

Final Exam Review

(AK)

Outcome 1A Review

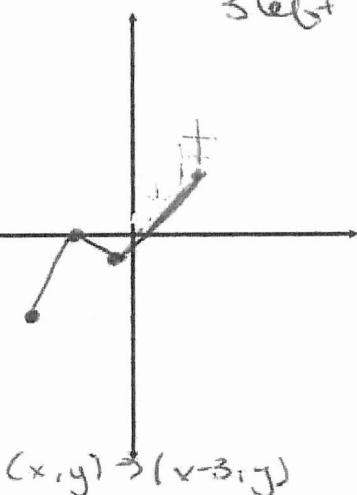
Level 2

1. Consider the graph of $y = f(x)$.

a) Sketch each of the transformed function:

i) $y = f(x+3)$

3 left



ii) $y = -2f(x)$

V.S. at x-axis

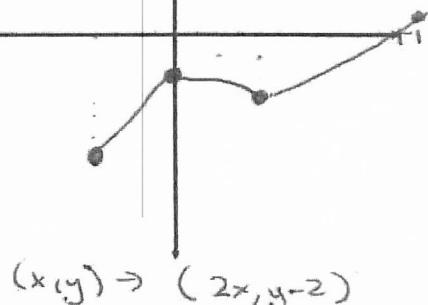
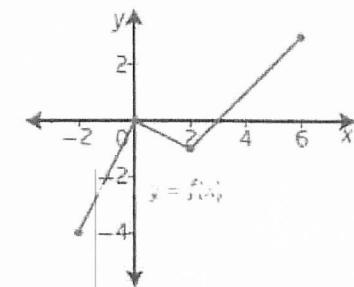
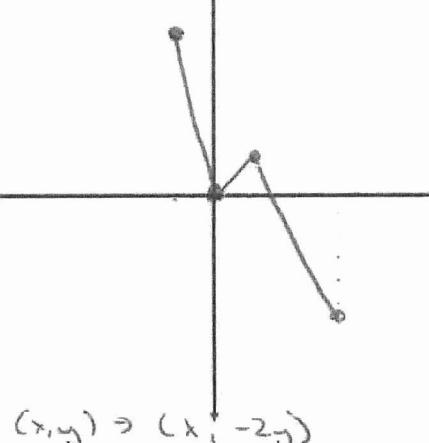
$y = f(-2x)$

U.reflection about x-axis

$y = f(\frac{1}{2}x) - 2$

H.S x2

Down 2



2. For each equation, describe how the graph was translated, reflected or stretched.

a) $y = -2f(3(x-4))$

V.stretch factor of 2

V.reflection about x-axis

H.stretch factor of $\frac{1}{3}$

H. translation 4 right

b) $y = f(x-5)-3$

H. translation 5 right

V. translation 3 down

c) $y = f(-2x)+5$

H.stretch factor $\frac{1}{2}$

H. reflection about x-axis

V.translation
5 down

3. Consider the graph of $y = f(x)$ and $y = g(x)$.

Determine the equation of the translated function in the form

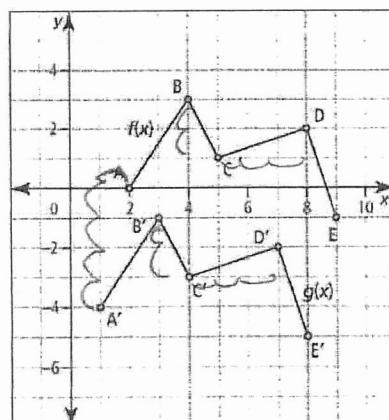
$y = af(b(x-h))+k$.

$y = f(x+1) - 4$

4 down $\Rightarrow k = -4$

1 left $\Rightarrow h = -1$

no V. or H
stretch.



5. Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$

$$\begin{aligned} x &= 3y - 6 \\ x + 6 &= 3y \\ y &= \frac{x+6}{3} \\ y &= \frac{1}{3}x + 2 \end{aligned}$$

b) $f(x) = x^2 - 7$

$$\begin{aligned} x &= y^2 - 7 \\ x + 7 &= y^2 \\ y &= \pm\sqrt{x+7} \end{aligned}$$

c) $y = (x - 5)^2 - 9$

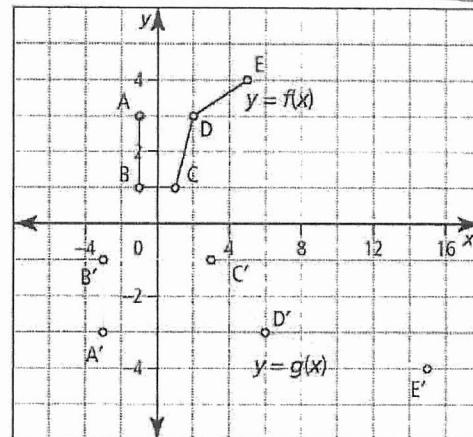
$$\begin{aligned} x &= (y - 5)^2 - 9 \\ \pm\sqrt{x+9} &= \sqrt{(y-5)^2} \\ y - 5 &= \pm\sqrt{x+9} \\ y &= \pm\sqrt{x+9} + 5 \end{aligned}$$

Level 3

6. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine an equation for $g(x)$.

$$y = -f(\frac{1}{3}x)$$

- V. reflection
 a \Rightarrow -ve
 II. stretch $\times 3$
 b $= \frac{1}{3}$



$$(x, y) \rightarrow (3x, -y)$$

7. Write the equation for each transformation of $y = x^2$ in the form

$$y = af(b(x - h)) + k$$

- a) a vertical stretch by a factor of 3, reflected in the y-axis, and translated 3 units left and 2 units down

$$y = 3f(-(x+3)) - 2$$

- b) a horizontal stretch by a factor of 2, reflected in the x-axis, and translated 7 units up

$$y = -f(\frac{1}{2}x) + 7$$

9. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form

$$y = af(b(x - h)) + k$$

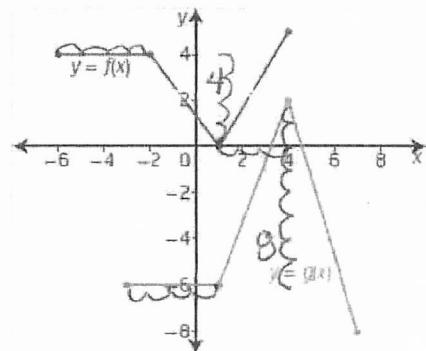
$$y = -2f(x-3) + 2$$

V. reflection

a \Rightarrow -ve

No H stretch

V. stretch $\times 2$



3 right, 2 up.

10. The key point $(-18, 12)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $-3f(x + 5) + 4$

$$(x, y) \rightarrow (x - 5, -3y + 4)$$

$$(-18, 12) \rightarrow (-18 - 5, -3(12) + 4)$$

$$\rightarrow \boxed{(-23, -32)}$$

b) $y = 2f(6x)$

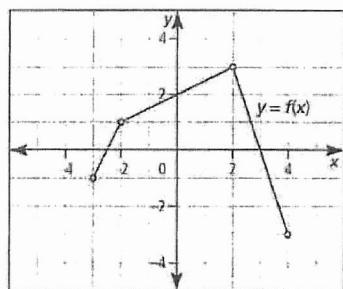
$$(x, y) \rightarrow (\frac{1}{6}x, 2y)$$

$$(-18, 12) \rightarrow (\frac{1}{6}(-18), 2(12))$$

$$\rightarrow \boxed{(-3, 24)}$$

11. Consider the graph of the function $y = f(x)$.

Sketch $y = f(x)$ to $y = 3f(-2(x - 1)) + 4$.



