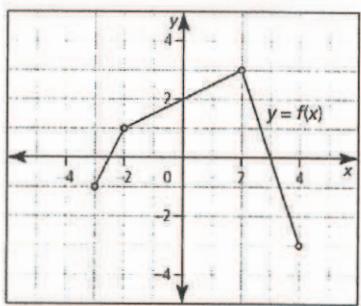
$$(x, y) \rightarrow (\frac{1}{6}x, 2y)$$

$$(-18, 12) \rightarrow (\frac{1}{6}(-18), 2(12))$$

$$\rightarrow (-3, 24)$$

11. Consider the graph of the function $y = f(x)$.

Sketch $y = f(x)$ to $y = 3f(-2(x - 1)) + 4$.



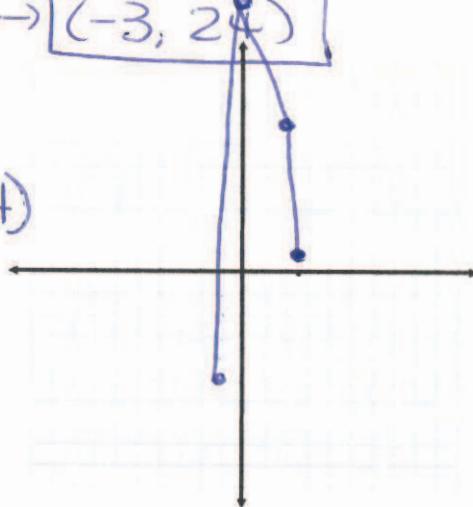
$$(x, y) \rightarrow (-\frac{1}{2}x + 1, 3y + 4)$$

$$(-3, 1) \rightarrow (\frac{5}{2}, 1)$$

$$(-2, 1) \rightarrow (2, 7)$$

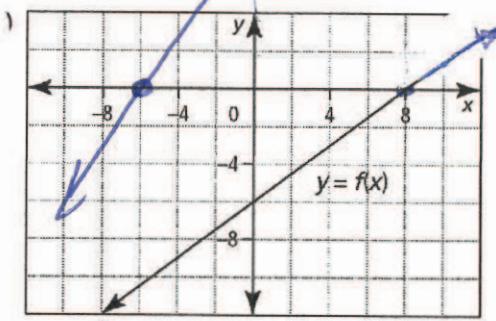
$$(2, 3) \rightarrow (0, 13)$$

$$(4, -3) \rightarrow (-1, -5)$$

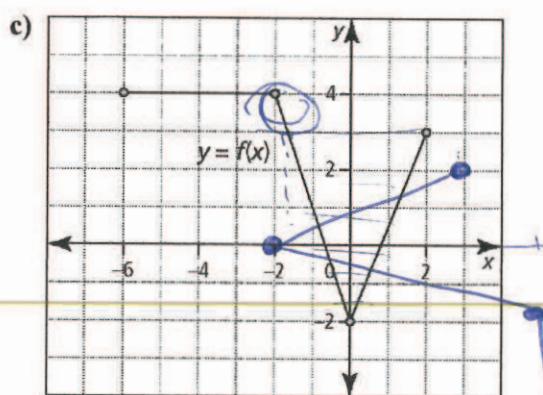
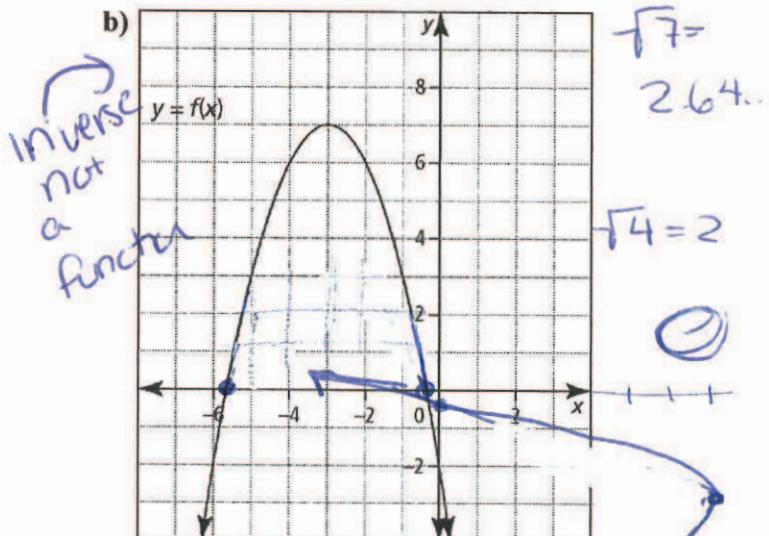


Level 4

12. Sketch the graph of its inverse, $x = f(y)$. Determine whether the inverse is a function. If the inverse is not a function, restrict the domain of the original graph to make it a function.



Inverse is a function



$(x, y) \rightarrow (y, x)$

Inverse Not a function

Restrict domain to $(-\infty, -3]$

or $[-3, \infty)$

Restrict domain to $[-2, 0]$ or $[0, 2]$

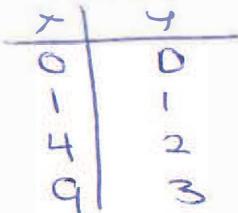
Chapter 2 Outcome 11a

1. Identify a, b, h and k for each of the following

a) $y = 5\sqrt{x+7} - 2$

$$\begin{aligned} a &= 5 \\ b &= 1 \\ h &= -7 \\ k &= -2 \end{aligned}$$

2. Graph $y = \sqrt{x}$



Level 3

3. Write the equation of a radical function that would result by applying each set of transformations to the graph of $f(x) = \sqrt{x}$

a) vertical stretch by a factor of 3, and horizontal stretch by a factor of 2

$$y = 3\sqrt{\frac{1}{2}x} \quad \text{or} \quad y = 3f(\frac{1}{2}x)$$

b) horizontal reflection in the y-axis, translation up 3 units, and translation left 2 units

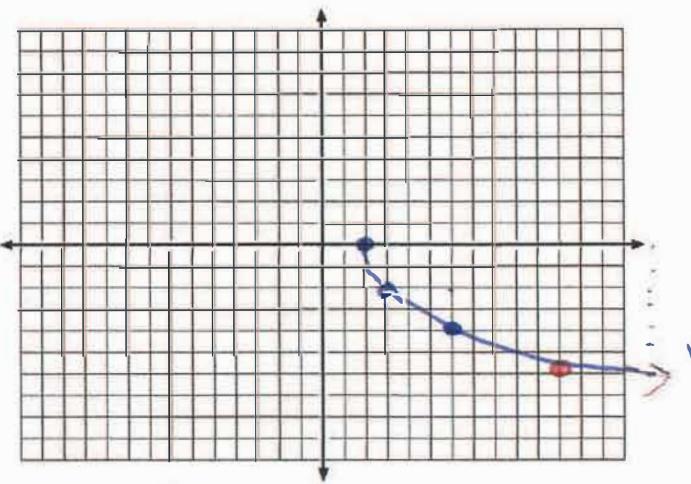
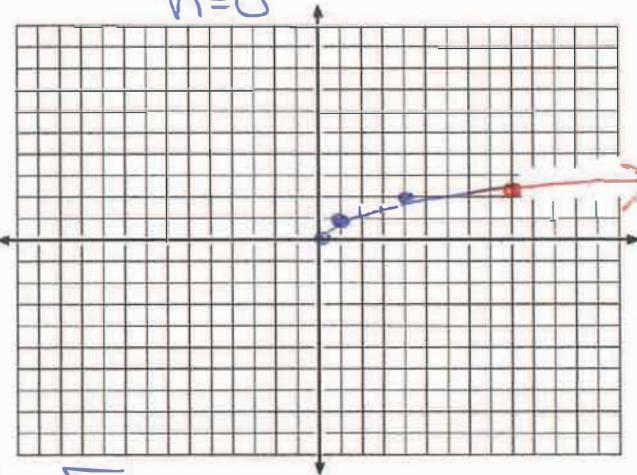
$$y = f(-(x+2)) + 3 \quad \text{or} \quad y = f(-((x+2))) + 3$$

4. Graph the functions below.

Then, identify the domain and range.

a) $y = -2\sqrt{x-2}$

$$\begin{aligned} (x,y) &\rightarrow (x+2, -2y) \\ (0,0) &\rightarrow (2, 0) \\ (1,1) &\rightarrow (3, -2) \\ (4,2) &\rightarrow (6, -4) \\ (9,3) &\rightarrow (11, -6) \end{aligned}$$



