1.1 ARITHMETIC SEQUENCES

To determine if a given sequence is Arithmetic, Geometric or neither. To determine a particular term of an arithmetic sequence.

SEQUENCE:
- An arithmetic sequence is an ordered list of terms in which the difference between consecutive terms is constant. This constant is called the common difference and uses the variable "d". The difference is found by subtracting the first number from the second number in the sequence. $d = t_2 - t_1$
- A geometric sequence is an ordered list of terms in which there is a common ratio between

  consecutive terms. The variable for the common ratio is r and can be found by: $r = \dfrac{t_2}{t_1}$

EX #1: In the sequence: 3, 5, 7, 9, . . . what is the common difference, d?

EX #2: In the sequence: 2, 4, 8, 16, ... what is the common ratio, r?

EX #3: Are the following sequences arithmetic, geometric or neither.

a) $\dfrac{1}{3}, \dfrac{1}{9}, \dfrac{1}{27}, ...$ b) 10, 20, 30,.....   c) $\dfrac{10}{9}, \dfrac{8}{7}, \dfrac{7}{6}, ...$ d) 2.5, 0.25, 0.025,...

GENERAL TERM OF AN ARITHMETIC SEQUENCE
- The formula to find a specific term in an arithmetic sequence is:
   $t_n = a + d(n - 1)$ where t_n is the value of a particular n^{th} term
                               n is the number of terms
                               a is the first term
                               d is the common difference

EX #4: A construction company hires a plumber to install pipes in new homes. The plumber will be paid $65 for the first hour of work, $110 for two hours of work, $155 for three hours of work, and so on.
a) State the first three terms of the sequence.

b) Is there a common difference, d?

d) What will the plumber get paid for 10 hours of work?

e) What is the GENERAL TERM?

f) Use the GENERAL TERM to determine how much did the plumber get paid for 30 hours of work?

EX #5: A farmer decides to plant an apple orchard. She plants 24 apple trees in the first year and 15 more apple trees in each subsequent year. During which year will she plant 204 apple trees?
a) How would you choose to solve this problem?

b) What does the sequence look like?

c) We can use the Arithmetic Sequence Formula:

EX #6: How many terms are in the following sequence? 5, 2, -1, ... , -142?

EX #7:   Find the first term and common difference of the arithmetic sequence whose third term is 1 and whose seventh term is -11.

EX #8:   An amphitheater has 25 seats in the second row and 65 seats in the seventh row. The last row has 209 seats. The numbers of seats in the rows produces an arithmetic sequence. How many rows of seats are in the amphitheater? How many seats are in the first row?

PRACTICE EX #9:   Determine the number of terms in the sequence: 3,8,13,...,58

PRACTICE EX #10: Determine the values of a and d. State the missing terms of the sequence: ___, ___, ___, 27, 39

Part A:

1. Determine if the sequence is arithmetic, geometric, or neither.
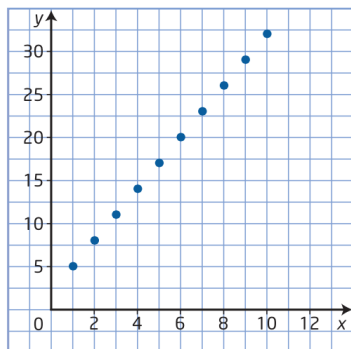
a) $16, 32, 48, 64, 80$

b) $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}$

c) $-4, -7, -10, -13, -16$

d) $1, 2, 4, 7, 11, 16, 22$

e) $x, x+y, x+2y, x+3y$

f) $6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$

2. Given the first term and the common difference, write the first 5 terms of the sequence.

a) $a = -30; d = -3$

b) $a = -37; d = 10$

c) $a = 3p - 5q; d = -p + 2q$

d) $a = -12; d = 200$

e) $a = 4; d = \frac{1}{5}$

f) $a = 0.4; d = -1.2$

3. Determine the general term of the sequence.

a) $22, 32, 42, 52, ...$

b) $-34, -234, -434, -634$

c) $\frac{7}{6}, \frac{5}{6}, \frac{1}{2}, ...$

d) $a = 5y; d = -3y$

e)

4. Determine the nth term of the arithmetic sequence.

a) $15, 21, 27, 33, ... \ t_{n38}$

b) $39, 32, 25, 18, ... \ t_{25}$

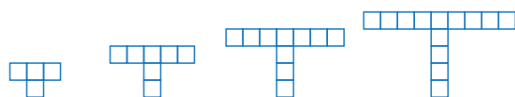5. Each square in this pattern has a side length of 1. Assume that the pattern continues.

Figure 1   Figure 2   Figure 3   Figure 4

a) Write the general form of the sequence that relates the figure number to the perimeter.

b) Determine the perimeter of figure #9.