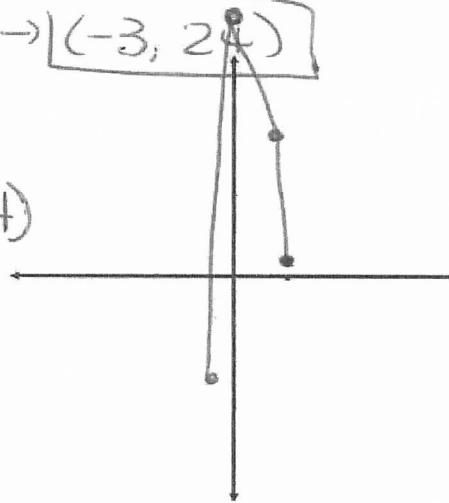
$$(x, y) \rightarrow (-\frac{1}{2}x + 1, 3y + 4)$$

$$(-3, -1) \rightarrow (\frac{5}{2}, 1)$$

$$(-2, 1) \rightarrow (2, 7)$$

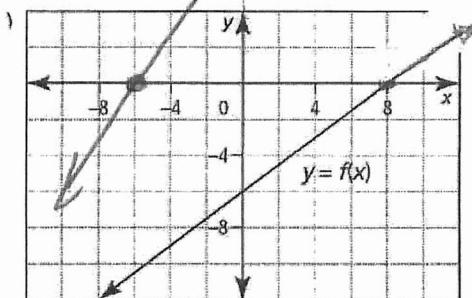
$$(2, 3) \rightarrow (0, 13)$$

$$(4, -3) \rightarrow (-1, -5)$$

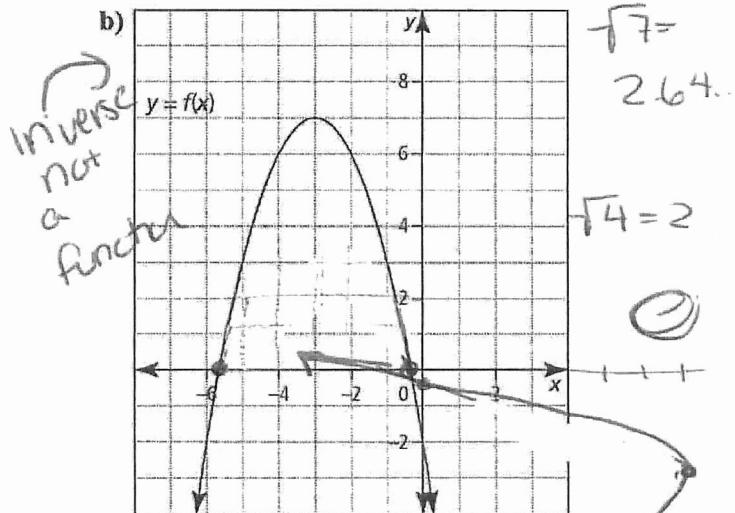


Level 4

12. Sketch the graph of its inverse, $x = f(y)$. Determine whether the inverse is a function. If the inverse is not a function, restrict the domain of the original graph to make it a function.

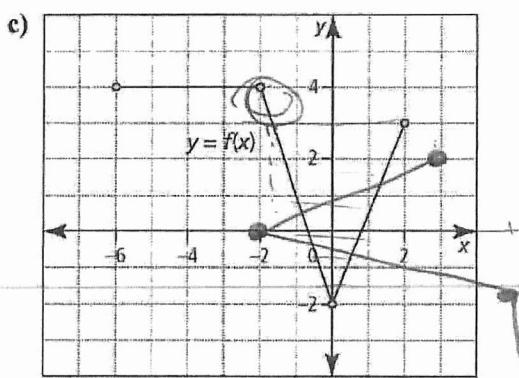


Inverse is a function



$$\sqrt{7} = 2.64$$

$$\sqrt{4} = 2$$



$$(x, y) \rightarrow (y, x)$$

Inverse
Not a function

Restrict domain
to $(-\infty, -3]$
or
 $[-2, 0]$ or
 $[0, 2]$

or
 $[-3, \infty)$

Outcome 2A

1. Identify a, b, h and k for each of the following

a) $y = 5\sqrt{x+7} - 2$

$$\begin{aligned} a &= 5 \\ b &= 1 \\ h &= -7 \\ k &= -2 \end{aligned}$$

2. Graph $y = \sqrt{x}$

x	0	1	4	9
	0	1	2	3
	0	1	2	3
	0	1	2	3
	0	1	2	3

Level 3

3. Write the equation of a radical function that would result by applying each set of transformations to the graph of $f(x) = \sqrt{x}$

- a) vertical stretch by a factor of 3, and horizontal stretch by a factor of 2

$$y = 3\sqrt{\frac{1}{2}x} \quad \text{or} \quad y = 3f(\frac{1}{2}x)$$

- b) horizontal reflection in the y-axis, translation up 3 units, and translation left 2 units

$$y = \sqrt{-(x+2)} + 3 \quad \text{or} \quad y = f(-x+2) + 3$$

4. Graph the functions below.

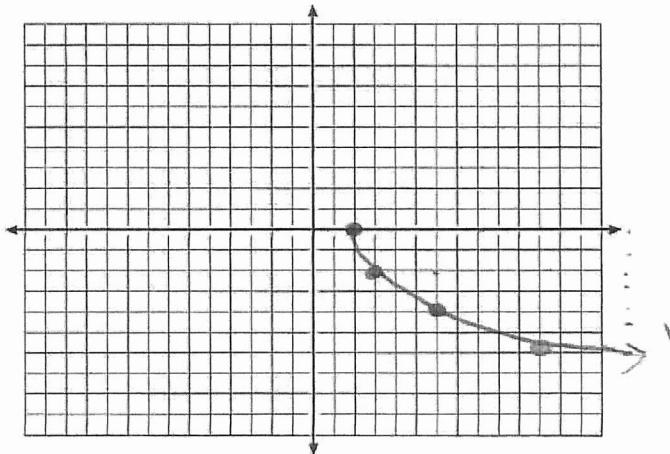
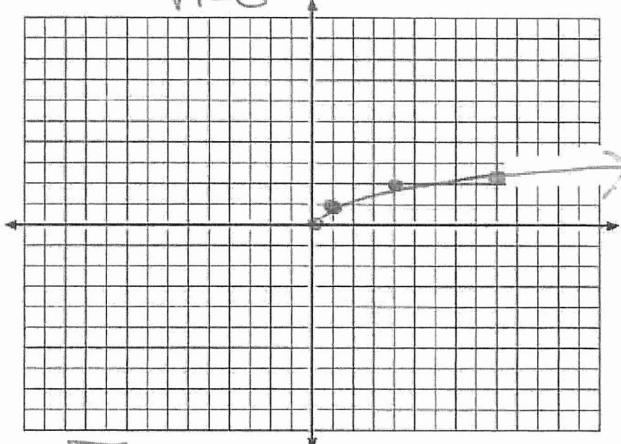
Then, identify the domain and range.

