4.1 GRAPHICAL SOLUTIONS OF QUADRATIC EQUATIONS

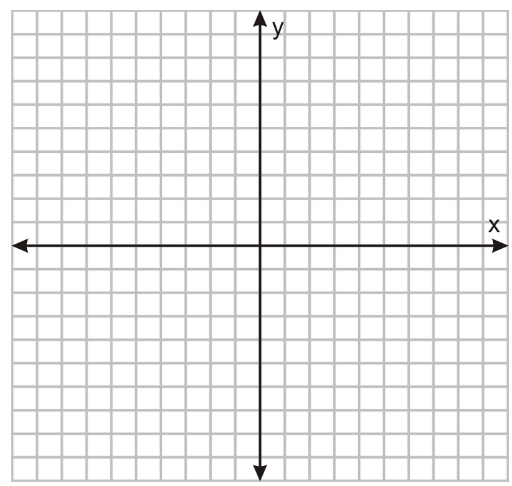
To determine the coordinates of the vertex, the domain and range, the axis of symmetry, the x and y intercepts and the direction of opening of the graph of f(x)=a(x – p)² + q without the use of technology.

REVIEW:

- A **QUADRATIC FUNCTION** is a function of degree two: $y = x^2$, $y = 2x^2 - 5x + 1$, $y = 2(x - 3)^2 - 3$, $y = (x + 1)^2$
- The place(s) where the quadratic function crosses the x axis are called the __________________
- A quadratic function may have ____________, ______________ or _____________ x intercepts
- There are four ways that a question may be asking you to find the x intercepts. They could ask you to:
    - Find the x intercepts
    - Find the ________________
    - Find the ________________
    - Find the ________________
- At this point, you may consider all four of the above terms to be exactly the same thing.

EX #1:
Using a table of values, sketch $y = 2x^2 + 4x - 6$ and identify the roots. Verify your answer(s).

x	y



EX #2: Using technology, find the zeros to the quadratic equation $y = -\frac{4}{17}x^2 + 9x - 5$

EX #4: Solve the equation $-4x^2 + 15 = -3x$ by graphing the corresponding function using technology.

- What is the difference between a QUADRATIC EQUATION, a QUADRATIC FUNCTION and a QUADRATIC EXPRESSION?

EX #3:

The manager of Jasmine's Fine Fashions is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100+15x-x^2$ gives the store's revenue R, in dollars, from dress sales, where x is the price change, in dollars. Use technology to determine the price changes that will result in no revenue?

EX #4:

The product of two consecutive positive numbers is 110. Represent this as an algebraic equation and graph to solve the equation to find the numbers.

EX #5:

- Is the equation $\frac{x^2-3}{5}+2=\frac{4x+9}{3}$ a quadratic equation? _______________
- If it is a quadratic equation, rewrite it in standard form.
- Is there more than one answer to writing the equation in standard form?
  - If so, will each form have different x intercepts?
  - What will be different between the forms?
- Graph to solve the equation.

4.1 ASSIGNMENT #1(Graphing Calculator Allowed)

4.1 FA#1: P215 #2, 3bd, 4ac(use a TABLE), 5, 6, 7, 8, 11, 12

4.1 ULA#1: P215 # 13, 14, 15, 17, 18 and the following:

1. Square a number add 9, divide the result by 5. The result is equal to twice the number. Write an equation to represent this and solve the equation.
2. A prize is divided equally among five people. If the same prize money is divided among six people each prize winner would get $2 less than previously. Write an equation to represent this and solve the equation.

4.2 DAY 1: SOLVING QUADRATICS BY FACTORING

PC20

To review factoring polynomial expressions.

REVIEW:

To factor a polynomial expression means to:

STEPS TO FOLLOW TO FACTOR AN EXPRESSION:

1. First, ALWAYS look for a GREATEST COMMON FACTOR (GCF)

EX #1: FACTOR OUT THE GCF OF THE FOLLOWING:

a) $2x + 4$

b) $9w^2 + 81$

c) $22bc + 33ab^2c^5$

d) $\frac{2}{3}x^2 - 8x$

e) $-\frac{1}{2}x^2 + \frac{5}{4}$

2. IF YOU CAN'T FACTOT OUT A GCF AND YOU HAVE A QUADRATIC IN THE FORM $ax^2 + bx + c$ where $a \neq 0$, USE THE WINDOW METHOD, DECOMOPOSITION OR TRIAL AND ERROR TO FACTOR THE EXPRESSION

- **Remember that within the WINDOW METHOD questions that there are some easier way to do some questions. Difference of square questions and quadratics where a = 1 can be done in your head!**

