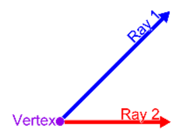
2.1 ANGLES IN STANDARD POSITION & 2.2 SPECIAL TRIANGLES

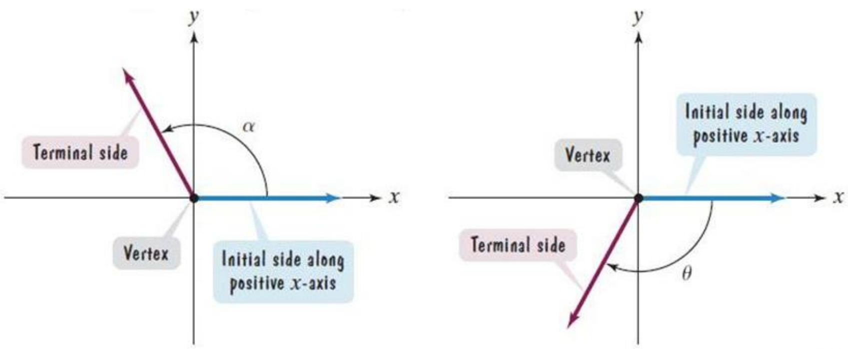
PC20

To sketch an angle in standard position, determine the quadrant in which the angle terminates and determine the reference angle. To determine the angle in standard position, when a given angle is reflected in the x and y axis. To determine the relationship between sides of special right triangles.

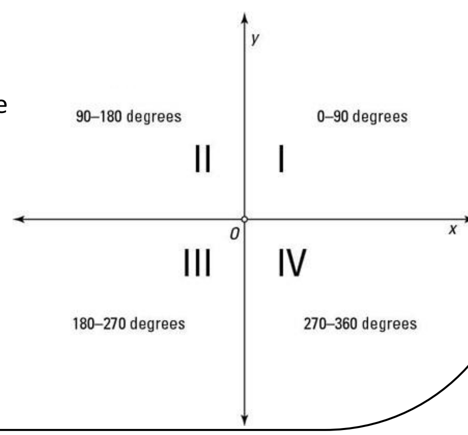
- In **GEOMETRY**, an angle is formed by two rays having a common endpoint



- In **TRIGONOMETRY** angles are often interpreted as rotations of a ray about a vertex.
  - These rotations are said to be in **STANDARD POSITION** when the vertex is located at the origin of the Cartesian plane.
  - In standard position the initial location of the ray is located on the positive x axis and is called the **INITIAL ARM**
  - Positive angles in standard position are created by rotating the initial arm counterclockwise about the vertex. The end location of the ray is called the **TERMINAL ARM.**
  - If the rotation of the terminal arm is clockwise, the angle has a negative measure



- We indicate the quadrant that an angle belongs in by the quadrant where the terminal arm is located
- If the terminal arm of an angle lies along an axis, then it is called a **QUADRANTAL ANGLE.**



EX #1: Classify the following angles by quadrant. Sketch.

a) 80° b) 120° c) 266° d) -110°

REFERENCE ANGLE:

- For every angle in standard position, there is a corresponding **ACUTE** angle called the **REFERENCE ANGLE** which is located at the vertex of the **REFERENCE TRIANGLE.**
  - The **REFERENCE TRIANGLE** can be drawn by connecting a point on the terminal arm of an angle in standard position to the x axis so that the connecting line is perpendicular to the x axis
  - The **REFERENCE ANGLE** is found within the reference triangle at the vertex (by the origin)
  - The reference angle is always between 0° and 90°.

EX #2: Sketch the following angles in standard position and determine the reference angle.

a) $\theta=70°$

b) $\theta=120°$

c) $\theta=250°$

d) $\theta=345°$

e) $\theta=-70°$

f) $\theta=-150°$

EX #3: Determine the measure of 3 other angles in standard position $0° < \theta < 360°$ that have a reference angle of 30°. Sketch.

EX #4: Determine the angle in standard position when an angle of 60° is reflected in:

a) The y-axis

b) The x-axis

d) The y axis and then in the x axis

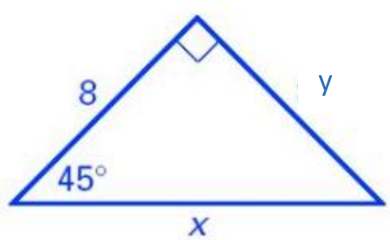
SPECIAL TRIANGLES:

- A **special right triangle** is a right triangle where the sides are related in a simple way.
- In trigonometry we recognize two triangles as being called **SPECIAL TRIANGLES.** Let's develop them:

- NOTE: You will need to memorize these two triangles! They will be used often.