Part B:

1. For each arithmetic sequence, determine the values of a and d. State the missing terms of the sequence.

a) ___, ___, ___, 19, 23

b) ___, ___, 3, $\frac{3}{2}$

c) ___, 4, ___, ___, 10

2. Identify which term each number is in the given arithmetic sequence. Use the formula.

a) 107 in the sequence 2, 5, 8, ...

b) -140 in the sequence 20, 16, 12

c) 21.2 in the sequence 6, 6.8, 7.6

d) $\frac{31}{2}$ in the sequence $\frac{1}{2}, \frac{7}{8}, \frac{5}{4}, ...$

3. What is the first term of an arithmetic sequence whose common difference is 4 and whose 24th term is 73? Use the formula.

4. What is the first term of an arithmetic sequence whose common difference is 3 and whose 16th term is 48? Use the formula.

5. Find the first term and the common difference of the arithmetic sequence whose 9th term is 21 and whose 15th term is 33.

6. Find the first term and the common difference of the arithmetic sequence whose 15th term is 91 and whose 20th term is 121.

7. Susan joined a fitness class at her local gym. She incorporated a sit-up routine that followed an arithmetic sequence. On the 6th day of the program, Susan performed 11 sit-ups. On the 15th day she did 29 sit-ups.

a) Write the general term that relates the number of sit-ups to the number of days.

b) If Susan's goal is to be able to do 100 sit-ups, on which day of her program will she accomplish this?

8. Hydrocarbons are the starting points in the formation of thousands of products, including fuels, plastics, and synthetic fibres. Some hydrocarbon compounds contain only carbon and hydrogen atoms. Alkanes are saturated hydrocarbons that have single carbon-to-carbon bonds. The diagrams below show the first three alkanes.

Methane   Ethane

Propane

a) Write the general term that relates the number of hydrogen atoms to the number of carbon atoms.

b) Hectane contains 202 hydrogen atoms. How many carbon atoms are required to support 202 hydrogen atoms?

Solutions:

Part A:

1. a) arithmetic b) geometric c) arithmetic d) neither e) arithmetic f) geometric

2.a) $-30, -33, -36, -39, -42$ b) $-37, -27, -17, -7, 3$ c) $3p - 5q, 2p - 3q, p - q, q, -p + 3q$

d) $-12, 188, 388, 588, 788$ e) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}, \frac{24}{5}$ f) $0.4, -0.8, -2, -3.2, -4.4$

3.a) $tn = 12 + 10n$ b) $tn = 166 - 200n$

c) $tn = \frac{3}{2} - \frac{1}{3}n$ d) $tn = 8y - 3yn$ e) $2 + 3n$

4.a) 237   b) -129

Part B:

1a)   a = 7, d = 4: 7, 11, 15, 19, 23   b) a = 6, d = $-\frac{3}{2}$: $6, \frac{9}{2}, 3, \frac{3}{2}$ c) a = 2, d = 2: 2, 4, 6, 8, 10

2a) 36   b) 41   c) 20   d) 41

3) a = -19   4) a = 3   5) a = 5; d = 2   6) a = 7, d = 6

7a) $tn = 2n - 1$ b) 51st day

8a) $tn = 2n + 2$ b) 100 carbon atoms

To determine the sum of an arithmetic series. To determine the values of a, d, n and t_n for an arithmetic series.

A series is a sum of terms that form an arithmetic sequence.

Sequence	Series
5, 8, 11, 14, 17, 20, …..	$5 + 8 + 11 + 14 + 17 + 20 + …..$
3, -6, 12, -24, 48, -96, …..	$3 - 6 + 12 - 24 + 48 - 96 + …..$

Compare a sequence to a series.

A series requires us to find the sum of all the terms involved.

EX #1: Determine the sum of the series: 2 + 4 + 6 + 8

EX #2: Determine the sum of the series: 57 + 60 + 63 + …… + 354

We can use the following formula to answer this question!

ARITHMETIC SERIES FORMULAS:

$$S_n = \frac{n(a+t_n)}{2} \qquad \text{or} \qquad S_n = \frac{n[2a+d(n-1)]}{2}$$

Only one formula will be chosen for each question depending on what is given in the question.

Use the formula to answer example 2:

EX #3: Determine the sum of indicated series: $3 + 8 + 13 + \ldots S_{12}$

EX #4: Determine the value of the first term if $d = 4$, $Sn = 1830$, and $n = 30$.

EX #5: For the arithmetic series, determine the value of n: $a = 42$, $t_n = 68$, $S_n = 330$

1. The bottom row in a trapezoid had 49 cans. Each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid.
   a) How many cans were in the trapezoid?
   b) How many cans were in the 10th row of the trapezoid?