a) $y = -2\sqrt{x-2}$

$$\begin{aligned} (x,y) &\rightarrow (x+2, -2y) \\ (0,0) &\rightarrow (2, 0) \\ (1,1) &\rightarrow (3, -2) \\ (4,2) &\rightarrow (6, -4) \\ (9,3) &\rightarrow (11, -6) \end{aligned}$$

b) $y = -4\sqrt{-x} + 8$

$$\begin{aligned} a &= -4 \\ b &= -1 \\ h &= 0 \\ k &= 8 \end{aligned}$$



$$\begin{aligned} D &= [2, \infty) \\ R &= [-\infty, 0] \end{aligned}$$

c) $y = \sqrt{2x} - 4$

$$(x, y) \rightarrow (\frac{1}{2}x, y+4)$$

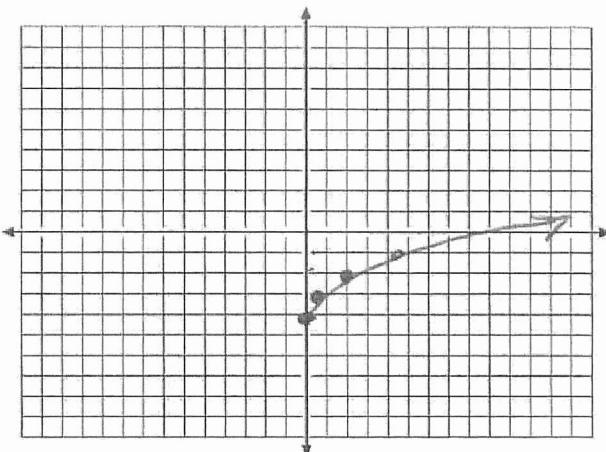
$$(0, 0) \rightarrow (0, -4)$$

$$(1, 1) \rightarrow (\frac{1}{2}, -3)$$

$$(4, 2) \rightarrow (2, -2)$$

$$(9, 3) \rightarrow (4.5, -1)$$

$$D = [0, \infty) \quad R = [-4, \infty)$$



c) $y = 2\sqrt{-(x-3)} + 1$

$$(x, y) \rightarrow (-x+3, 2y+1)$$

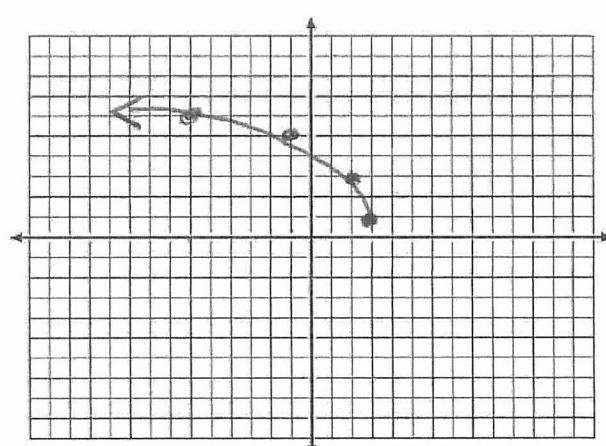
$$(0, 0) \rightarrow (3, 1)$$

$$(1, 1) \rightarrow (2, 3)$$

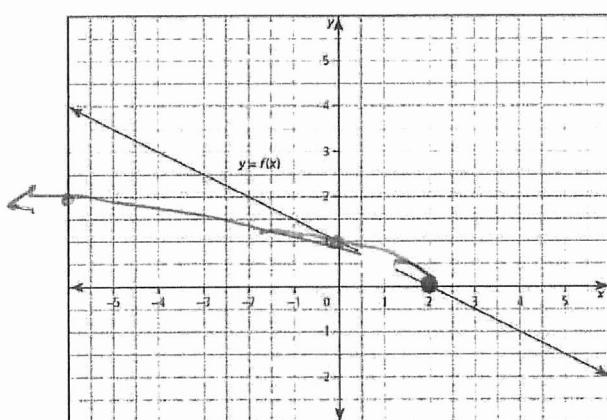
$$(4, 2) \rightarrow (-1, 5)$$

$$(9, 3) \rightarrow (-6, 7)$$

$$D = (-\infty, 3] \quad R = [1, \infty)$$

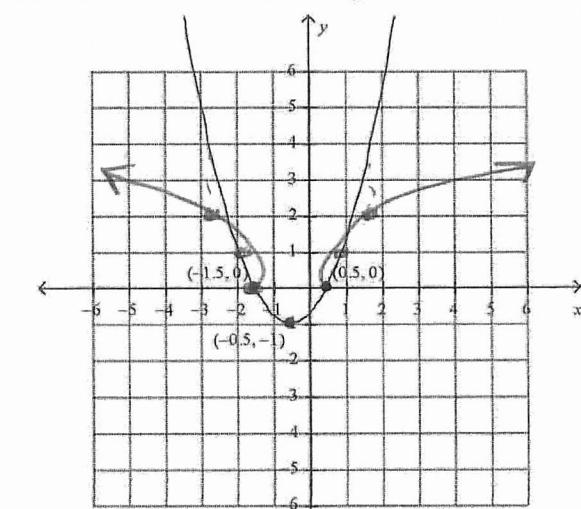


5. Graph $\sqrt{f(x)}$ from the following graphs of $f(x)$ and state the domain and range



$$D = (-\infty, 2]$$

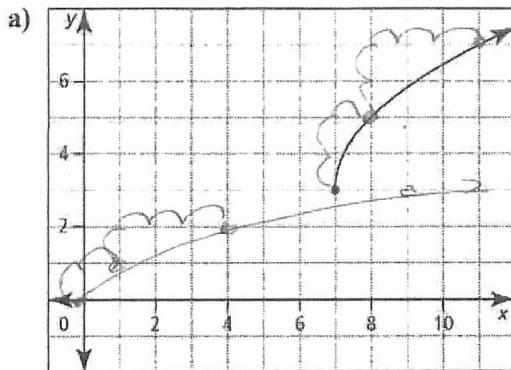
$$R = [0, \infty)$$



$$D = (-\infty, -1.5] \cup [0.5, \infty)$$

$$R = [0, \infty)$$

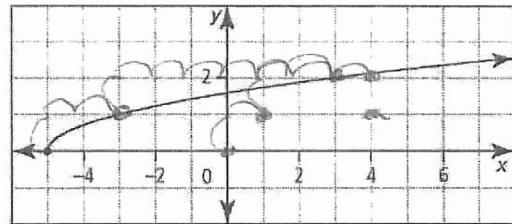
6. For each function, write an equation of a radical function of the form $y = a\sqrt{b(x - h)} + k$.



$$y = 2f(x-7) + 3$$

$$y = 2\sqrt{x-7} + 3$$

b)



H. stretch $\times 2$

$$y = f(\frac{1}{2}(x+5))$$

$$y = \sqrt{\frac{1}{2}(x+5)}$$

Outcome 3A Review

1. Divide the following using long division or synthetic division.

a) $(2w^4 + 3w^3 - 5w^2 + 2w - 27) \div (w+3)$

$$\begin{array}{r} 2 \ 3 \ -5 \ 2 \ -2 \\ \downarrow \quad \quad \quad \quad \quad \\ 6 \ -9 \ 12 \ -30 \\ 2 \ -3 \ 4 \ -10 \ 3 \end{array}$$

$$\boxed{2w^3 - 3w^2 + 4w - 10 + \left(\frac{3}{w+3}\right)}$$

b) $\frac{2x^3 - 10x^2 - 15x - 20}{x+5}$

$$\begin{array}{r} 2 \ -10 \ -15 \ -20 \\ \downarrow \quad \quad \quad \quad \\ 10 \ -100 \ 425 \\ 2 \ -20 \ 85 \ -445 \end{array}$$

$$2x^2 - 20x + 85 + \left(\frac{-445}{x+5}\right)$$

2. Determine the remainder when $x^3 + x^2 - 16x - 16$ is divided by

a) $x+2$

b) $x-4$

$P(-2) = 12$

$P(4) = 0$

b) Are any of the binomials above a factor of $x^3 + x^2 - 16x - 16$?