$$\begin{aligned} D &= [2, \infty) \\ R &= (-\infty, 0] \end{aligned}$$

c) $y = \sqrt{2x} - 4$

$$(x, y) \rightarrow (\frac{1}{2}x, y - 4)$$

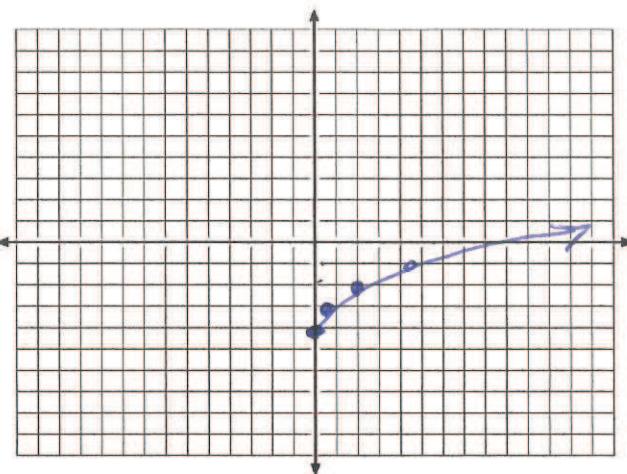
$$(0, 0) \rightarrow (0, -4)$$

$$(1, 1) \rightarrow (\frac{1}{2}, -3)$$

$$(4, 2) \rightarrow (2, -2)$$

$$(9, 3) \rightarrow (4.5, -1)$$

$$D = [0, \infty) \quad R = [-4, \infty)$$



c) $y = 2\sqrt{-(x-3)} + 1$

$$(x, y) \rightarrow (-x+3, 2y+1)$$

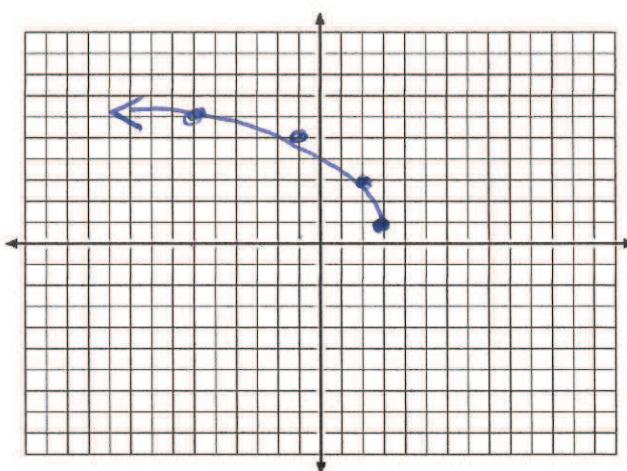
$$(0, 0) \rightarrow (3, 1)$$

$$(1, 1) \rightarrow (2, 3)$$

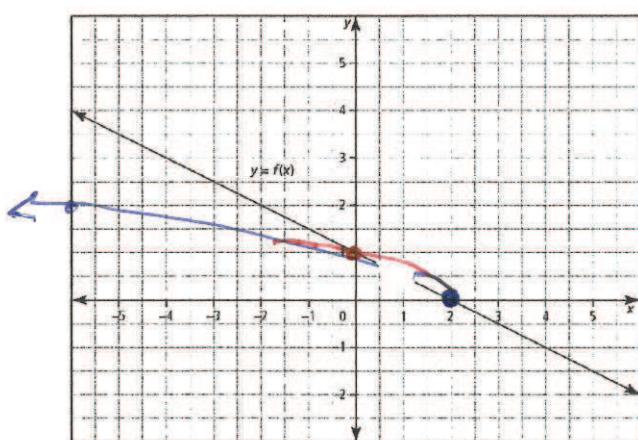
$$(4, 2) \rightarrow (-1, 5)$$

$$(9, 3) \rightarrow (-6, 7)$$

$$D = (-\infty, 3] \quad R = [1, \infty)$$

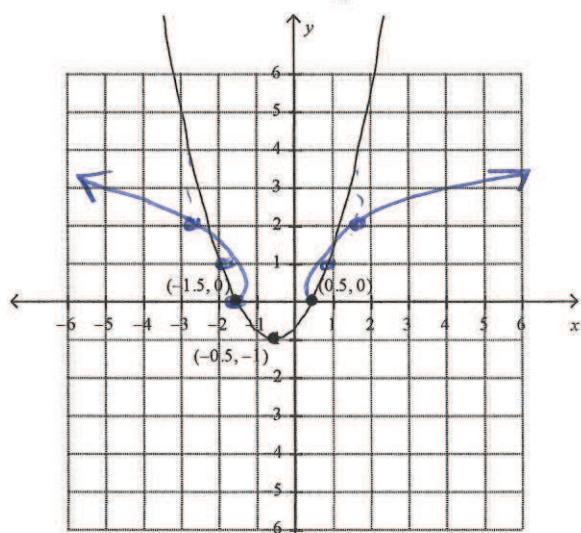


5. Graph $\sqrt{f(x)}$ from the following graphs of $f(x)$ and state the **domain and range**



$$D = (-\infty, 2]$$

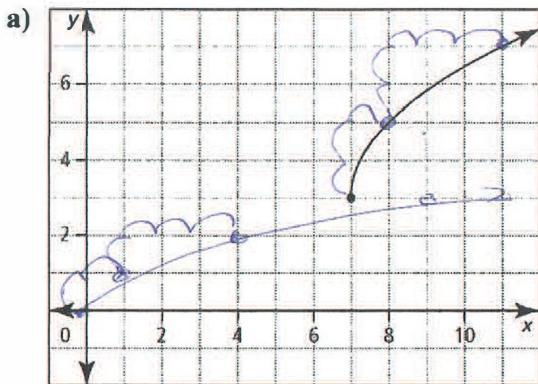
$$R = [0, \infty)$$



$$D = (-\infty, -1.5] \cup [0.5, \infty)$$

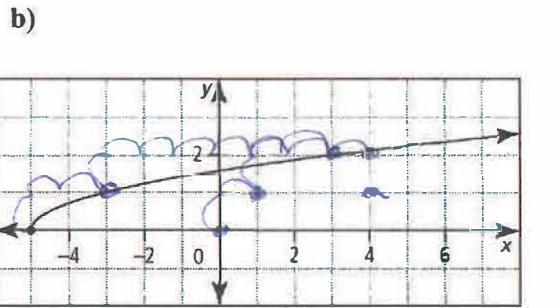
$$R = [0, \infty)$$

6. For each function, write an equation of a radical function of the form $y = a\sqrt{b(x - h)} + k$.



$$y = 2f(x-7) + 3$$

$$y = 2\sqrt{x-7} + 3$$



H. stretch $\times 2$

$$y = f(\frac{1}{2}(x+5))$$

$$y = \sqrt{\frac{1}{2}(x+5)}$$

Chapter 3 Outcome 10a

1. Divide the following using long division or synthetic division.

a) $(2w^4 + 3w^3 - 5w^2 + 2w - 27) \div (w+3)$

$$\begin{array}{r} 2 \quad 3 \quad -5 \quad 2 \quad -27 \\ \downarrow \quad 6 \quad -9 \quad 12 \quad -30 \\ 2 \quad -3 \quad 4 \quad -10 \quad 3 \end{array}$$

$$2w^3 - 3w^2 + 4w - 10 + \left(\frac{3}{w+3} \right)$$

b) $\frac{2x^3 - 10x^2 - 15x - 20}{x+5}$

$$\begin{array}{r} 2 \quad -10 \quad -15 \quad -20 \\ \downarrow \quad 10 \quad -100 \quad 425 \\ 2 \quad -20 \quad 85 \quad -445 \end{array}$$

$$2x^2 - 20x + 85 + \left(\frac{-445}{x+5} \right)$$

2. Determine the remainder when $x^3 + x^2 - 16x - 16$ is divided by

a) $x + 2$

b) $x - 4$

$P(-2) = 12$

$P(4) = 0$

- b) Are any of the binomials above a factor of $x^3 + x^2 - 16x - 16$?

$x - 4$

- ### 3. Factor completely

a. $x^3 + 2x^2 - 13x + 10$

$$\text{b. } x^4 - 26x^2 + 25$$

$$P(1) = 1 + 2 - 13 + 10$$

三〇

$x-1$ a factor

$$\begin{array}{c|ccccc} -1 & 1 & 2 & -13 & 10 \\ & \downarrow & & -1 & -3 & 10 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$$(x-1)(x^2 + 3x - 10)$$

$$(x+5)(x-2)$$

4. Determine the value(s) of k so that the binomial is a factor of the polynomial: $x^2 - 8x - 20$, $x + k$

$$P(-k) = 0$$

$$(-k)^2 - 8(-\frac{1}{2}) - 20 = 0$$

$$k^2 + 8k - 20 = 0$$

$$(k+10)(k-2)=0$$

$$15 = 10 \quad 5 = 3$$

5. The following polynomial has a factor of $x - 3$. What is the value of k ? $kx^3 - 10x^2 + 2x + 3$

$$P(3) = 6$$

$$k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$$

$$275 - 90 + 6 + 3 = 274$$

$$27K - 81 = 0$$

$$\frac{27k=81}{27}$$

k=3

Chapter 3 Outcome 10b

1. Determine which of the following are polynomials. For each polynomial function, state the degree.

a) $h(x) = 5 - \frac{1}{x}$

No

b) $y = 4x^2 - 5x$
Yes
 $d = 2$

yes
d-6

10

2. What is the leading coefficient, degree and constant term of each polynomial function?

a) $f(x) = -x^3 + 6x - 8$

$LC = -1$

$\deg = 3$

$con = -8$

c) $g(x) = 7x^3 + 3x^5 - 8x + 10$

$LC = 3$

$\deg = 5$

$con = 10$

b) $y = 5 + 2x^2$

$LC = 2$

$\deg = 2$

$con = 5$

d) $k(x) = 9x - 2x^2$

$LC = -2$

$\deg = 2$

$con = 0$

3. Identify the following characteristics for each polynomial function:

- the type of function and whether it is of even or odd degree
- the end behaviour of the graph of the function
- the number of possible x -intercepts
- the y -intercept

a) $g(x) = -2x^4 + 6x^2 - 7x - 5$

Even $0-4 \times \text{int}$

$\text{III} \rightarrow \text{IV}$ $y_{\text{int}} = -5$

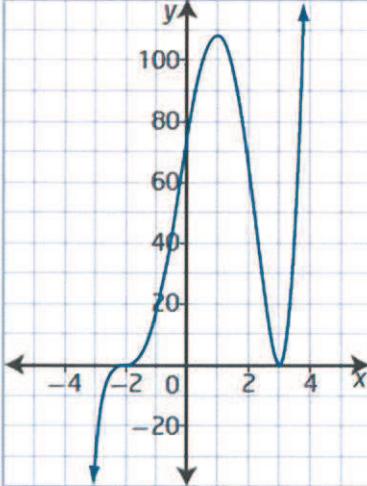
b) $f(x) = 2x^5 + 1x^3 - 12$

odd

$1-5 \times \text{int}$

$\text{I} \rightarrow \text{II}$ $y_{\text{int}} = -12$

4. Fill in the table below for the following graphs

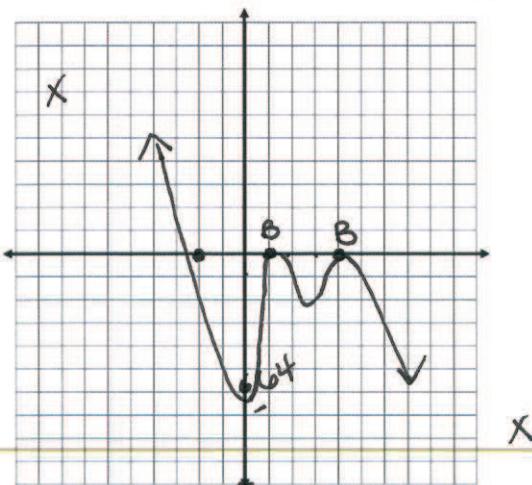
Graph	Odd or Even	Sign of Leading Coefficient	Number of x -intercepts	Intervals where the function is positive	Intervals where the function is negative
	ODD	-	3	(-∞, -2) ∪ (0, ∞)	(-2, 0)

	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
	O	-	3	(-\infty, -4) ∪ (-4, -1) ∪ (-1, 3)	(-4, -1) ∪ (3, \infty)
	E	-	4	(-2, -1) ∪ (0, 2) ∪ (2, 3) ∪ (3, \infty)	(-\infty, -2) ∪ (-1, 0) ∪ (1, 2) ∪ (3, \infty)
	E	+	3	(-\infty, -1) ∪ (1, 3) ∪ (3, \infty)	(-1, 1)

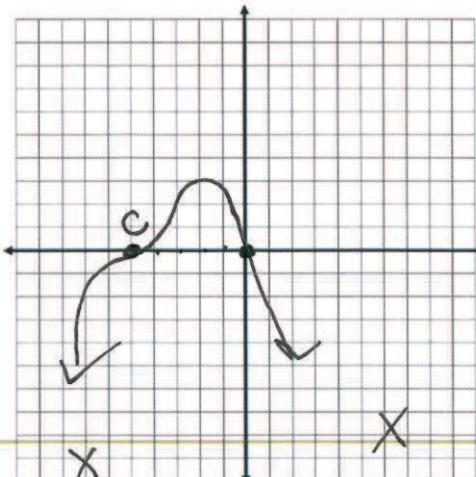
4. Graph the following polynomial functions. The first three have already been factored for you.