EX #4: Use special triangles to determine the missing lengths.

a)



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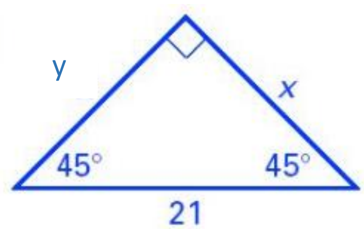

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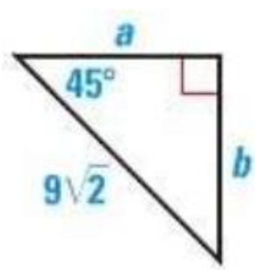

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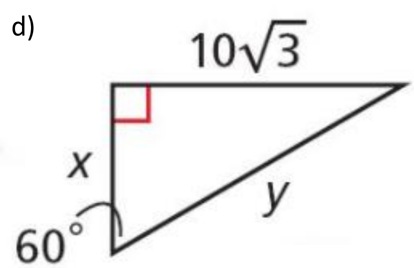

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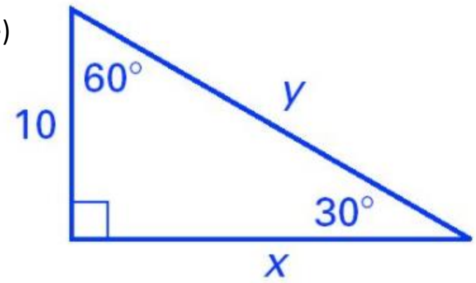

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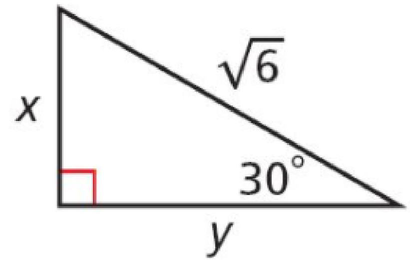

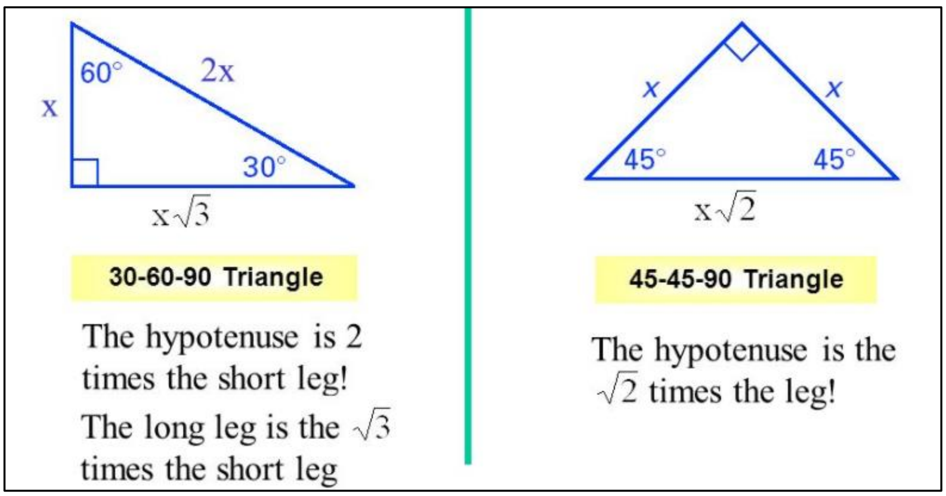

EX #5: An 8m boom is used to horizontally move a bundle of piping from point A to point B. Determine the exact horizontal displacement of the end of the boom when the operator raises it from 30° to 60°. Hint: Use two triangles in your drawing!