$x-4$

3. Factor completely

a. $x^3 + 2x^2 - 13x + 10$

$$P(1) = 1+2-13+10 \\ = 0$$

$x-1$ a factor

$$\begin{array}{r} \boxed{1} & 2 & -13 & 10 \\ -1 & \downarrow & -1 & -3 & 10 \\ \hline 1 & 3 & -10 & 0 \end{array}$$

$$(x-1)(x^2 + 3x - 10)$$

$$(x-1)(x+5)(x-2)$$

4. Determine the value(s) of k so that the binomial is a factor of the polynomial: $x^2 - 8x - 20, x+k$

$$P(-k) = 0$$

$$(-k)^2 - 8(-k) - 20 = 0$$

$$k^2 + 8k - 20 = 0$$

$$(k+10)(k-2) = 0$$

$$k=10 \quad k=2$$

5. The following polynomial has a factor of $x-3$. What is the value of k ? $kx^3 - 10x^2 + 2x + 3$

$$P(3) = 0$$

$$k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$$

$$27k - 90 + 6 + 3 = 0$$

$$27k - 81 = 0$$

Outcome 3B Review

$$\frac{27k-81}{27}$$

$$k=3$$

1. Determine which of the following are polynomials. For each polynomial function, state the degree.

a) $h(x) = 5 - \frac{1}{x}$

No

b) $y = 4x^2 - 3x + 8$

Yes
 $d=2$

c) $g(x) = -9x^6$

Yes
 $d=6$

d) $f(x) = \sqrt[3]{x}$

No

2. What is the leading coefficient, degree and constant term of each polynomial function?

a) $f(x) = -x^3 + 6x - 8$

$LC = -1$

$\deg = 3$

$Con = -8$

c) $g(x) = 7x^3 + 3x^5 - 8x + 10$

$LC = 3$

$\deg = 5$

$Con = 10$

b) $y = 5 + 2x^2$

$LC = 2$

$\deg = 2$

$Con = 5$

d) $k(x) = 9x - 2x^2$

$LC = -2$

$\deg = 2$

$Con = 0$

3. Identify the following characteristics for each polynomial function:

- the type of function and whether it is of even or odd degree
- the end behaviour of the graph of the function
- the number of possible x -intercepts
- the y -intercept

a) $g(x) = -2x^4 + 6x^2 - 7x - 5$

Even $0-4 \times \text{int}$

$\text{III} \rightarrow \text{IV}$ $y_{\text{int}} = -5$

b) $f(x) = 2x^5 + 1x^3 - 12$

Odd

$\text{II} \rightarrow \text{I}$

$1-5 \times \text{int}$

$y_{\text{int}} = -12$

4. Fill in the table below for the following graphs

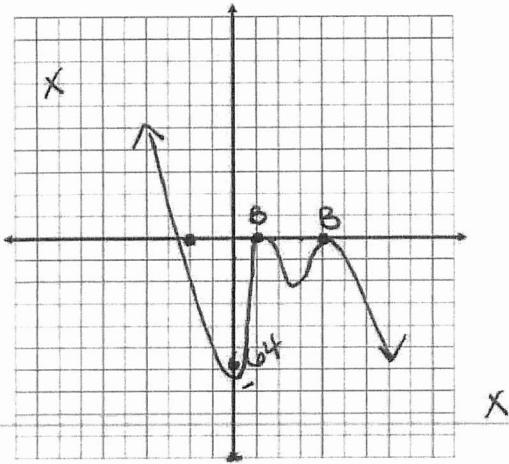
Graph	Odd or Even	Sign of Leading Coefficient	Number of x -intercepts	Intervals where the function is positive	Intervals where the function is negative
	ODD	+	3	(-∞, -4) ∪ (-4, -2) ∪ (0, 3)	(-2, 0) ∪ (3, ∞)

	Odd or Even	Sign of Leading Coefficient	Number of x-intercepts	Intervals where the function is positive	Intervals where the function is negative
	O	-	3	(-∞, -4) ∪ (-1, 3) ∪ (3, ∞)	(-4, -1)
	E	-	4	(-2, -1) ∪ (-1, 2) ∪ (2, 3) ∪ (3, ∞)	(-∞, -2)
	E	+	3	(-∞, -1) ∪ (1, 3) ∪ (3, ∞)	(-1, 1)

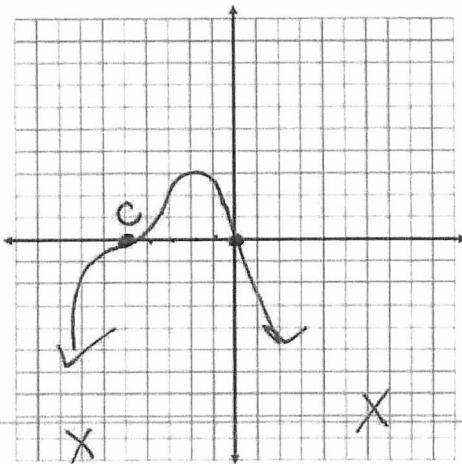
4. Graph the following polynomial functions. The first three have already been factored for you.