$$y = -2(x-1)^2(x+2)(x-4)^2 \quad d=5, \text{ UC}$$

$$y = -2x(x+5)^3 \quad d=4, \text{ LC}$$



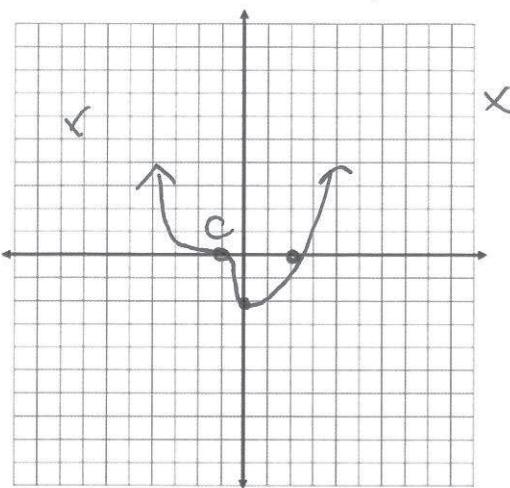
$$f(0) = -2(-1)^2(2)(-4)^2$$



$$f(0) = (-1)^3(-2) = -2$$

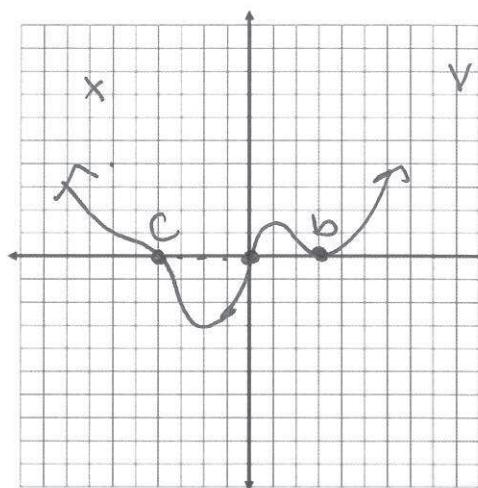
$$y = (x+1)^3(x-2)$$

$d=4$
+ LC



$$y = x(x+4)^3(x-3)^2$$

$d=6$
+ LC

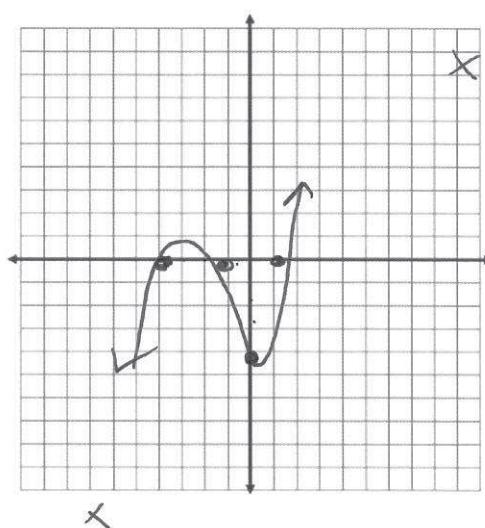
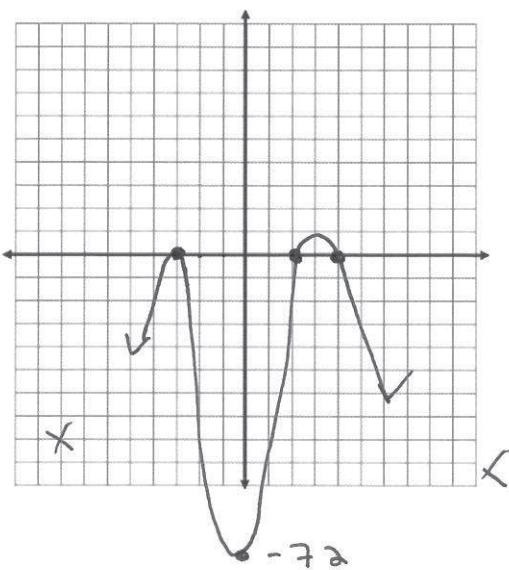


$$f(x) = -x^4 + 19x^2 + 6x - 72$$

$$y = x^3 + 4x^2 - x - 4$$

$$\begin{aligned} & -(x-2)(x+3)(x-4)(x+3) \\ & -(x-2)(x+3)^2 \quad \cancel{(x-4)} \end{aligned}$$

$$(x-1)(x+4)(x+1)$$



Final Exam Part 2 Review

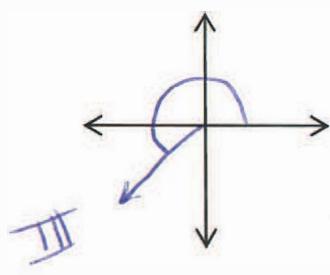
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Chapter 4 Outcome 1

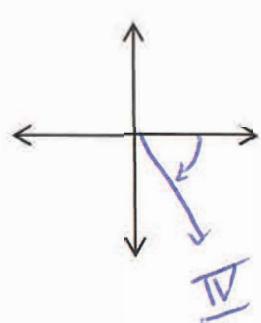
Level 2

1. Draw each angle in standard position. In what quadrant does each angle lie?

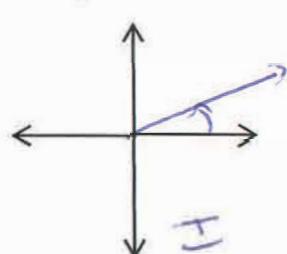
a) 215°



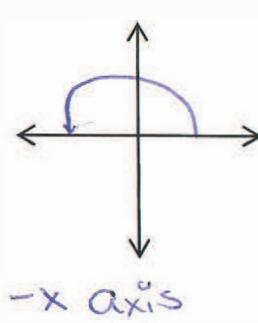
b) -70°



c) $\frac{\pi}{6}$



d) π



2. Change the degree measures to radians. Give answers as both exact and approximate measures to the nearest hundredth of a unit.

a) 150°

$$x = \frac{5\pi}{6}$$

$$\frac{x}{150} = \frac{\pi}{180}$$

$$x = \frac{150\pi}{180}$$

3. Change the radian measures to degrees. Round to two decimal places if necessary.

a) $\frac{4\pi}{5}$

$$144^\circ$$

b) $\frac{5\pi}{6}$

$$150^\circ$$

c) 3.8

$$\frac{x}{3.8} = \frac{180}{\pi} \quad x = 217.72^\circ$$

4. Determine the one positive and one negative angle that are coterminal with the given angle.

a) 450°

$$-360^\circ$$

b) $\frac{\pi}{5} \pm 2\pi$

$$\pm \frac{10\pi}{5}$$

$$\frac{11\pi}{5}, -\frac{9\pi}{5}$$

Level 3

5. Write an expression for all the angles that are coterminal with each given angle.

a) 75°

$$75^\circ + 360^\circ n, n \in \mathbb{N}$$

b) $\frac{\pi}{3}$

$$\frac{\pi}{3} + 2\pi n, n \in \mathbb{N}$$

c) 1

$$1 + 2\pi n, n \in \mathbb{N}$$

Chapter 4 Outcome 1b Unit Circle $x^2 + y^2 = 1^2$

$$x^2 + y^2 = 1$$

1. Which point(s) lies on the unit circle? Explain how you know.

$$\left(-\frac{5}{13}, \frac{12}{13}\right)$$

$$\left(\frac{5}{6}, \frac{1}{2}\right)$$

$$\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$$

$$\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$$

$$\frac{25}{169} + \frac{144}{169} = 1 \quad \text{Yes}$$

$$\left(\frac{5}{6}\right)^2 + \left(\frac{1}{2}\right)^2 \neq 1$$

No

$$\left(-\frac{2}{3}\right)^2 + \left(-\frac{\sqrt{5}}{3}\right)^2 = 1$$

$$\frac{4}{9} + \frac{5}{9} = 1 \quad \text{Yes}$$

2. Each of the following points lies on the unit circle. Find the missing coordinate satisfying the given conditions.

a) $\left(-\frac{2}{3}, y\right)$ in quadrant III



$$\left(-\frac{2}{3}\right)^2 + y^2 = 1$$

$$\frac{4}{9} + y^2 = \frac{9}{9}$$

$$y^2 = \frac{9}{9} - \frac{4}{9}$$

$$y = \pm \frac{\sqrt{5}}{3}$$



b) $(x, \frac{4}{5})$ in quadrant II

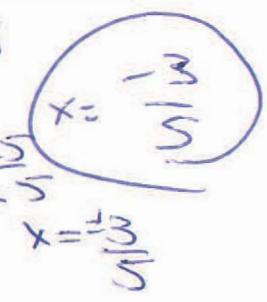


$$x^2 + \left(\frac{4}{5}\right)^2 = 1$$

$$x^2 + \frac{16}{25} = \frac{25}{25}$$

$$\sqrt{x^2} = \sqrt{\frac{9}{25}}$$

$$x = \pm \frac{3}{5}$$



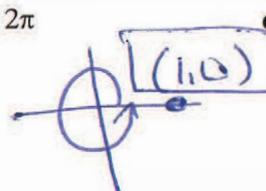
3. The point (x, y) is located where the unit circle. Determine the coordinates of point for the given angle.

$\cos \theta, \sin \theta$

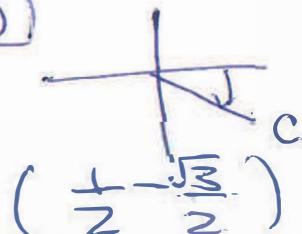
a) $\theta = 45^\circ$

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

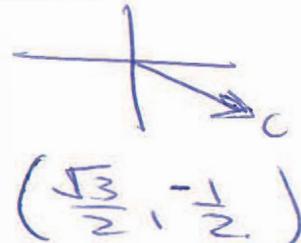
b) 2π



c) $\theta = -60^\circ$



d) $\frac{11\pi}{6}$



5. Identify a measure for θ in the interval $0 \leq \theta < 2\pi$ for is the given point.

a) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

d) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

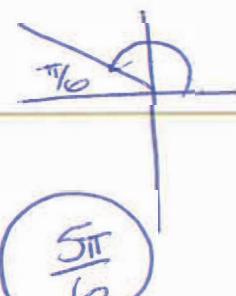
$\theta_R = 45^\circ$

$\frac{\pi}{4}$

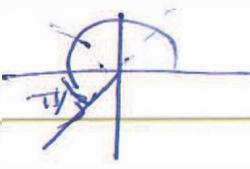


π

$\theta_R = 30^\circ, \frac{\pi}{6}$



$\theta_R = 60^\circ, \frac{\pi}{3}$

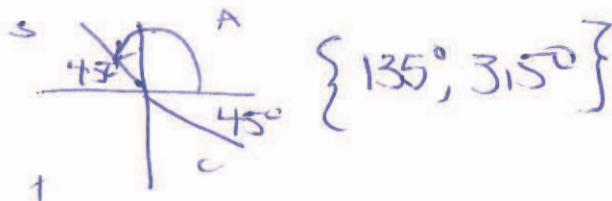


$\frac{4\pi}{3}$

7. Determine the measure of all angles that satisfy the given conditions. Use exact values when possible

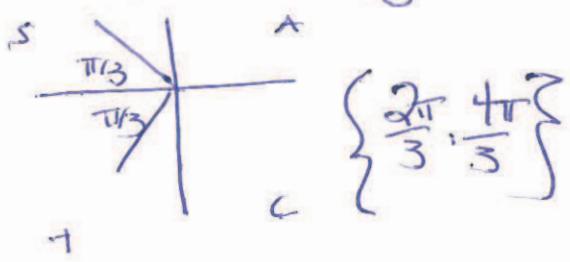
a) $\tan \theta = -1$, domain $0^\circ \leq \theta < 360^\circ$

$$\Theta_R = \tan^{-1}(1) = 45^\circ$$



c) $\cos \theta = -\frac{1}{2}$, domain $0 \leq \theta < 2\pi$

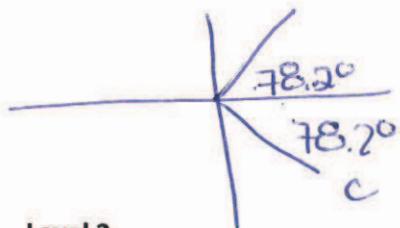
$$\Theta_R = 60^\circ = \frac{\pi}{3}$$



e) $\sec \theta = 4.87$, domain $0^\circ \leq \theta < 360^\circ$

$$\cos^{-1}(1/4.87)$$

$$\Theta_R = 78.2^\circ$$



Level 3

8. Determine the value of the following. Use exact values when possible

a) $\csc 60^\circ$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

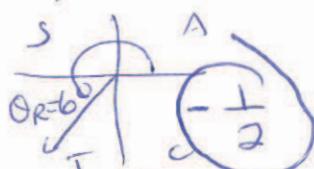
$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

d) $\cot 137^\circ$

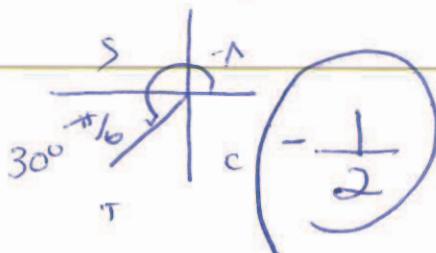
$$(\tan 137) -1$$

$$-1.07$$

b) $\cos 240^\circ$

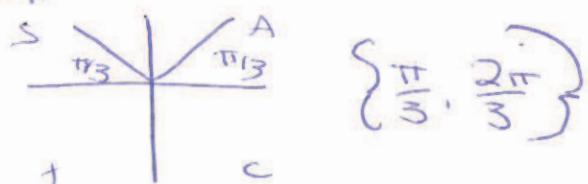


e) $\sin \frac{7\pi}{6}$



b) $\sin \theta = \frac{\sqrt{3}}{2}$, domain $0 \leq \theta < 2\pi$ Radian.