2.1/2.2 Day 1 ASSIGNMENT

2.1/2.2 Day 1 FA: P83 #1-7 PLUS EXTRA QUESTIONS ON NEXT PAGE

2.1/2.2 Day 1ULA: P83 #9,13, 14, 12, 17, 19, 16

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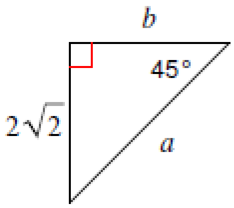

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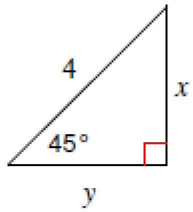

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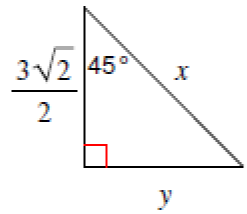

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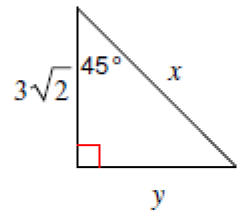

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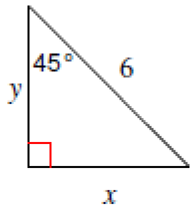

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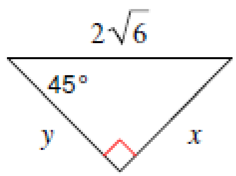

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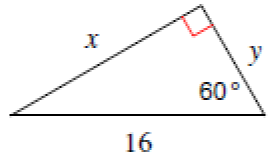

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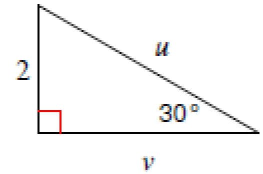

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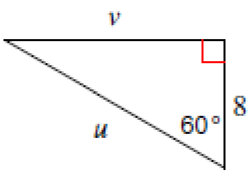

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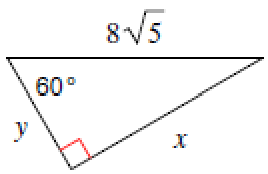

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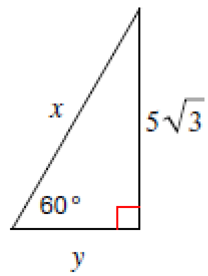

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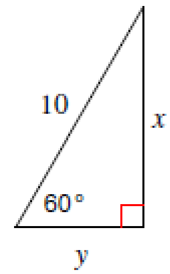

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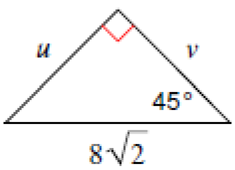

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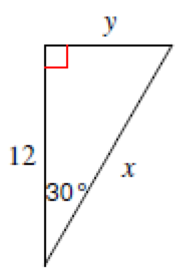

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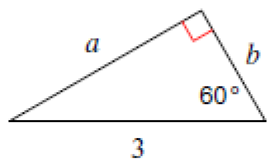

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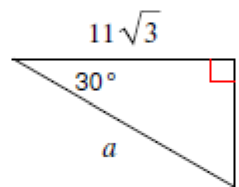

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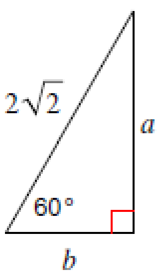

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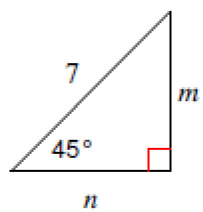


SOLUTIONS TO EXTRA QUESTIONS

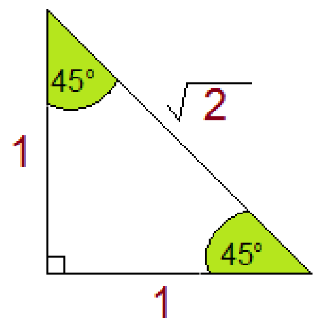
1. $a = 4; b = 2\sqrt{2}$
2. $x = 2\sqrt{2}; y = 2\sqrt{2}$
3. $x = 3; y = \frac{3\sqrt{2}}{2}$
4. $x = 6; y = 3\sqrt{2}$
5. $x = 3\sqrt{2}; y = 3\sqrt{2}$
6. $x = 2\sqrt{3}; y = 2\sqrt{3}$
7. $x = 8\sqrt{3}; y = 8$
8. $u = 4; v = 2\sqrt{3}$
9. $u = 16; v = 8\sqrt{3}$
10. $x = 4\sqrt{15}; y = 4\sqrt{5}$
11. $x = 10; y = 5$
12. $x = 5\sqrt{3}; y = 5$
13. $u = 8; v = 8$
14. $x = 8\sqrt{3}; y = 4\sqrt{3}$
15. $a = \frac{3\sqrt{3}}{2}; b = \frac{3}{2}$
16. $a = 22; b = 11$
17. $a = \sqrt{6}; b = \sqrt{2}$
18. $m = \frac{7\sqrt{2}}{2}; n = \frac{7\sqrt{2}}{2}$

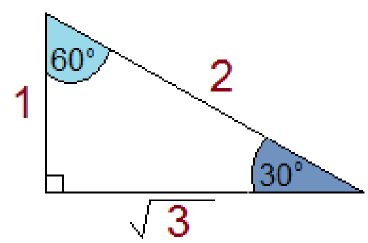
2.1/2.2 Day 2: EXACT VALUES OF TRIGONOMETRIC RATIOS

To determine the exact values of the trigonometric ratios of any angle that has a reference angle of 30°, 60° or 45° or any quadrantal angles.