$$y = -2(x - 1)^2(x + 2)(x - 4)^2 \quad \text{d=5, } \text{even}$$

$$y = -2x(x+5)^3 \quad \text{c=3, } \text{odd}$$



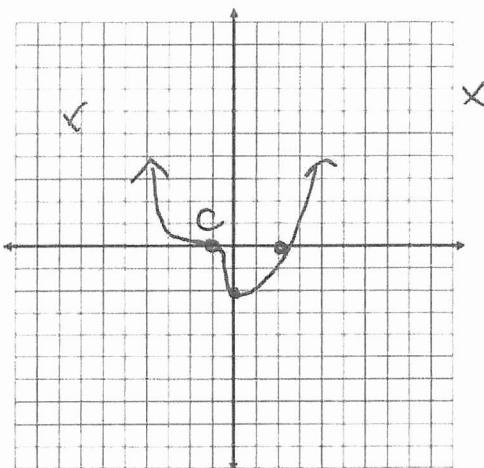
$$f(0) = -2(-1)^2(2)(-4)^2$$



$$f(0) = (1)^3(-2) = -2$$

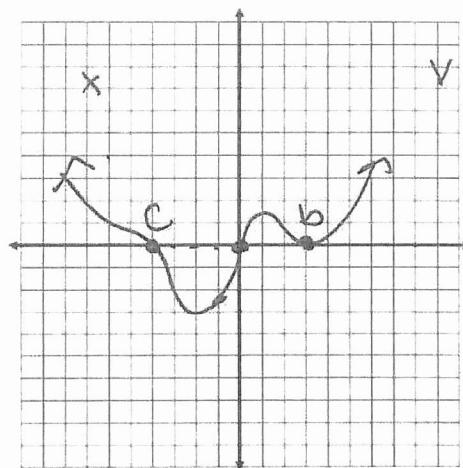
$$y = (x+1)^3(x-2)$$

$\alpha = 4$
+ LC



$$y = x(x+4)^3(x-3)^2$$

$\alpha = 6$
+ LC

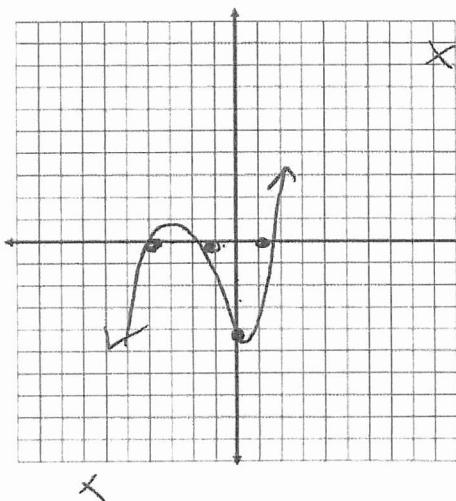
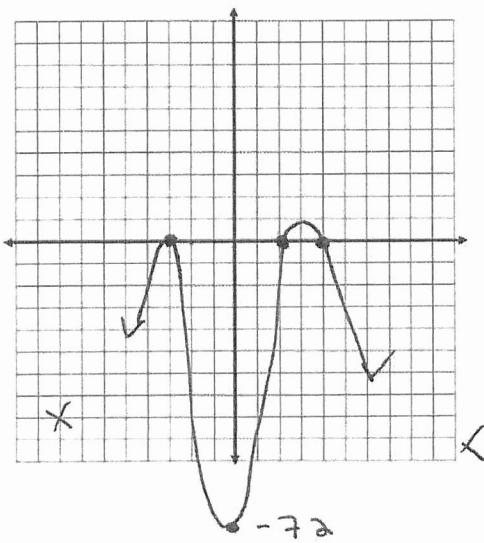


$$f(x) = -x^4 + 19x^2 + 6x - 72$$

$$y = x^3 + 4x^2 - x - 4$$

$$\begin{aligned} & -(x-2)(x+3)(x-4)(x+3) \\ & -(x-2)(x+3)^2 \quad (\cancel{(x-4)}) \end{aligned}$$

$$(x-1)(x+4)(x+1)$$



Chapter 7 – Outcome 30.9c

Level 2

1. Solve

a) $2^x = 64$

$$2^x = 2^6 \quad \{6\}$$

$$x = 6$$

c) $8^{2x} = 16^{x+3}$

$$2^{3(2x)} = 2^{4(x+3)}$$

$$6x = 4x + 12 \quad x = 6$$

$$2x = 12 \quad \{6\}$$

b) $3^x = 27^{x-2}$

$$3^x = 3^{3(x-2)} \quad x = 3 \quad \{3\}$$

$$x = 3x - 6$$

$$-2x = -6$$

d) $9^{2x-5} = 27^{x+6}$

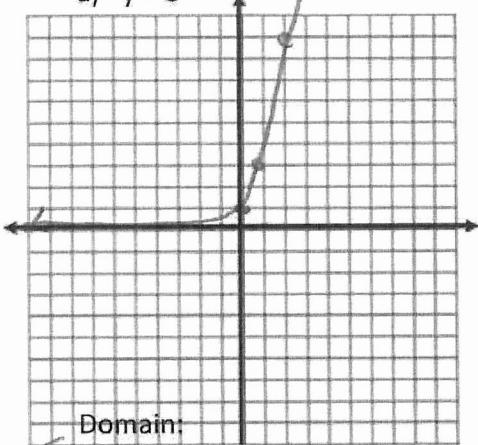
$$3^{2(2x-5)} = 3^{3(x+6)}$$

$$4x - 10 = 3x + 18$$

$$x = 28 \quad \{28\}$$

2. Graph each of the following, and then determine the:

a) $y = 3^x$



Domain: $(-\infty, \infty)$

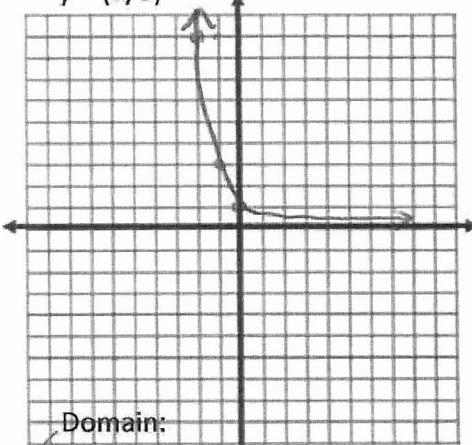
Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

$y = (1/3)^x$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: $y = 0$

Y intercept: $(0, 1)$

Increasing or Decreasing:

2. Identify all of the transformations of the following: (ie vertical translation up 2)

$$\text{Base } y = 3^x$$

a) $f(x) = 3^{-x} + 5$

$$\text{Base } y = \left(\frac{1}{3}\right)^x$$

b) $h(x) = -2\left(\frac{1}{3}\right)^{x+1}$

$b = -1$ h-reflection about y-axis

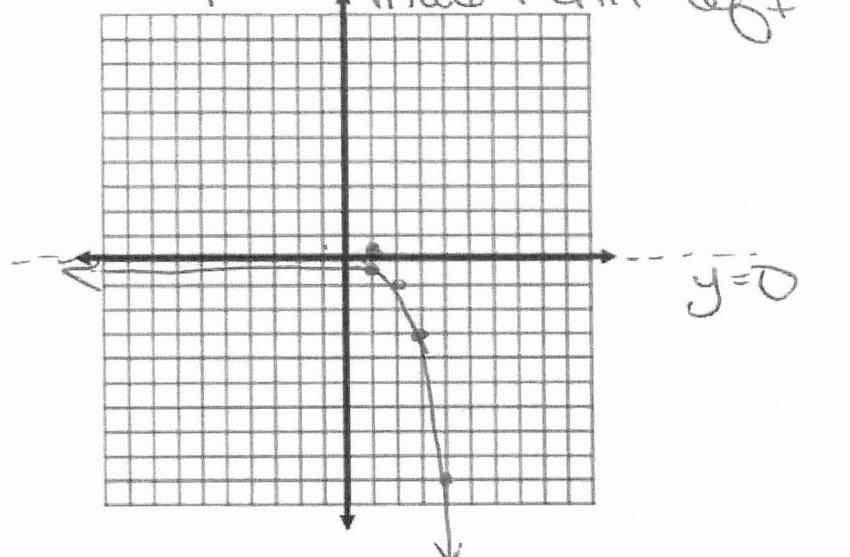
$K = 5$ v. trans up 5

Level 3

4. Sketch the graph of

$$y = -3^{x-2}$$

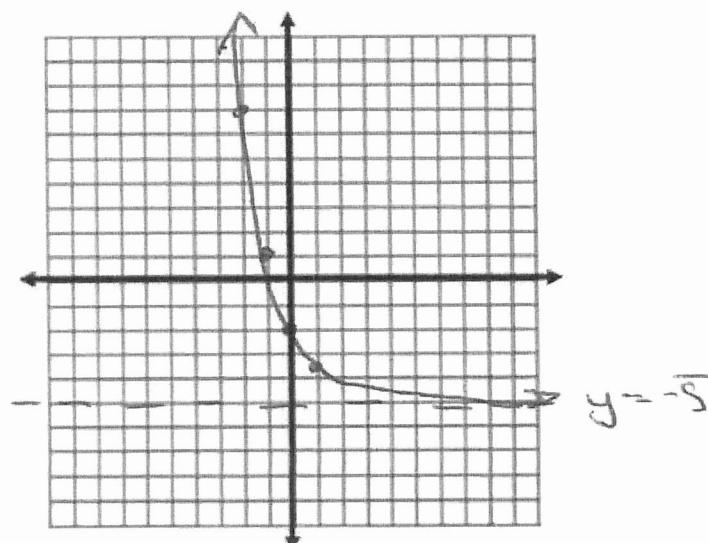
Base $y = 3^x$	
$(x, y) \rightarrow (x+2, -y)$	
$(-1, \frac{1}{3})$	$(1, -\frac{1}{3})$
$(0, 1)$	$(2, -1)$
$(1, 3)$	$(3, -3)$
$(2, 9)$	$(4, -9)$
$(3, 27)$	$(5, -27)$