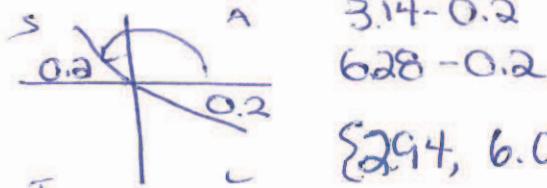
$$\Theta_R = 60^\circ = \frac{\pi}{3}$$



d) $\cot \theta = -4.87$, domain $0 \leq \theta < 2\pi$ Radian

$$\tan^{-1}(1/4.87)$$

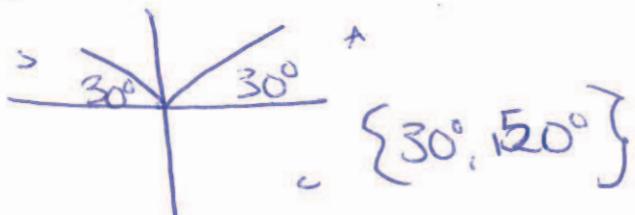
$$\Theta_R = 0.20$$



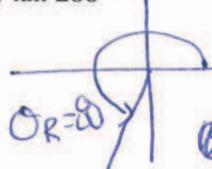
f) $\csc \theta = 2$, domain $0^\circ \leq \theta < 360^\circ$

$$\sin^{-1}(\frac{1}{2})$$

$$\Theta_R = 30^\circ$$



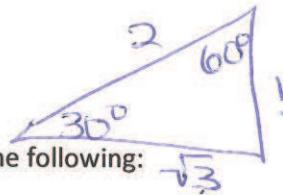
c) $\tan 260^\circ$



f) $\sec 4.5$

$$(\cos 4.5)^{-1}$$

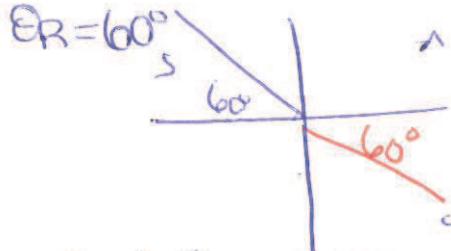
$$-4.74$$



9. Determine the exact value of each of the following:

a) $\tan \theta + \sqrt{3} = 0, 0 \leq \theta \leq 360^\circ$.

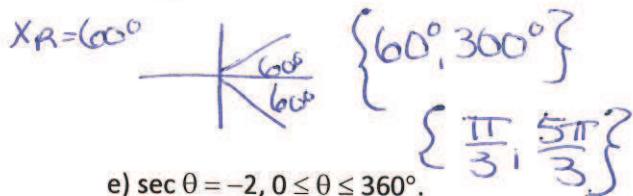
$$\tan \theta = -\sqrt{3}$$



c) $2 \cos^2 x - 5 \cos x + 2 = 0, 0 \leq x \leq 2\pi$.

$$(2\cos x - 1)(\cos x - 2) = 0$$

$$\cos x = \frac{1}{2} \quad (\cos x \neq 2)$$

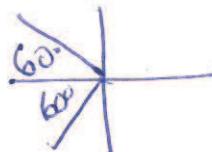


e) $\sec \theta = -2, 0 \leq \theta \leq 360^\circ$.

$$\cos \theta = -\frac{1}{2}$$

$$\theta_R = 60^\circ$$

$$\{120^\circ, 240^\circ\}$$



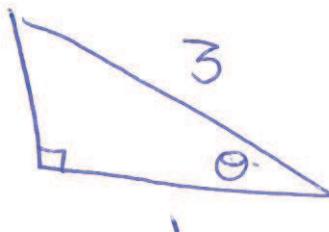
10. The point $\left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ is on the unit circle. Determine the exact value for each of the 6 trigonometric ratios.

$$\cos \theta = \frac{1}{3}$$

$$\sin \theta = \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

$$2\sqrt{2}$$



$$\sec \theta = \frac{3}{1} = 3$$

$$\csc \theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$= \frac{3\sqrt{2}}{4}$$

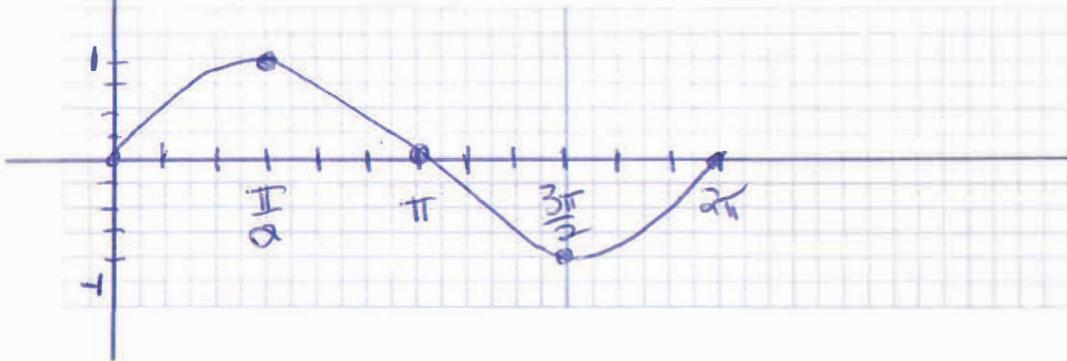
$$\cot \theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Chapter 5 Outcome 3

Level 2

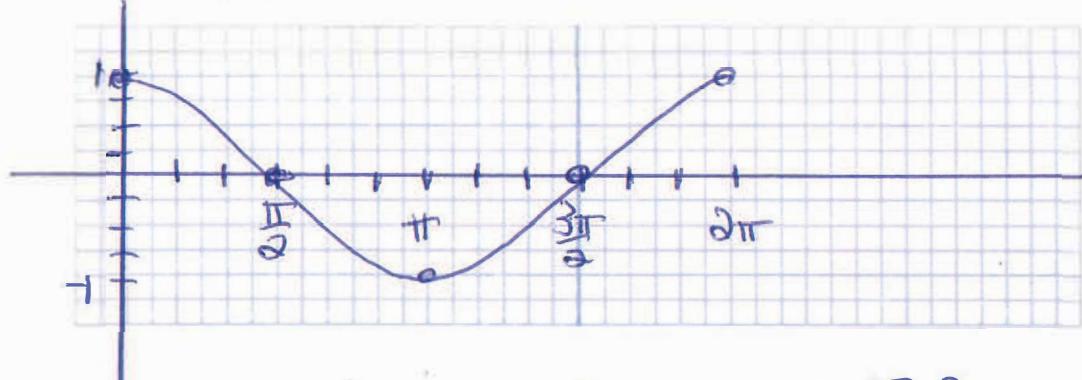
1.

- a) Sketch the following:
 $y = \sin x, 0 \leq x \leq 2\pi$



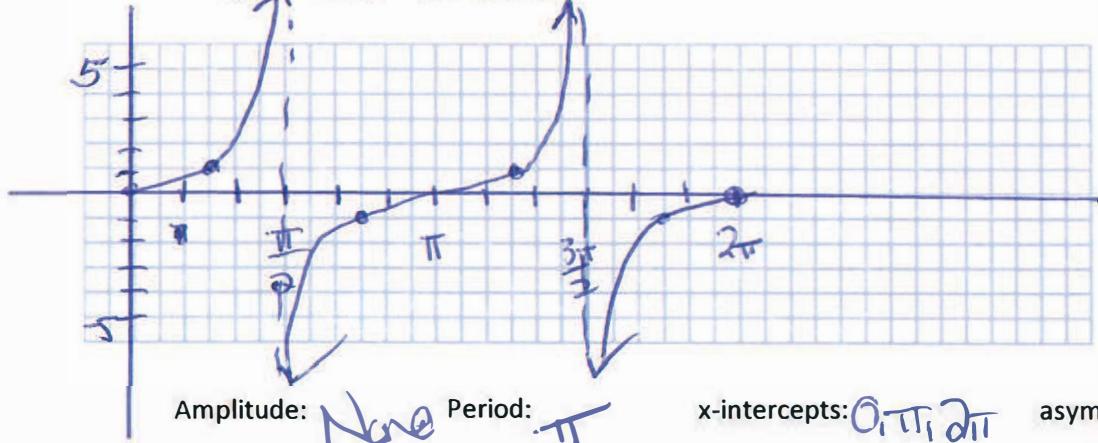
Amplitude: 1 Period: 2π x-intercepts: $0, \pi, 2\pi$ asymptotes: None

- b) $y = \cos x, 0 \leq x \leq 2\pi$



Amplitude: 1 Period: 2π x-intercepts: $\frac{\pi}{2}, \frac{3\pi}{2}$ asymptotes: None

- c) $y = \tan x, 0 \leq x \leq 2\pi$



Amplitude: None Period: π x-intercepts: $0, \pi, 2\pi$ asymptotes: $\frac{\pi}{2}, \frac{3\pi}{2}$