REVIEW:

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\qquad \cos\theta = \dfrac{adjacent}{hypotenuse}$ $\qquad \tan\theta = \dfrac{opposite}{adjacent}$





EX #1: Use your calculator to determine the value of each of the following.

a) sin30° b) cos 30° c) tan30°

e) sin60° b) cos60° f) tan60°

g) sin 45° h) cos45° i) tan45°

Are any of these answers an EXACT VALUE?

EX #2: Use the special triangles to determine the **EXACT VALUE** of each of the following.

a) sin30° b) cos 30° c) tan30°

e) sin60° b) cos60° f) tan60°

g) sin 45° h) cos45° i) tan45°

Check to see that these exact values turn into the same decimals as the answers in example 1.

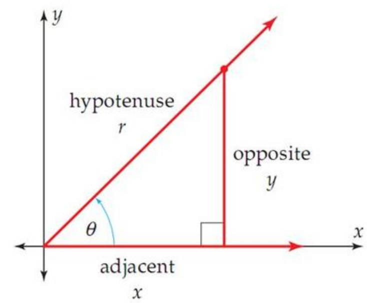
USING SPECIAL TRIANGLES TO FIND EXACT VALUES OF TRIG RATIOS:

- If an angle has a reference angle that is 30°, 60° or 45°, we can find the exact value of that angle by using the reference triangle with sides being labelled with the appropriate length (and using a negative value for that length in appropriate quadrants)
- We often modify the formula for sin, cos and tan when the triangle is placed in the coordinate plane

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{x}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{y}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$$



EX #3: Determine the exact value of the 3 trig ratios of 120°

STEPS

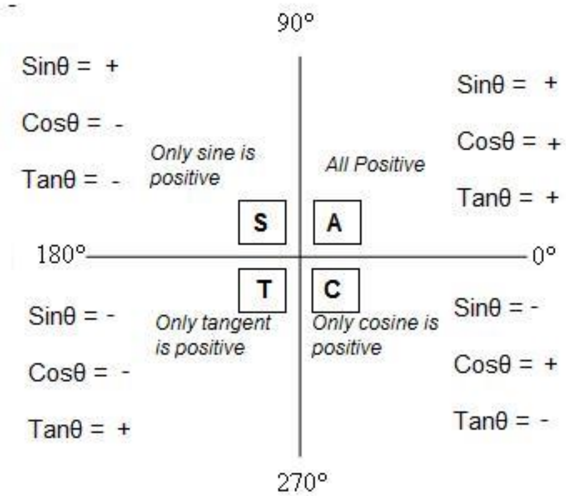
- Sketch the angle, sketch the reference triangle and determine the reference angle
- Determine the values of x, y and r and write those values on your triangle (remember to include any negative distances!)
- Write the trig ratios in EXACT FORM.

Angles in standard position share the same trig values as their reference angles, but the signs may be different depending on the quadrant that the angle is in.

EX #4: Determine which trig ratios will be positive in each quadrant. Justify your answers with diagrams.

CAST RULE:

The CAST rule is a rule that helps you remember which trigonometric ratios will be positive in each quadrant.



EX #5: Determine if sin230° will be positive or negative without using a calculator.

EX #6: In which quadrant does the terminal arm of angle ϴ lie if cosϴ>0 and sinϴ<0