$$y = 3(2^{-x}) - 5$$

Base $y = 2^x$

$(x, y) \rightarrow (-x, 3y - 5)$	
$(-1, \frac{1}{2})$	$(1, -3.5)$
$(0, 1)$	$(0, -2)$
$(1, 2)$	$(-1, 1)$
$(2, 4)$	$(-2, 7)$
$(3, 8)$	$(-3, 19)$



$$y = 2^{2x+4} - 1$$

$$y = 2^{2(x+2)} - 1$$

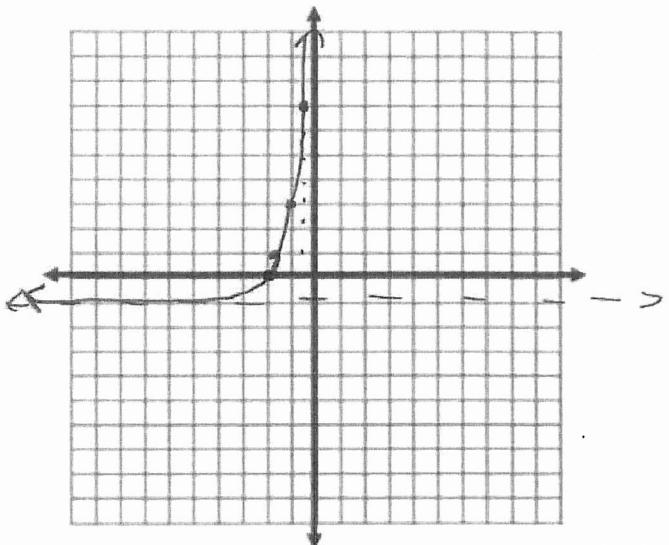
$$(x, y) \rightarrow (\frac{1}{2}x - 2, y - 1)$$

$$(0, 1) \rightarrow (-2, 0)$$

$$(1, 2) \rightarrow (-1.5, 1)$$

$$(2, 4) \rightarrow (-1, 3)$$

$$(3, 8) \rightarrow (-0.5, 7)$$



Chapter 8 – Part 1

Level 2

- Express as a logarithmic statement.

$$2^3 = 8$$

$$\log_2 8 = 3$$

- Express as an exponential statement.

$$\log_3 81 = 4$$

$$3^4 = 81$$

- Determine the value of each logarithm.

$$a) \log_5 25$$

2

$$c) \log_9 1$$

0

$$b) \log_2 \frac{1}{8}$$

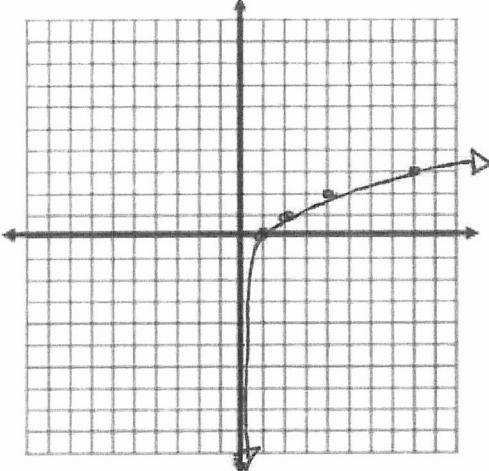
-3

$$d) \log_6 6$$

1

4. Graph each of the following and determine

b) $y = \log_2 x$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical asymptote $x = 0$

x intercept: $(1, 0)$

y intercept: N/A

5. Identify all of the transformations of the following: (state all stretches/reflections/translations up, down left or right)

a) $y = -2\log_3(x - 5) + 2$

$$\begin{aligned} a &= -2 && \text{v. stretch } \times 2 \\ b &= 1 && \text{v. ref. about } x \text{ axis} \\ h &= 5 && \text{right 5} \\ k &= 2 && \text{up 2} \end{aligned}$$

$y = 2\log_3(-x) + 1$

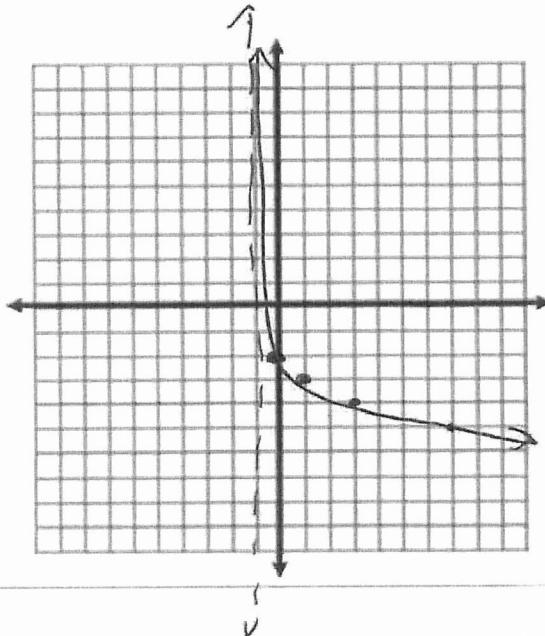
$$\begin{aligned} a &= 2 && \text{v. stretch } \times 2 \\ b &= -1 && \text{h. reflect. about } y \\ h &= 0 && \text{left 1} \\ k &= 1 && \text{up 1} \end{aligned}$$

Level 3

6. Sketch

$y = -\log_2(x + 1) - 2$

$$\begin{aligned} (x, y) &\rightarrow (x-1, -y-2) \\ (1, 0) &\rightarrow (0, -2) \\ (2, 1) &\rightarrow (1, -3) \\ (4, 2) &\rightarrow (3, -4) \\ (8, 3) &\rightarrow (7, -5) \end{aligned}$$