Level 3

2. Determine the following for each graph

a) Amplitude: 4

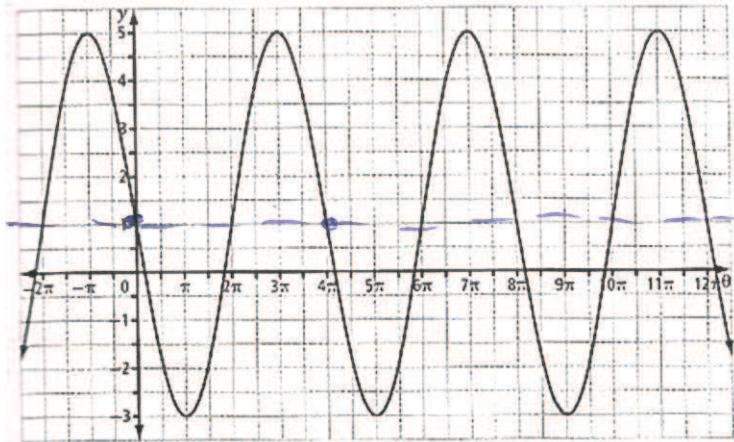
Domain: $x \in \mathbb{R}$

Range: $[-3, 5]$

Period: 4π $b = \frac{1}{2}$

Write the equation of the graph in form $y = a \cos b(x - c) + d$

$$y = 4 \cos \frac{1}{2}(x - 3\pi) + 1$$



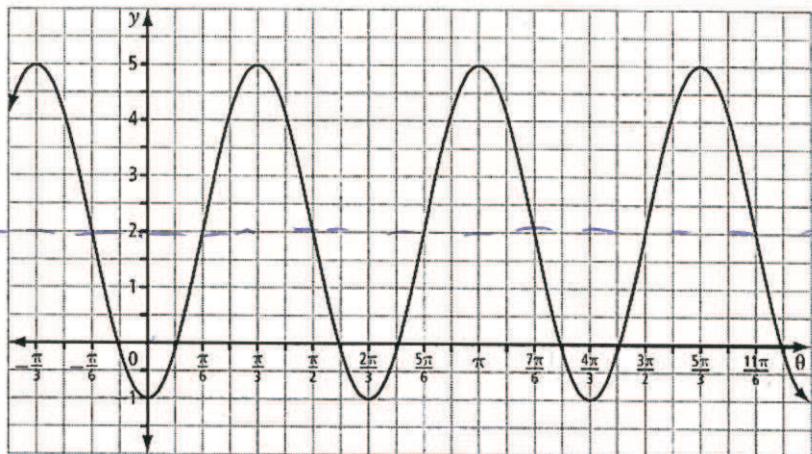
b) Amplitude: 3

Domain: $x \in \mathbb{R}$

Range: $[-1, 5]$

Period: $\frac{2\pi}{3}$ $b = 3$

Write the equation of the graph in form $y = a \sin b(x - c) + d$



c) Amplitude: 3

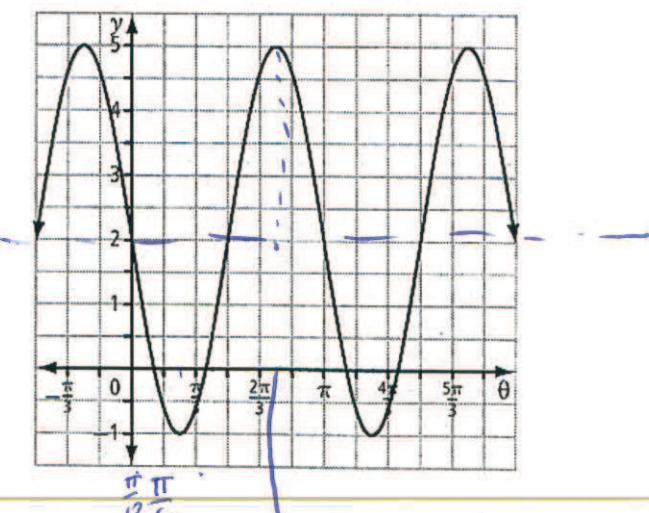
Domain: $x \in \mathbb{R}$

Range: $[-1, 5]$

Period: π $b = 2$

Write the equation of the graph in form $y = a \cos b(x - c) + d$

$$y = 3 \cos 2(x - \frac{3\pi}{4}) + 2$$

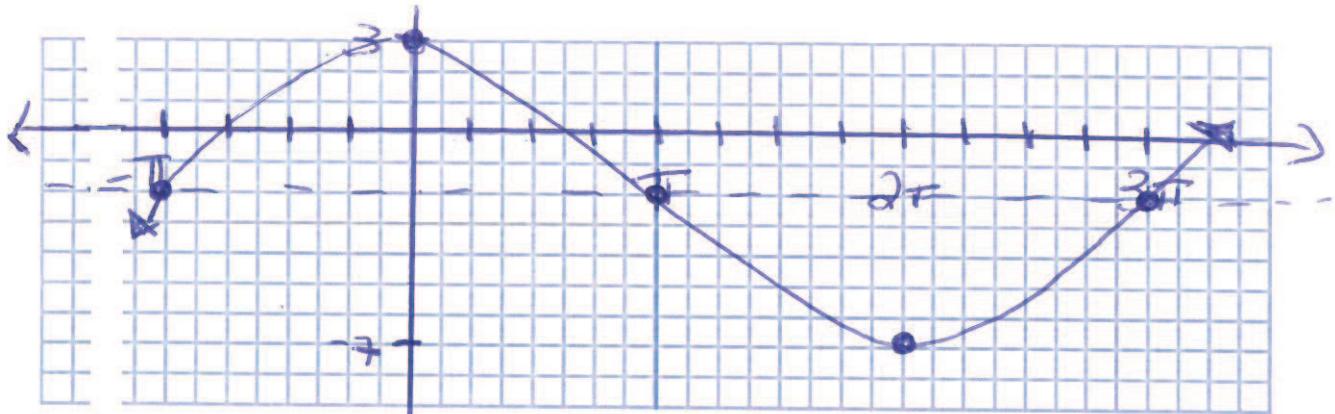


$$\frac{9\pi}{12} = \frac{3\pi}{4}$$

3. Graph each of the following for at least one cycle. For each state the domain, range, amplitude, and period

a) $y = 5\sin\frac{1}{2}(x + \pi) - 2$

Start @ $-\pi$ end $-\pi + 4\pi = 3\pi$



$D = (-\infty, \infty)$

$R = [-7, 3]$

Amp = 5

Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

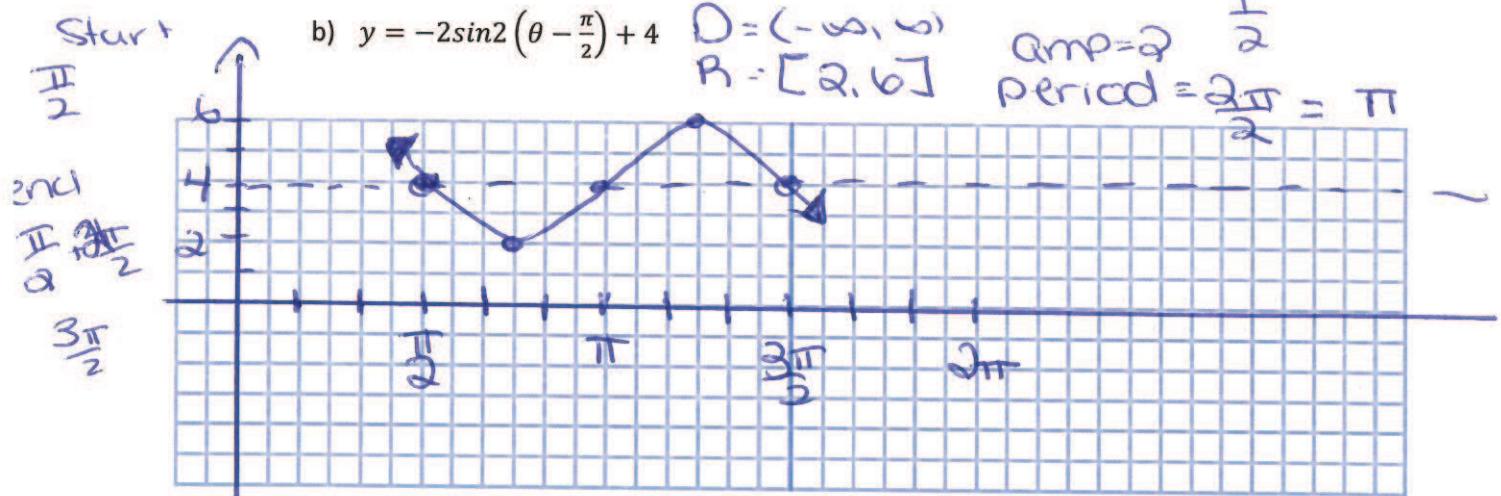
b) $y = -2\sin 2\left(\theta - \frac{\pi}{2}\right) + 4$

$D = (-\infty, \infty)$

$R = [2, 6]$

Amp = 2

Period = $\frac{2\pi}{2} = \pi$

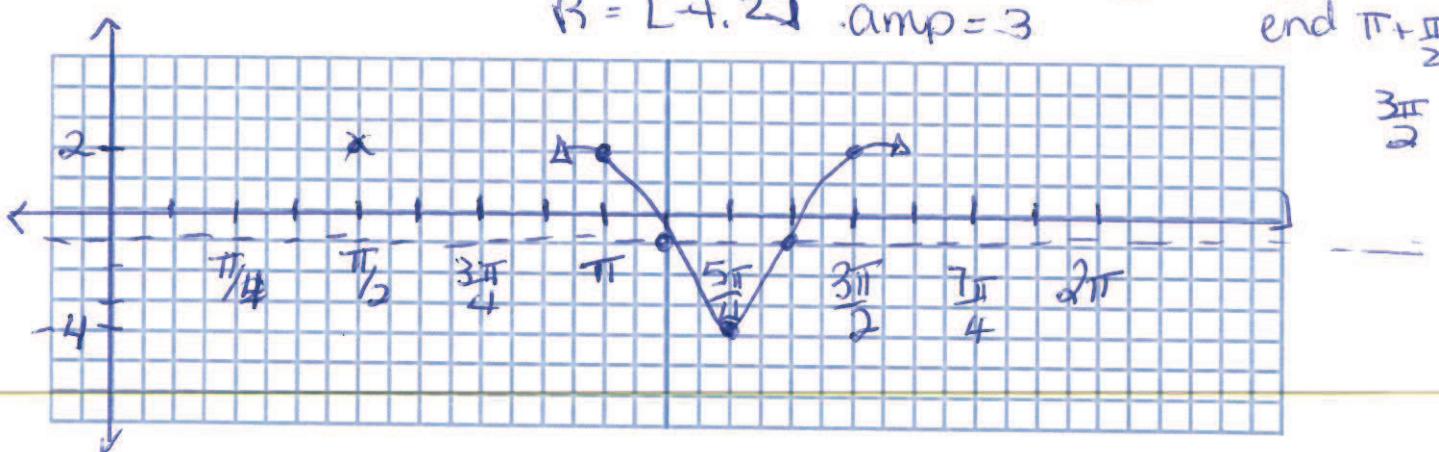


c) $y = 3\cos 4(x - \pi) - 1$

$D = (-\infty, \infty)$ period = $\frac{2\pi}{4} = \frac{\pi}{2}$

$R = [-4, 2]$ Amp = 3

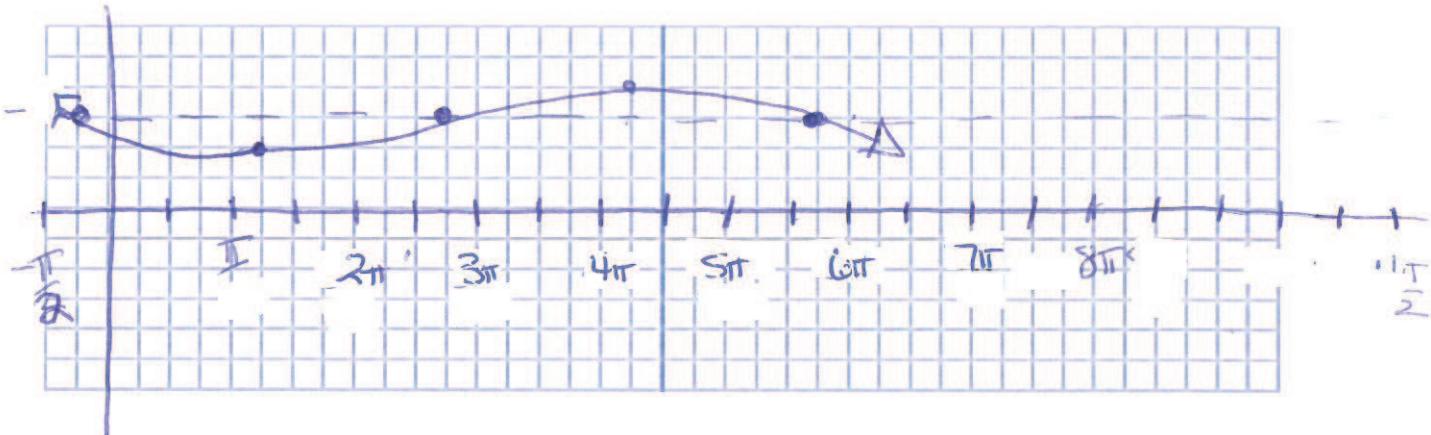
Start @ π end $\pi + \frac{\pi}{2}$



d) $y = -\sin \frac{1}{3}(\theta + \frac{\pi}{4}) + 3$

$$\text{Period} = 6\pi$$

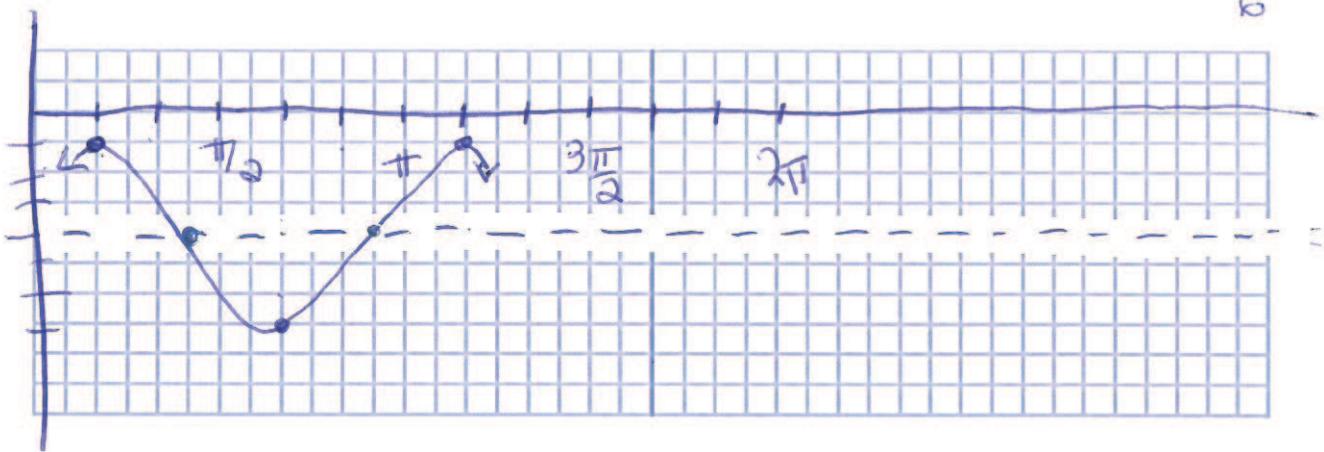
$$\frac{\pi}{4} + 6\pi = \frac{25}{4}\pi$$



e) $y = 3\cos\left(2\theta - \frac{\pi}{3}\right) - 4$

$$y = 3\cos 2(\theta - \frac{\pi}{6}) - 4$$

$$\frac{\pi}{6} + \pi = \frac{7\pi}{6}$$



Chapter 6 - Outcome 30.5

Level 2:

- 1) Verify that the equation $(\sec x + \tan x)\cos x - 1 = \sin x$ is true for $x = 30^\circ$

$$(\sec 30^\circ + \tan 30^\circ) \cos 30^\circ - 1 = \sin 30^\circ$$

$$\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) - 1 \quad \frac{1}{2} = \frac{1}{2}$$

$$\frac{3}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right) - 1 = \frac{1}{2}$$

$$\frac{3}{2} - 1 = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} \checkmark$$

2) Prove the following identities:

a) $\frac{\cos x \csc x}{\sec x \cot x} = \cos x$

$$\begin{aligned} \text{LHS} &= \frac{\cancel{\cos x} \cancel{\cos x} \sin x}{\sin x \cancel{\cos x}} \\ &= \cos x \\ \text{LHS} &= \text{RHS} \quad \square \end{aligned}$$

b) $\cot x \sin x = \cos x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x \cdot \sin x}{\sin x} \\ &= \cos x \\ \text{LHS} &= \text{RHS} \quad \square \end{aligned}$$

c) $\csc x \tan x \sec x \cos x = \sec x$

$$\begin{aligned} \text{LHS} &= \frac{\sin x \cdot \cos x}{\sin x \cdot (\cos x \cdot \cos x)} \\ &= \frac{1}{\cos x} \\ &= \sec x \\ \text{LHS} &= \text{RHS} \quad \square \end{aligned}$$

3) Determine the exact value of each trigonometric expression

a) $\sin 105^\circ$

$$\sin(15+60)$$

$$\sin 45 \cos 60 + \cos 45 \sin 60$$

$$\frac{\sqrt{2}}{2} \left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2} + \sqrt{6}}{4}$$

b) $\cos \frac{\pi}{9} \cos \frac{2\pi}{9} - \sin \frac{\pi}{9} \sin \frac{2\pi}{9}$

$$\cos(\frac{\pi}{9} + 2\frac{\pi}{9})$$

$$\cos(\frac{3\pi}{9})$$

$$\cos(\frac{\pi}{3})$$

Level 3:

4) Prove the following identities.

a) $\sin \theta (\cot \theta + 1) = \sin \theta + \cos \theta$

$$\begin{aligned} &\sin \theta (\cot \theta + 1) \\ &= \sin \theta \cdot \frac{\cos \theta}{\sin \theta} + \sin \theta \\ &= \cos \theta + \sin \theta \quad \square \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

b) $\frac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = \tan x$

$$\begin{aligned} &\frac{\sin x (1 + \sin x)}{\cos x (1 + \sin x)} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

$$\text{LHS} = \text{RHS} \quad \rightarrow$$

$$c) \sin 2x = \tan x + \tan x \cos 2x$$

$$\begin{aligned} \text{RHS} &= \tan x (1 + \cos 2x) \\ &= \tan x (1 + 2\cos^2 x - 1) \\ &= \tan x (2\cos^2 x) \\ &= \frac{\sin x (2\cos^2 x)}{\cos x} \\ &= 2\sin x \cos x \\ &\quad - \frac{1(1+\cos 2x)}{(1-\cos x)} \end{aligned}$$

$$e) \frac{1(1+\cos 2x)}{(1-\cos x)} \frac{1(1-\cos x)}{1+\cos x} = 2 \cot x \csc x$$

$$d) \frac{\tan \theta}{\cos \theta + \cos \theta \tan^2 \theta} = \sin x$$

$$\frac{\tan}{\cos(\tan^2 \theta)}$$

$$\frac{\tan \theta}{\cos \theta \cdot \sec^2 \theta}$$

$$\frac{\sin \theta \cdot \cos \theta}{\cos \theta \cdot \cos \theta}$$

$$\left. \begin{array}{l} \text{Sinx} \\ \text{LHS} = \text{RHS} \end{array} \right\} \square$$

$$\frac{1 + \cos \theta - (1 - \cos \theta)}{\sin^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{2 \cos \theta}$$

$$\frac{2 \cos \theta}{\sin \theta \sin \theta}$$

$$2 \cot \theta \sec \theta$$

LHS = RHS \square

Level 4:

5) Prove:

$$\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$$

$$\frac{1 - (1 - 2\sin^2 \theta)}{2\sin x \cos x}$$

$$\frac{2\sin^2 \theta}{2\sin \theta \cos \theta}$$

$$\left. \begin{array}{l} \tan \theta \\ \text{LHS} = \text{RHS} \end{array} \right\} \square$$

6) State the non-permissible values for questions 4a, 4d and 5.

$$4a) \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin \theta \neq 0$$

$$\theta \neq 0, \pi, 2\pi$$

$$4d) \cos \theta (1 + \tan^2 \theta) \neq 0$$

$$\cos \theta \neq 0 \quad \tan^2 \theta \neq -1$$

$$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\tan \theta \neq \pm \infty \quad \text{no restriction}$$

$$\theta \neq \frac{\pi}{2} + \pi n, n \in \mathbb{N}$$

$$5. \sin x \neq 0$$

$$\theta \neq 0 \pm \pi n, n \in \mathbb{N}$$

$$\cos x \neq 0$$

$$\theta \neq \frac{\pi}{2} \pm \pi n, n \in \mathbb{N}$$

$$\frac{\tan \theta}{\sin \theta} \neq 0 \quad \cos \theta \neq 0$$

Chapter 7 – Outcome 30.9c

Level 2

1. Solve

a) $2^x = 64$

$$2^x = 2^6 \quad \{6\}$$

$x = 6$

c) $8^{2x} = 16^{x+3}$

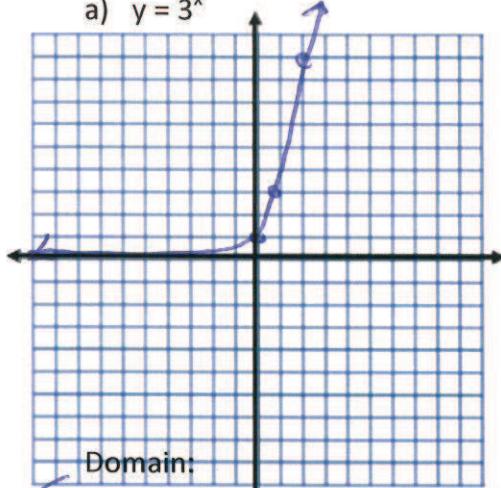
$$2^{3(2x)} = 2^{4(x+3)}$$

$$6x = 4x + 12 \quad x = 6$$

$$2x = 12 \quad \{6\}$$

2. Graph each of the following, and then determine the:

a) $y = 3^x$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

b) $3^x = 27^{x-2}$

$$3^x = 3^{3(x-2)} \quad x = 3 \quad \{3\}$$

$x = 3x - 6$

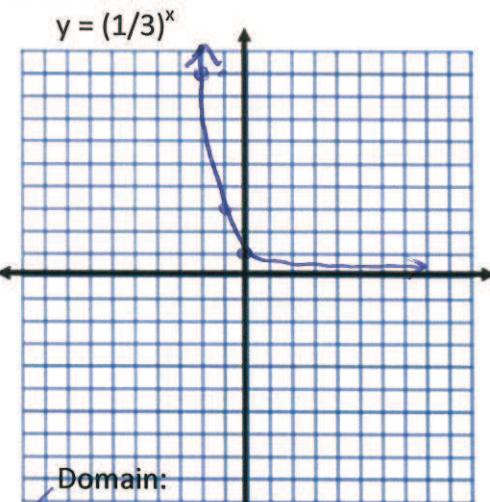
$$-2x = -6$$

d) $9^{2x-5} = 27^{x+6}$

$$3^{2(2x-5)} = 3^{3(x+6)}$$

$$4x - 10 = 3x + 18$$

$$x = 28 \quad \{28\}$$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

2.. Identify all of the transformations of the following: (ie vertical translation up 2)

$$\text{Base } y = 3^x$$

$$a) f(x) = 3^{-x} + 5$$

$b = -1$ h-reflection about y-axis

$K = 5$ v. trans up 5

Level 3

4. Sketch the graph of

$$y = -3^{x-2}$$

$$\text{Base } y = 3^x$$

$$(x, y) \rightarrow (x+2, -y)$$

$$(-1, \frac{1}{3}) \quad (1, -\frac{1}{3})$$

$$(0, 1) \quad (2, -1)$$

$$(1, 3) \quad (3, -3)$$

$$(2, 9) \quad (4, -9)$$

$$(3, 27) \quad (5, -27)$$

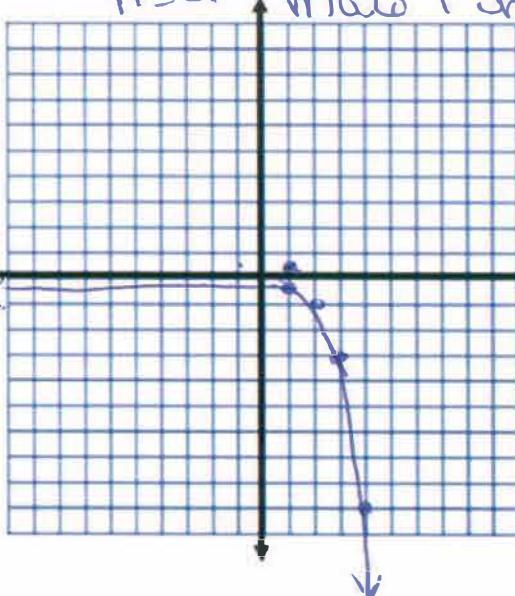
$$\text{Base } y = (\frac{1}{3})^x$$

$$b) h(x) = -2(\frac{1}{3})^{x+1}$$

$$a = -2$$

v. stretch x 2

$h = -1$ v. reflection about x-axis
move 1 unit left



$$y = 3(2^{-x}) - 5$$

$$\text{Base } y = 2^x$$

$$(x, y) \rightarrow (-x, 3y-5)$$

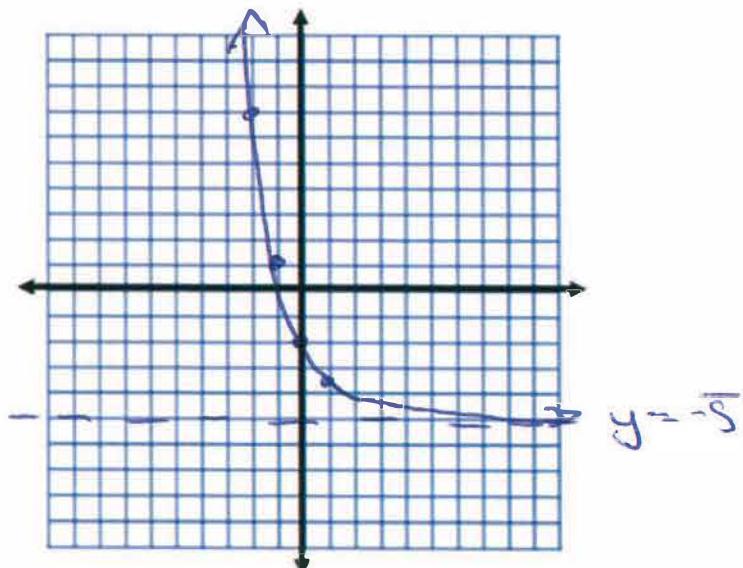
$$(-1, \frac{1}{2}) \rightarrow (1, -3.5)$$