2. Ryan's grandparents loaned him the money to purchase a new bike. He agreed to repay $25 at the end of the first month, $30 at the end of the second month, $35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much money did he pay back to his grandparents?

3. Kaitlyn makes $10 per hour for her first hour of work, $10.50 for her second hour of work, $11 for her third hour and so on.
   a) How much money will she make during her 8th hour of work?
   b) How much money will she make after an 8-hour day of work?

4. During a free fall, a skydiver falls 16 feet in the first second, 48 feet in the 2nd second, and 80 feet in the third second. If she continues to fall at this rate:
   a) How many feet will she fall during the 8th second?
   b) How many feet will she fall altogether during the 20 second free-fall?

5. The Chinese zodiac associates years with animals. Addison was born in 1994, the Year of the Dog.
   a) The Year of the Dog repeats every 12 years. List the first three years that Addison will celebrate his birthday in the Year of the Dog.
   b) In 2099, Nunavut will celebrate its 100th birthday. Will that year also be the year of the Dog? Explain.

6. Write the first four numbers that are multiples of 4 between 1 and 999.
   b) Determine the sum of all the multiples of 4 between 1 and 999.
   c) Determine the 25th multiple of 4 between 1 and 999.

7. *It's About Time*, in Langley, BC, is Canada's largest custom clock manufacturer. They have a grandfather clock that, on the hours, chimes the number of times that corresponds to the time of day. For example, at 4:00 p.m., it chimes 4 times. How many times does the clock chime in a 24-hour period?

Answers:
1a) 319   b) 13
2) $630
3a) $13.50/hour   b) $94.00
4a) 240 ft.   b) 6400 ft.
5a) 2006, 2018, 2030   b) No: 2099 is not a term in the sequence. If you solve for n, you will get 9.75, which is not a whole number.
6a) 4, 8, 12, 16   b) 124 500   c) 100
7. 156 times

1.3 GEOMETRIC SEQUENCES

To use the formula for a geometric sequence to find t_n, a, r, n and the general term. To solve situational questions involving geometric sequences.

GEOMETRIC SEQUENCE:

$t_n = ar^{n-1}$ Where: *a* is the first term

r is the common ratio

n is the number of terms

EX #1: Find the 12th term of the sequence 2, 6, 18,…..

EX #2: Find 11th term (in exact form) of the sequence 3125, 2500, 2000,……

EX #3: a) How many terms are in the sequence 2, 10, 50, … 156250?

b) How many terms are in the sequence -2, 6, -18, … -1458?

EX #4: What is the general term for the geometric sequence that has a first term of 3 and a common ratio of 4.

EX #5: Determine the missing value for each geometric sequence with the following properties.

a) If n = 7, t_7 = 12288, t_1 = 3, find r

b) If n = 5, $t_5 = \dfrac{128}{81}$, $r = \dfrac{4}{3}$, find t_1

EX #6: In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r and list the first 4 terms of the sequence.

EX #7: A ball is dropped from a height of 4m. After each bounce, it rises to 60% of its previous height.
a) Model this question as a geometric sequence

b) Write a general term for the sequence.

b) What height does the ball reach after the 6th bounce?

If a sequence increases by a constant PERCENTAGE, it is geometric
- The **r** value will be **1 + (The Percentage as a decimal)**. For example, if property taxes increase 1% per year, the **r** value would be **1.01**

If a sequence decreases by a constant PERCENTAGE, it is also geometric
- The **r** value will be **1 - (The Percentage as a decimal)**. For example, if a car's value depreciates by 15% per year, the **r** value would be **0.85** because the car will only be worth 85% of what is was the year prior

EX #7: The population of a city increases by 6.5% each year for 10 years. If the initial population is 200 000, what is the population after 10 years? Note: Be careful about the value of n!!

1.3 ASSIGNMENT

1.3 FA: P39 #1c-f, 3bd, 4, 5bc, 6abc, 8, 9, PLUS THE FOLLOWING

1. Use the formula to find the required term of the given sequence.
   a) 7th term of 1, 3, 9, ...   b) 20th term of $1, -2, 4, ...$ c) 13th term of $-8, -4, -2, ...$
   d) 7th term of $64, -16, 4, ...$ e) 6th term of 64, 48, 36, ...   f) 18th term of $x, 2x^2, 4x^3, ...$

2. Liam just started a new job earning $45 000 per year. Assuming good performance, he is promised a raise of 2%. How much money will Liam be making in his 10th year on the job?

3. A certain type of new car depreciates 20% per year. How much will the car be worth when it is 8 years old, if it is $35 000 brand new?