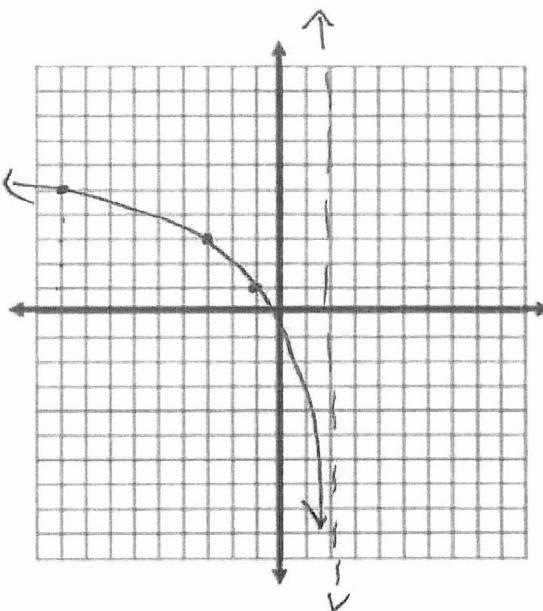
$$y = 2 \log_3(x - 2) + 1$$

$$(x, y) \rightarrow (-x, 2y + 1)$$

$$(1, 0) \rightarrow (-1, 1)$$

$$(3, 1) \rightarrow (-3, 3)$$

$$(9, 2) \rightarrow (-9, 5)$$



Chapter 8 Part 2

Level 2

1. Use your laws of logarithms to expand each of the following:

a) $\log_4 \frac{x}{3}$

b) $\log_4 x^5$

c) $\log_2 yx^5$

$\log_4 x - \log_4 3$

$5 \log_4 x$

$\log_2 y + 5 \log_2 x$

2. Use the laws of logarithms to simplify each of the following:

a) $\log 2 + \log 7$

b) $4 \log_3 5$

c) $\log_2 42 - \log_2 6$

$\log 14$

$\log_3 625$

$\log_2 7$

3. Determine the value of x.

a) $\log_2 x = 3$

b) $3 \log_5 x = \log_5 125$

$2^3 = x$

$\log_5 x^3 = \log_5 5^3$

$8 = x$

$x = 5$

c) $6^x = 216$

d) $4^{x+1} = 64$

$6^x = 6^3$

$4^{x+1} = 4^3$

$x = 3$

$x = 2$

Level 3

4. Use the laws of logarithms to simplify and then evaluate each of the following:

a) $\log_3 270 - (\log_3 2 + \log_3 5)$

$$\log_3 \frac{270}{2 \cdot 5}$$

$$\log_3 \frac{270}{10}$$

$$\log_3 27$$

$$3$$

$$\log_2 \frac{246}{27}$$

$$\log_2 8$$

$$3$$

5. Write each expression in terms of individual logarithms.

a) $\log_2 \frac{x^5 \sqrt[3]{y}}{7z}$

b) $\log_5 \sqrt{xy^3}$

$$5 \log_2 x + \frac{1}{3} \log_2 y - \log_2 7 - \log_2 z \quad \frac{1}{2} \log_5 x + \frac{3}{2} \log_5 y$$

6. Write each expression as a single logarithm.

a) $3 \log w + \log \sqrt{w} - 2 \log w$

b) $\log_2(x+6) + \log_2(x-1)$

$$\log w^3 + \log w^{1/2} - \log w^2$$

$$\log_2 (x+6)(x-1)$$

$$\frac{3}{2} + \frac{1}{2} - 2$$

$$\log \frac{w^3 \cdot w^{1/2}}{w^2}$$

$$\log_2 (x^2 + 5x - 6)$$

$$\frac{1}{2}$$

$$\frac{3}{2}$$

7. Solve for x.

a) $\log_5 x + 6 = 8$

$$\log_5 x = 2$$

$$5^2 = x$$

$$25 = x$$

b) $\log_4 x + \frac{2}{3} \log_4 x = 6$

$$\log_4 x^3 = 6$$

$$\sqrt[3]{4^6} = \sqrt[3]{x^3}$$

$$4^2 = x$$

$$16 = x$$

$$c) \log_2 x^2 - \log_2 5 = \log_2 20$$

$$d) \log_3(x+7) - \log_3(x-3) = 2$$

$$\log_2 \frac{x^2}{5} = \log_2 20$$

$$\log_3 \frac{(x+7)}{(x-3)} = 2$$

$$\begin{aligned}\frac{x^2}{5} &= 20 \\ \sqrt{\frac{x^2}{5}} &= \sqrt{100} \\ x &= \pm 10\end{aligned}$$

* -10 works b/c $\log_2(-10)^2 = \log_2 100$

$$e) 3^x = 100$$

$$\begin{aligned}\log 3^x &= \log 100 \\ x \log 3 &= \log 100 \\ x &= \frac{\log 100}{\log 3}\end{aligned}$$

$$x \approx 4.1918$$

$$f) 7^{x-3} = 517 \quad \boxed{x = \frac{34}{7} = \frac{17}{4} = 4.25}$$

$$\begin{aligned}\log 7^{x-3} &= \log 517 \\ (x-3) \frac{\log 7}{\log 7} &= \log 517\end{aligned}$$

$$x-3 = \frac{\log 517}{\log 7} + 3$$

$$x = \frac{\log 517}{\log 7} + 3$$

Level 4

8. Solve the following. State any restrictions

$$\log_6(x+3) - 2 = -\log_6(x-2) \quad x \approx 6.21086$$

$$x+3 > 0$$

$$\log_6(x+3) + \log_6(x-2) = 2$$

~~$x > 3$~~

$$\log_6(x+3)(x-2) = 2$$

$$x-2 > 0$$

$$\boxed{x > 2}$$

$$6^2 = (x+3)(x-2)$$

$$36 = x^2 + 3x - 2x - 6$$

$$-36$$

$$0 = x^2 + x - 42$$

$$0 = (x+7)(x-6)$$

~~$x = 6$~~

9. Use what you have learned about logarithms to show how you could use two different transformations to graph the logarithmic function $y = \log_2 8x$

$$\textcircled{1} \quad y = \log_2 8x \rightarrow \text{h. stretch of } \frac{1}{8}$$

$$2. \quad y = \log_2 8 + \log_2 x$$

$$y = 3 + \log_2 x$$

v. trans up 3

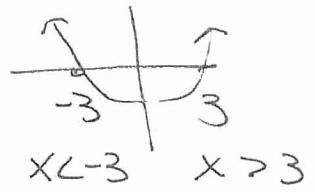
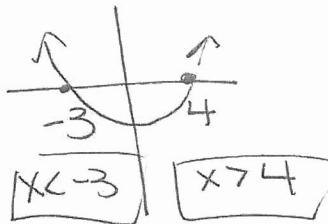
10. Simplify the following logarithm. State the restrictions

$$\log(x^2 - x - 12) - \log(x^2 - 9)$$

$$\log \left(\frac{x^2 - x - 12}{x^2 - 9} \right)$$

$$\log \frac{(x+4)(x-3)}{(x+3)(x-3)}$$

$$\log \left(\frac{x-4}{x-3} \right), \quad x < -3, \quad x > 4$$



Chapter 9 Review

Level 2

1. Determine the characteristics of the following functions:

$$a) y = \frac{2x-1}{x-4}$$

Equation of Vertical Asymptotes: $x = 4$

Points of Discontinuity (holes): NA

Equation of Horizontal Asymptote: $y = 2$

$$b) y = \frac{x+5}{(x+5)(x-3)} = \frac{1}{x-3}$$

Equation of Vertical Asymptotes: $x = 3$

Points of Discontinuity (holes): $(-5, -\frac{1}{8})$

Equation of Horizontal Asymptote: $y = 0$

$$c) y = \frac{x^2-4}{x^2+3x+2} = \frac{(x-2)(x+2)}{(x+2)(x+1)} = \frac{x-2}{x+1}$$

Equation of Vertical Asymptotes: $x = -1$

Points of Discontinuity (holes): $(-2, 4)$

Equation of Horizontal Asymptote: $y = 1$

Level 3/Level 4 (Level 4 Questions will have an oblique asymptote. You will need to determine that on your own.)