$$(0, 1) \rightarrow (0, -2)$$

$$(1, 2) \rightarrow (-1, 1)$$

$$(2, 4) \rightarrow (-2, 7)$$

$$(3, 8) \rightarrow (-3, 19)$$



$$y = 2^{2x+4} - 1$$

$$y = 2^{2(x+2)} - 1$$

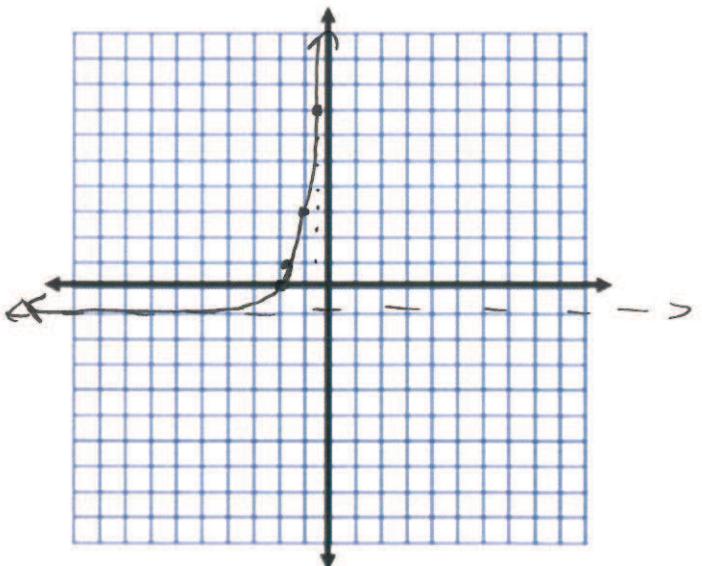
$$(x, y) \rightarrow (\frac{1}{2}x - 2, y - 1)$$

$$(0, 1) \rightarrow (-2, 0)$$

$$(1, 2) \rightarrow (-1.5, 1)$$

$$(2, 4) \rightarrow (-1, 3)$$

$$(3, 8) \rightarrow (-0.5, 7)$$



Chapter 8 – Part 1

Level 2

- Express as a logarithmic statement.

$$2^3 = 8$$

$$\log_2 8 = 3$$

- Express as an exponential statement.

$$\log_3 81 = 4$$

$$3^4 = 81$$

- Determine the value of each logarithm.

a) $\log_5 25$

2

c) $\log_9 1$

0

b) $\log_2 \frac{1}{8}$

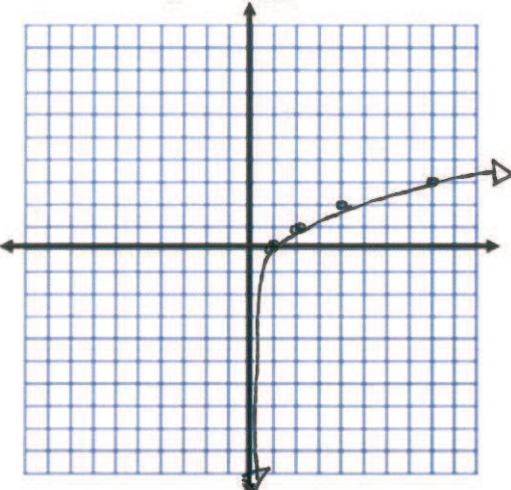
-3

d) $\log_6 6$

1

4. Graph each of the following and determine

b) $y = \log_2 x$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical asymptote $x = 0$

x intercept: $(1, 0)$

y intercept: N/A

5. Identify all of the transformations of the following: (state all stretches/reflections/translations up, down left or right)

a) $y = -2\log_3(x - 5) + 2$

$a = -2$ v. stretch $\times 2$
 $b = 1$ v. ref. about x axis
 $h = 5$ right 5
 $k = 2$ up 2

$y = 2\log_3(-x) + 1$

$a = 2$ v. stretch $\times 2$
 $b = -1$ h. reflect. about y
 $h = 0$
 $k = 1$ up 1

Level 3

6. Sketch

$y = -\log_2(x + 1) - 2$

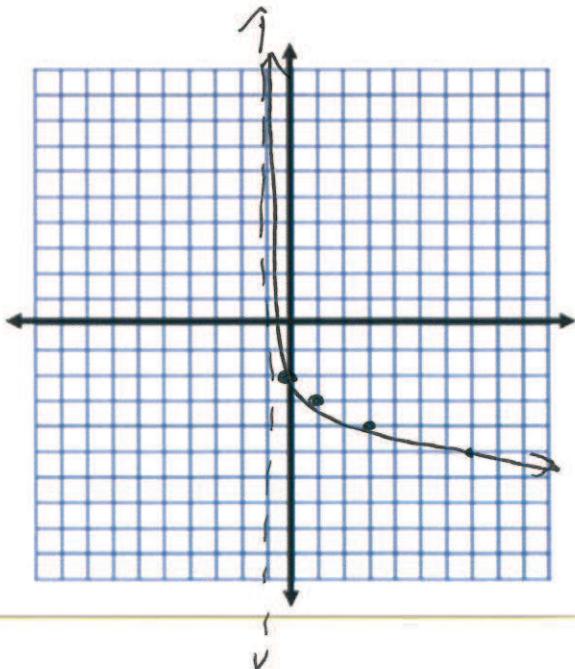
$$(x, y) \rightarrow (x-1, -y-2)$$

$$(1, 0) \rightarrow (0, -2)$$

$$(2, 1) \rightarrow (1, -3)$$

$$(4, 2) \rightarrow (3, -4)$$

$$(8, 3) \rightarrow (7, -5)$$



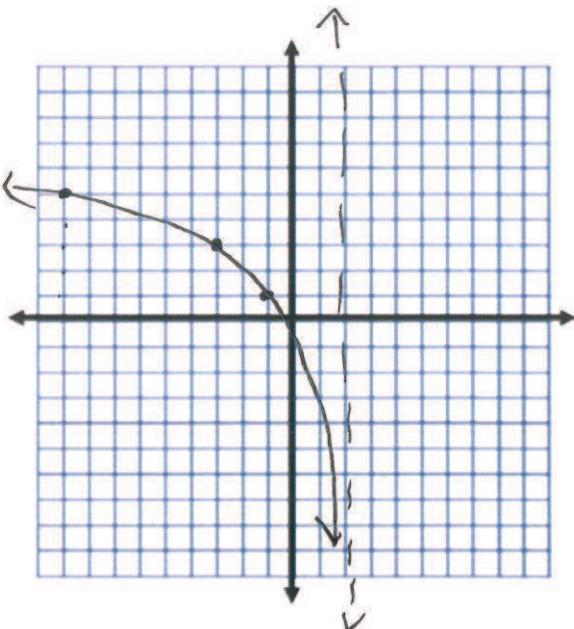
$$y = 2 \log_3(x - 2) + 1$$

$$(x, y) \rightarrow (-x, 2y + 1)$$

$$(1, 0) \rightarrow (-1, 1)$$

$$(3, 1) \rightarrow (-3, 3)$$

$$(9, 2) \rightarrow (-9, 5)$$



Chapter 8 Part 2

Level 2

1. Use your laws of logarithms to expand each of the following:

a) $\log_4 \frac{x}{3}$

b) $\log_4 x^5$

c) $\log_2 yx^5$

$\log_4 x - \log_4 3$

$5 \log_4 x$

$\log_2 y + 5 \log_2 x$

2. Use the laws of logarithms to simplify each of the following:

a) $\log 2 + \log 7$

b) $4 \log_3 5$

c) $\log_2 42 - \log_2 6$

$\log 14$

$\log_3 625$

$\log_2 7$

3. Determine the value of x.

a) $\log_2 x = 3$

$$\begin{aligned} 2^3 &= x \\ 8 &= x \end{aligned}$$

b) $3 \log_5 x = \log_5 125$

$$\begin{aligned} \log_5 x^3 &= \log_5 5^3 \\ x^3 &= 5^3 \\ x &= 5 \end{aligned}$$

c) $6^x = 216$

$$6^x = 6^3$$

d) $4^{x+1} = 64$

$$4^{x+1} = 4^3$$

$x = 3$

$x = 2$

Level 3

4. Use the laws of logarithms to simplify and then evaluate each of the following:

a) $\log_3 270 - (\log_3 2 + \log_3 5)$

$$\log_3 \frac{270}{2 \cdot 5}$$

$$\log_3 \frac{270}{10}$$

$$\log_3 27$$

$$3$$

$$\log_2 \frac{270}{27}$$

$$\log_2 8$$

$$3$$

5. Write each expression in terms of individual logarithms.

a) $\log_2 \frac{x^5 \sqrt[3]{y}}{z^2}$

b) $\log_5 \sqrt{xy^3}$

$$5 \log_2 x + \frac{1}{3} \log_2 y - \log_2 z - \log_2 x = \frac{1}{2} \log_5 x + \frac{3}{2} \log_5 y$$

6. Write each expression as a single logarithm.

a) $3 \log w + \log \sqrt{w} - 2 \log w$

b) $\log_2(x+6) + \log_2(x-1)$

$$\log w^3 + \log w^{1/2} - \log w^2$$

$$\log \frac{w^3 \cdot w^{1/2}}{w^2}$$

$$\log_2 (x+6)(x-1)$$

$$\log_2 (x^2 + 5x - 6)$$

$$3 + \frac{1}{2} - 2$$

$$1 \frac{1}{2}$$

$$\frac{3}{2}$$

7. Solve for x.

a) $\log_5 x + 6 = 8$

$$\log_5 x = 2$$

$$5^2 = x$$

$$25 = x$$

$$\log_4 x^3 = 6$$

$$\sqrt[3]{4^6} = \sqrt[3]{x^3}$$

$$4^2 = x$$

$$16 = x$$

$$16 = x$$

$$c) \log_2 x^2 - \log_2 5 = \log_2 20$$

$$\log_2 \frac{x^2}{5} = \log_2 20$$

$$\sqrt{\frac{x^2}{5}} = \sqrt{20}$$

$$\sqrt{x^2} = \pm\sqrt{100}$$

$$x = \pm 10$$

*-10 works b/c $\log_2(-10)^2 = \log_2 100$

$$e) 3^x = 100$$

$$\log 3^x = \log 100$$

$$x \log 3 = \log 100$$

$$x = \frac{\log 100}{\log 3}$$

$$x \approx 4.1918$$

$$d) \log_3(x+7) - \log_3(x-3) = 2$$

$$\log_3 \frac{(x+7)}{(x-3)} = 2$$

$$3^2 = \frac{x+7}{x-3}$$

$$9 = \frac{x+7}{x-3}$$

$$9x - 27 = x + 7$$

$$8x = 34$$

$$f) 7^{x-3} = 517$$

$$x = \frac{34}{8} = \frac{17}{4} = 4.25$$

$$\log 7^{x-3} = \log 517$$

$$(x-3) \frac{\log 7}{\log 7} = \log 517$$

$$x-3 = \frac{\log 517}{\log 7} + 3$$

$$x = \frac{\log 517}{\log 7} + 3$$

Level 4

8. Solve the following. State any restrictions

$$\log_6(x+3) - 2 = -\log_6(x-2)$$

$$x \approx 6.21086$$

$$x+3 > 0$$

~~$x > -3$~~

$$\log_6(x+3) + \log_6(x-2) = 2$$

$$\log_6 (x+3)(x-2) = 2$$

$$x-2 > 0$$

$$\textcircled{X} > 2$$

$$6^2 = (x+3)(x-2)$$

$$36 = x^2 + 3x - 2x - 6$$

$$-36$$

$$0 = x^2 + x - 42$$

$$0 = (x+7)(x-6)$$

~~$x = 6$~~

9. Use what you have learned about logarithms to show how you could use two different transformations to graph the logarithmic function $y = \log_2 8x$