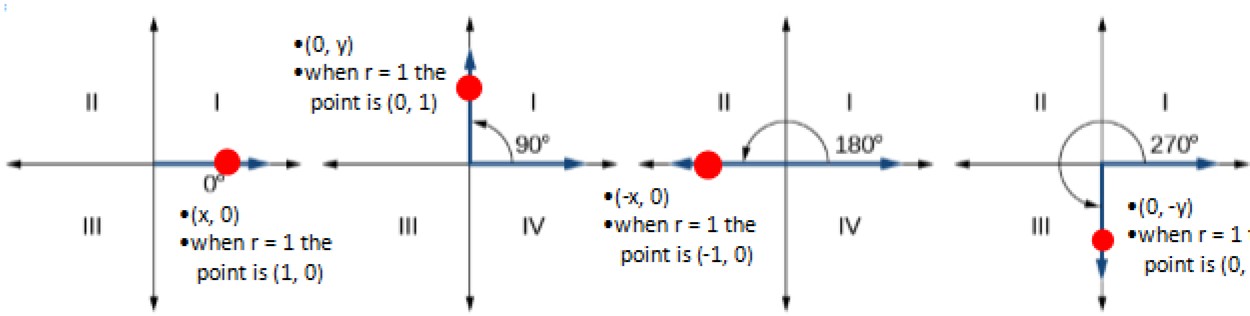
EX #7: Determine the exact values of the trig ratios for 315°.

EX #8: Solve $\sin\theta = -\frac{1}{\sqrt{2}}$ for 0° $\leq$ ϴ $\leq$ 360° using a diagram involving a special right triangle

EX #9: Without using a calculator, predict if the following is true or false: sin183°=sin3°

TRIGONOMETRIC VALUES OF QUADRANTAL ANGLES:

- Quadrantal angles are angles that lie on the arms of the quadrants such as 0°, 90°, 180°, 270° and 360° or their negative counterparts
- We can't draw reference triangles for these angles so instead we use assumptions about the value of x and y within ordered pairs found on each arm of the four quadrants (notice that to simplify we use r = 1)



And we use this version of our formulas: $\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

EX #10: Using the above diagram, determine the exact value of the following:

a) sin0° b) cos0° c) tan0°

d) sin90° c) cos90° d) tan90°

e) sin180° f) cos180° g) tan180°

2.1/2.2 Day 2 ASSIGNMENT

2.1/2.2 Day 1 FA: P83 #8, 11, P96 #2, 4, 6, 9, 10 (do 270° and 360° only)
2.1/2.2 Day 1ULA: P83 # 23 P96#17, 18, 19, 20, 29

Day 3: 2.2 Cont: TRIGONOMETRIC RATIOS OF ANY ANGLE

To determine the three trigonometric ratios of an angle, given a point P that lies on the terminal arm of the angle or given one of the other trig ratios. To find angle(s) in standard position given a point on the terminal arm or a trigonometric ratio.

EX #1: Determine the 3 trig ratios for the angle whose terminal arm passes through the point (3, 4)

What is the value of ϴ?

STEPS TO FINDING THE RATIOS:

- Sketch the point and draw the angle in standard position
- Sketch the reference triangle
- Label the lengths of the reference triangle using the given ordered pair
- Use Pythagorean theorem to calculate the length of r (the radius vector) in EXACT FORM.
- Use our formulas to find the trig ratios

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

STEPS TO FINDING THE VALUE OF ϴ:

- Use the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ function and one of your ratios to find the value of the REFERENCE ANGLE $ϴ_R$. Use the value of $ϴ_R$ and in which quadrant it is located to find the value of ϴ (the angle in standard position)

EX #2: The point (12, -9) lies on the terminal arm of an angle in standard position. Determine the exact trigonometric ratios for ϴ.

EX #3: Suppose Θ is an angle in standard position with a terminal arm in QIII and where $\cos\theta = -\frac{3}{4}$. What are the exact values of the other trig ratios?

What is the value of Θ?

EX #4: Solve for Θ.

a) tan Θ = -0.9004, $0° \leq \Theta \leq 360°$

$\cos\theta = -\frac{\sqrt{2}}{2}$, $0° \leq \Theta \leq 360°$

EX #5: Point (4, -6) is on the terminal arm of an angle in standard position. Determine the measure of the angle.

Day 3: 2.2 Cont. ASSIGNMENT

2.2 Day 3 FA P 96 #3, 5, 7, 8, 11, 12, 13,
2.2 Day 3 ULA P 96 #14, 15, 16, 21

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

Section 2.1

https://goo.gl/svVSy5

https://goo.gl/fhWjry

Section 2.2

https://goo.gl/3kyj88 (Exact Values)

https://goo.gl/4UdVBa (Trig Ratios)

https://goo.gl/YSc8Xx

https://goo.gl/ozuQTG

Section 2.3

https://goo.gl/zqWdPe

https://goo.gl/1EGLAf

THE AMBIGUOUS CASE:

https://goo.gl/BvUoJK

https://goo.gl/ueGu8Q

Section 2.4

https://goo.gl/d25tyH

https://goo.gl/yMaJSJ