2. Graph the following functions. Be sure to give the equations of all asymptotes.

a) $y = \frac{-2x+4}{x+5} = -\frac{2(x-2)}{x+5}$

V.A @ $x = -5$

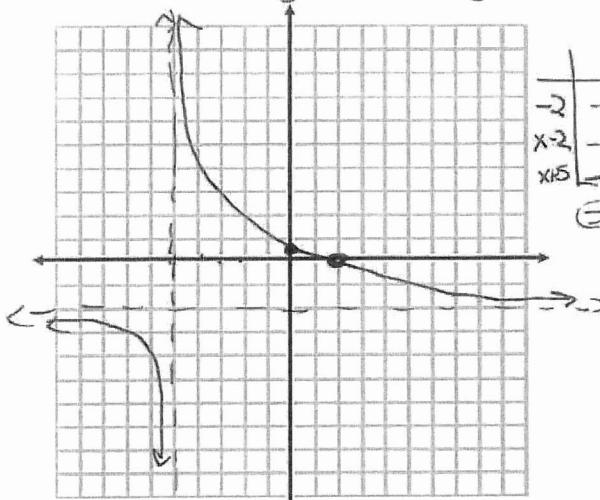
HA @ $y = -2$

x int (2, 0)

y int (0, 4)

b) $y = \frac{x^2-16}{x+4} = \frac{(x-4)(x+4)}{(x+4)} = x - 4$

hole (-4, -8)



c) $y = \frac{x-5}{x^2-2x-15} = \frac{(x-5)}{(x-5)(x+3)}$

V.A @ $x = -3$

HA @ $y = 0$

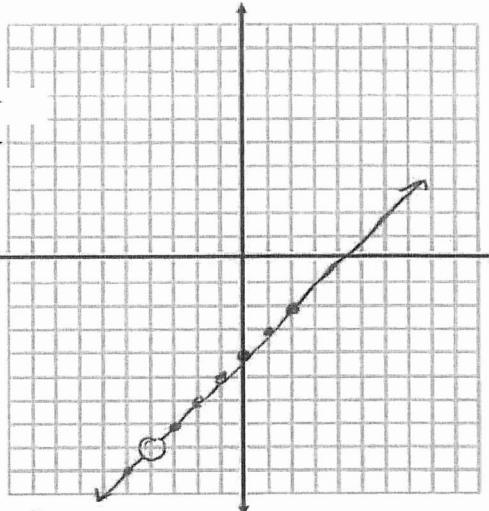
y int (0, 1/3)

no x int

$$= \frac{1}{x+3}$$

-5	2
-2	-
x-2	-
x+5	+
(x-5)	+
(x+3)	+

(-) (+) (-)



d) $y = \frac{x^2-3x-18}{x^2+7x+12} = \frac{(x-6)(x+3)}{(x+3)(x+4)}$

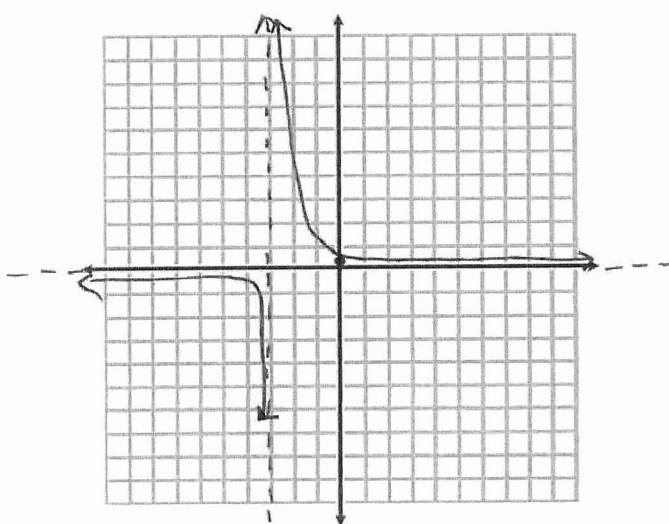
V.A @ $x = -4$

HA @ $y = 1$

x int (6, 0)

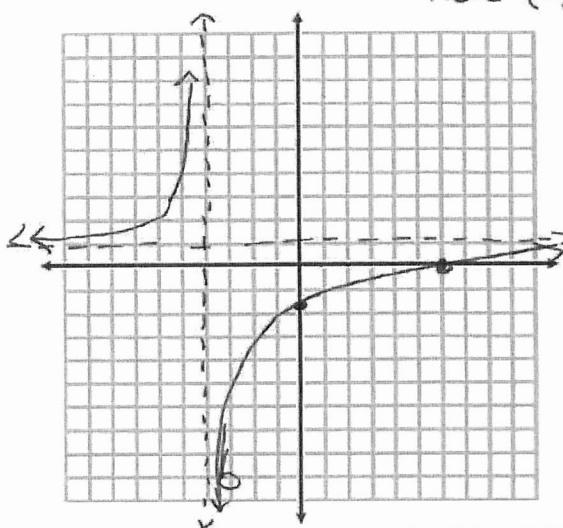
y int (0, -1.5)

Hole (-3, -9)



$x+3$

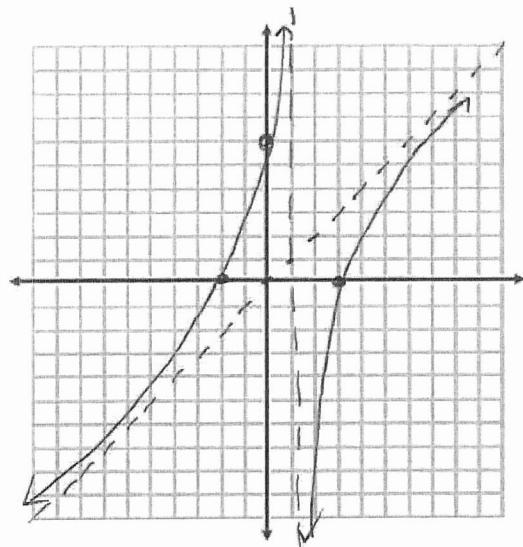
$$\begin{array}{c|cc} & -1 & + \\ \hline - & | & | \\ (-) & | & (+) \end{array}$$



$x-6$ $x+4$

-	-	-	-
(+)	(-)	(-)	(+)

$$y = \frac{x^2 - x - 6}{x-1} = \frac{(x-3)(x+2)}{(x-1)}$$

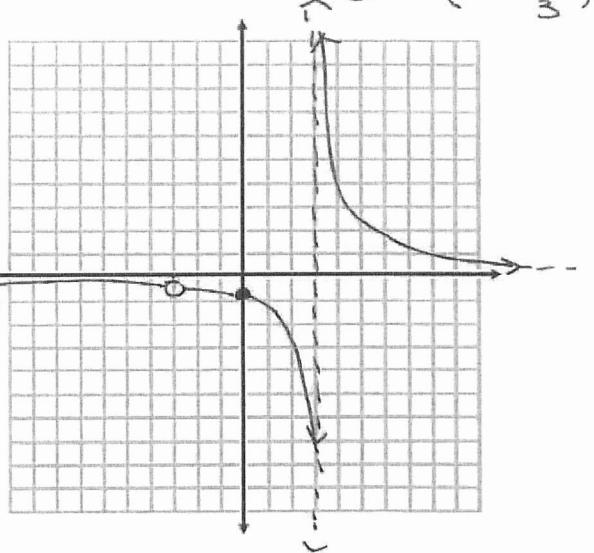


O.A.

$$\begin{array}{r} \cancel{-1} \\ \begin{array}{r} 1 & -1 & -6 \\ \downarrow & -1 & 0 \\ 1 & 0 & \boxed{-6} \end{array} \end{array} \quad \begin{array}{l} x_{\text{int}} (-2, 0) \\ (3, 0) \end{array} \quad \begin{array}{l} y_{\text{int}} (0, 6) \end{array}$$

$$y = \frac{2x+6}{x^2-9} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$$

Hole $(-3, -\frac{1}{3})$
 $y \text{ int } (0, -\frac{2}{3})$



$$\begin{array}{c} 2 \\ x-3 \\ \hline + & - & + \\ \ominus & \oplus & \ominus \end{array}$$

$\cap @ y = x$

$$\begin{array}{c|ccccc} & -2 & 1 & 3 \\ \hline x-3 & - & 1 & -1 & -1 & + \\ x+2 & - & 1 & + & 1 & + \\ x-1 & - & 1 & -1 & +1 & + \\ \hline & \ominus & \oplus & \ominus & \ominus & \oplus \end{array}$$