$$\textcircled{1} \quad y = \log_2 8x \rightarrow \text{h. stretch of } \frac{1}{8}$$

$$2. \quad y = \log_2 8 + \log_2 x$$

$$y = 3 + \log_2 x$$

v. trans up 3

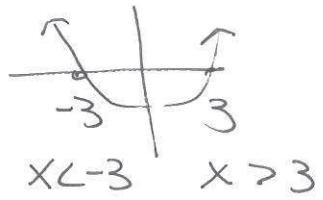
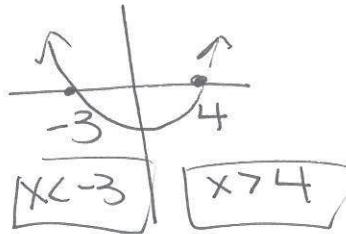
10. Simplify the following logarithm. State the restrictions

$$\log(x^2 - x - 12) - \log(x^2 - 9)$$

$$\log \left(\frac{x^2 - x - 12}{x^2 - 9} \right)$$

$$\log \left(\frac{(x+4)(x-3)}{(x+3)(x-3)} \right)$$

$$\log \left(\frac{x+4}{x-3} \right), \quad x < -3, \quad x > 4$$



Chapter 9 Review

Level 2

1. Determine the characteristics of the following functions:

a) $y = \frac{2x-1}{x-4}$

Equation of Vertical Asymptotes: $x = 4$

Points of Discontinuity (holes): NA

Equation of Horizontal Asymptote: $y = 2$

b) $y = \frac{x+5}{(x+5)(x-3)} = \frac{1}{x-3}$

Equation of Vertical Asymptotes: $x = 3$

Points of Discontinuity (holes): $(-5, -\frac{1}{8})$

Equation of Horizontal Asymptote: $y = 0$

c) $y = \frac{x^2-4}{x^2+3x+2} = \frac{(x-2)(x+2)}{(x+2)(x+1)} = \frac{x-2}{x+1}$

Equation of Vertical Asymptotes: $x = -1$

Points of Discontinuity (holes): $(-2, 4)$

Equation of Horizontal Asymptote: $y = 1$

Level 3/Level 4 (Level 4 Questions will have an oblique asymptote. You will need to determine that on your own.)

2. Graph the following functions. Be sure to give the equations of all asymptotes.

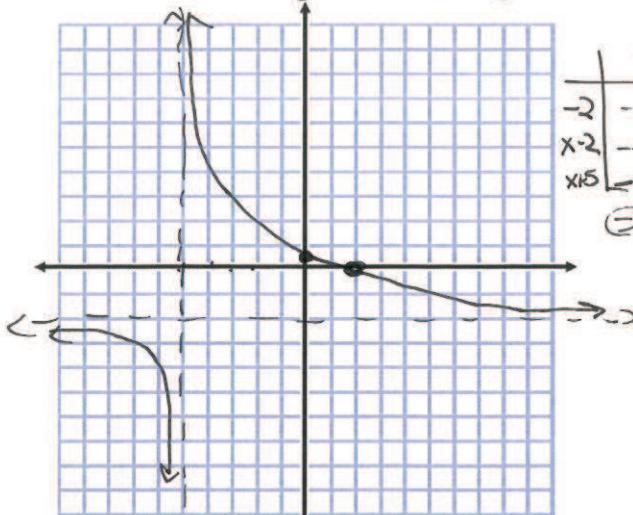
a) $y = \frac{-2x+4}{x+5} = -\frac{2(x-2)}{x+5}$

V.A @ $x = -5$

HA @ $y = -2$

x int (2, 0)

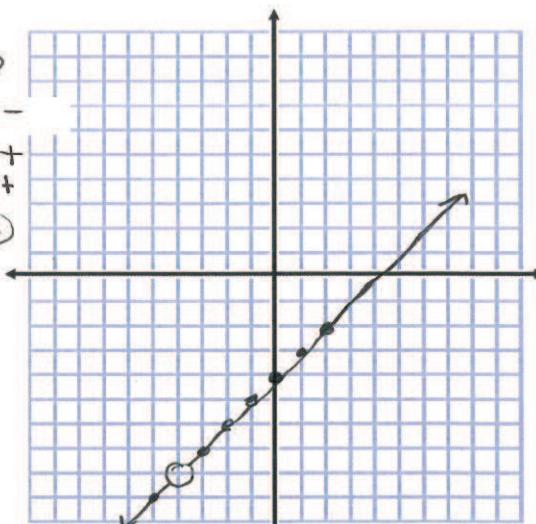
y int (0, 4)



b) $y = \frac{x^2-16}{x+4} = \frac{(x-4)(x+4)}{x+4} = x-4$

hole (-4, -8)

-5	2
-2	-1
x-2	-1
x+5	1
⊕	+
⊖	-



c) $y = \frac{x-5}{x^2-2x-15} = \frac{(x-5)}{(x-5)(x+3)}$

VA @ $x = -3$

HA @ $y = 0$

y int (0, 1/3)

no x int

$$= \frac{1}{x+3}$$

d) $y = \frac{x^2-3x-18}{x^2+7x+12} = \frac{(x-6)(x+3)}{(x+3)(x+4)}$

$$= \frac{x-6}{x+4}$$

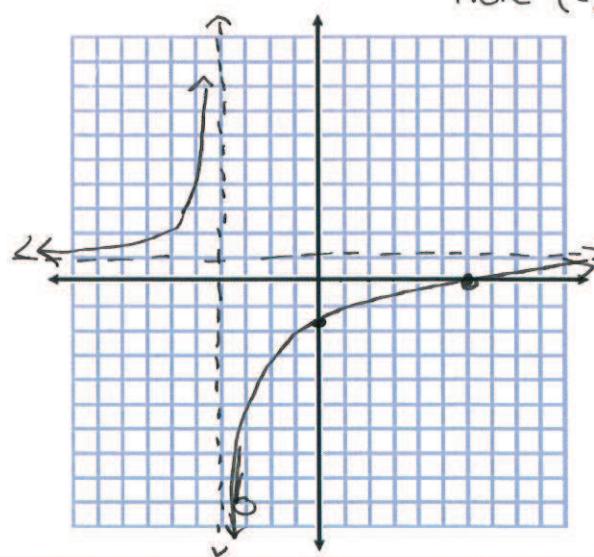
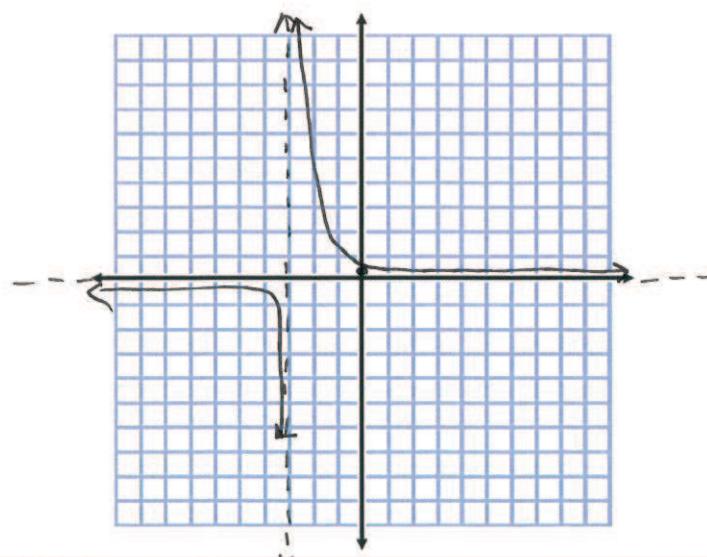
V.A @ $x = -4$

HA @ $y = 1$

x int (6, 0)

y int (0, -1.5)

Hole (-3, -9)



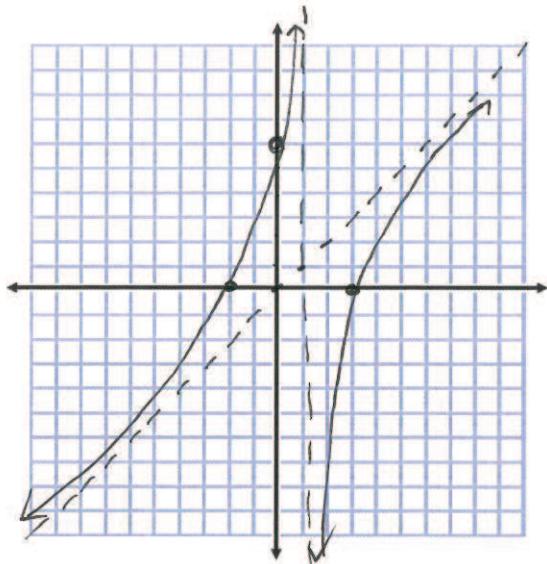
$x+3$

-	1	+
⊖	⊕	⊕
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$x-6$

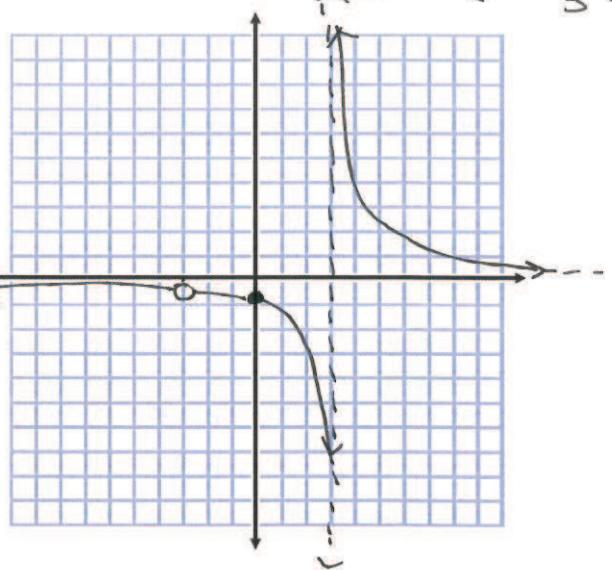
-	1	-	1	+
⊖	⊕	⊖	⊕	⊕
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$$y = \frac{x^2 - x - 6}{x-1} = \frac{(x-3)(x+2)}{(x-1)}$$



$$y = \frac{2x+6}{x^2-9} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$$

Hole $(-3, -\frac{1}{3})$
 $y \text{ int } (0, -\frac{2}{3})$



O.A.

$$\begin{array}{c} x_{\text{int}} \\ (-2, 0) \\ (3, 0) \end{array}$$

$$\begin{array}{c} \cancel{-1} \quad \boxed{1} \quad -1 \quad -6 \\ \downarrow \quad \downarrow \quad -1 \quad 0 \\ 1x \quad 0 \quad \boxed{-6} \end{array} \quad y_{\text{int}} (0, 6)$$

$$\begin{array}{c} 2 \\ x-3 \\ \hline + \\ - \\ \ominus \end{array} \quad \begin{array}{c} -3 \\ + \\ + \\ \oplus \end{array}$$

C.A @ $y = x$

$$\begin{array}{c} -2 \quad 1 \quad 3 \\ \hline - \quad + \quad - \quad - \quad + \\ - \quad + \quad + \quad + \quad + \\ - \quad + \quad - \quad + \quad + \\ \ominus \quad \oplus \quad \ominus \quad \ominus \quad \oplus \end{array}$$