4. Identify what term each number is in the given geometric sequence. Use the nth term formula.

   a)   156250 in the sequence 2, 10, 50, ...   d) 262144 in the sequence $\dfrac{1}{64}, \dfrac{1}{16,} \dfrac{1}{4}, ...$

   b) $\dfrac{1}{32}$ in the sequence 32, -16, 8, ...   e) $\dfrac{4}{81}$ in the sequence $\dfrac{81}{64}, \dfrac{27}{32}, \dfrac{9}{16}, ...$

   c)   -1458 in the sequence -2, 6, -18, ...   f) -10240 in the sequence -5, -10, -20, ...

5.   In a certain region, the number of highway accidents increased by 5% per year. How many accidents will there be in 2018 if there were 5120 in 1978?

1.3 ULA: P39 #7, 10, 14, 17, 22, 23,

SOLUTIONS TO EXTRA QUESTIONS

1a) 729   b) -524288   c) $-\dfrac{1}{512}$ d) $\dfrac{1}{64}$ e) $\dfrac{243}{16}$ f) $131072x^{18}$

2. $53 779.17

3. $5872.03

4a) 8th term   b) 11th term   c) 7th term   d) 13th term   e) 9th term   f) 12th term

5. about 36 045

To use the formula for a geometric series to find S$_n$, a, r and n. To solve situational questions involving geometric series.

GEOMETRIC SERIES FORMULA:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Where: S_n is the sum of the first n terms

a is the first term
r is the common ratio
n is the number of terms

EX #1: Determine the sum of the first 10 terms of each geometric series.

a) 5+15+ 45+ ...

b) $t_1 = 64, r = \dfrac{1}{4}$

EX #2: Determine the sum of each geometric series.

a) −2+ 4 − 8 + ...− 8192

b) $\dfrac{1}{64} + \dfrac{1}{16} + \dfrac{1}{4} + ... + 1024$

EX #3: A phone tree is used to contact a large number of people in a short period of time. In a particular phone tree, the first person contacts 3 people, who each contact 3 more people, and so on.

a) Draw a diagram and write a series to represent the total number of people in the phone tree.

b) How many people are contacted after 6 levels of the tree (assuming the first level has 1 person)?

c) After how many levels will the total number of people contacted reach 2,391,484?

To find the sum of an infinite geometric series.

Video on You Tube " Zeno's Paradox – numberphile"   https://goo.gl/3vTguF

> **INFINITE GEOMETRIC SERIES:**
> - An **INFINITE GEOMETRIC SERIES** has an INFINITE number of terms.
>   Question: How can you calculate the sum of something infinite????
> - You can only calculate the sum of an infinite geometric series if the series is CONVERGENT. A convergent series is a series in which the absolute value of the terms are getting successfully smaller as the series progresses. **A CONVERGENT series happens when: -1 < r < 1**
> - A **DIVERGENT** series is when the r > 1 or when r < -1. You cannot find the sum of an infinite geometric series that is divergent.

EX #1: Determine if the following series are convergent or divergent. Discuss what the sum might look like if you tried to add up all the terms (to infinity and beyond….. ☺)

a) $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots$

b) 2 + 4 + 6 + 8+…….

> **SUM OF AN INFINITE CONVERGENT GEOMETRIC SERIES:**
>
> $$S_\infty = \frac{a}{1-r} \quad \text{where -1 < r < 1}$$

EX #2: A Find the sum of the following series if possible. If not possible explain why.

a) $1 - \frac{1}{3} + \frac{1}{9} - \cdots$ b) $2 - 4 + 8 - \cdots$

c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

d) $1.11 + 0.222 + 0.0444 + \ldots$

e) $4 + 6 + 9 + \dfrac{27}{2} + \dfrac{81}{4} + \ldots$

EX #3: Convert $0.\overline{3}$ to a fraction by changing it to an infinite geometric series

EX #4: Convert $0.\overline{17}$ to a fraction by changing it to an infinite geometric series

EX #5: A ball is dropped from a height of 4m to the floor. After each bounce, the ball rises to 50% of its previous height.

a) What is the total vertical distance travelled after 6 bounces?

b) What is the vertical distance travelled when the ball comes to rest?

1.5 ASSIGNMENT

1.5 FA: **P63 # 1, 2, 3, 6, 7, 13, 16**

Convert $0.\overline{24}$ to a fraction by changing it to an infinite geometric series

1.5 ULA: P63 #5, 8, 9, 10, 16, 17

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

Section 1.1

https://goo.gl/eUi69M

https://goo.gl/MnP5wt

Section 1.2

https://goo.gl/BFGEwW

https://goo.gl/UeRvLj

Section 1.3

https://goo.gl/8Kzsjp

https://goo.gl/mfNvH1

Section 1.4

https://goo.gl/zoxnUY

https://goo.gl/qwUKrJ

Section 1.5

https://goo.gl/y38gez

https://goo.gl/M7g9Li