EX #2: FACTOR THE FOLLOWING USING THE METHOD OF YOUR CHOICE:

a) $x^2 + 6x - 16$

b) $x^2 - 4$

c) $3x^2 - 7x - 6$

d) $4m^2 - 36$

e) $x^2 - 5x - 24$

f) $4x^2 + 4x - 15$

g) $x^2 - y^2$

h) $\frac{4}{49}x^2 - \frac{25}{81}y^2$

c) $16x^2 + 25y^2$

d) $x^2 + 10x + 25$

e) $x^2 - 7x - 10$

f) $36x^2 - 12x + 1$

3. IF YOU CAN FACTOR OUT A GCF YOU ALSO HAVE TO CHECK TO SEE IF THE EXPRESSION IN BRACKETS CAN CONTINUE TO BE FACTORED USING THE WINDOW METHOD/DECOMPOSTION/TRIAL AND ERROR.

EX #3: FACTOR THE FOLLOWING

a) $2x^2 + 10x - 28$

b) $5x^2 - 20$

c) $-3x^2 + 42x - 147$

d) $-6x^2 - 13x + 5$

e) $-\frac{3}{10}x^2 + \frac{11}{10}x + 2$

f) $0.4x^2 - 1.8x - 1 = 0$

g) $3x^2 = \frac{29}{2}x - 14$

h) $-x^2 + \frac{625}{121}$

NEW: HOW TO FACTOR QUESTIONS THAT AREN'T QUADRATIC BUT IN THE FORM OF A QUADRATIC EQUATION

EX #4 : FACTOR $x^2 + 4x + 3$

FACTOR ☺^{2} + 4 ☺ + 3 using a substitution of m = ________

EX #5: Factor the following polynomials in quadratic form using substitution

a) $-2(x + 3)^2 + 12(x + 3) + 14$

b) $4(x - 2)^2 - 0.25(y - 4)^2$

4.2 ASSIGNMENT #1(NO Graphing Calculator)

4.1 FA#1: P 229 # 1-4, 10 AND #1, 2, 3 BELOW (NOTE: EXTRA BASIC FACTORING WORKSHEETS FOR PRACTICE ARE ON MY WEBSITE)

4.2 ULA #1 P 229 # 5, 6, 18, 26, 27 and #4 BELOW

1. Factor.

a) $2x^2 - 50y^2$ b) $0.1x^2 - 0.001$ c) $20x^2 - 125y^2$ d) $\frac{1}{100}x^2 - \frac{1}{25}y^2$

2. Factor.

a) $2x^2 + 16x + 24$ b) $3x^2 - 9x - 30$ c) $x^2 + \frac{5}{2}x - 6$ d) $x^2 + 2.5x - 1.5$

3. Factor each polynomial.

a) $\frac{x^2}{9} - \frac{4}{25}$ b) $6 + 5x - x^2$ c) $-x^2 + \frac{121}{64}$ d) $7 - \frac{5}{3}x - 2x^2$

4. Factor each polynomial expression.

a) i) $9x^2 - 4y^2$ ii) $9(x - 3)^2 - 4(2y + 1)^2$

b) i) $50x^2 - 162y^2$ ii) $50(2x - 5)^2 - 162(3y - 2)^2$

SOLUTIONS TO EXTRA QUESTIONS IN 4.2 DAY 1

a) $2(x - 5y)(x + 5y)$ b) $0.001(10x - 1)(10x + 1)$ c) $5(2x - 5y)(2x + 5y)$ d) $\frac{1}{100}(x - 2y)(x + 2y)$

a) $2(x + 6)(x + 2)$ b) $3(x - 5)(x + 2)$ c) $\frac{1}{2}(2x - 3)(x + 4)$ d) $0.5(2x - 1)(x + 3)$

a) $\left(\frac{x}{3} - \frac{2}{5}\right)\left(\frac{x}{3} + \frac{2}{5}\right)$ b) $(1 + x)(6 - x)$ c) $\left(\frac{11}{8} - x\right)\left(\frac{11}{8} + x\right)$ d) $\frac{1}{3}(7 + 3x)(3 - 2x)$

a) i) $(3x - 2y)(3x + 2y)$ ii) $(3x - 4y - 11)(3x + 4y - 7)$ b) i) $2(5x - 9y)(5x + 9y)$

4.2 DAY 2: SOLVING QUADRATICS BY FACTORING

To solve quadratic equations by factoring.

Zero Product Property: If $a \times b = 0$ then a = 0 or b= 0

STEPS to solving equations by FACTORING:

1. Set equation = 0 (write left side in standard form)
2. Factor fully
3. Set each factor = 0 and solve each. These solutions are called the roots.
   (Note: the only factors that will produce solutions are those factors that contain the variable)
4. Write the solution by either listing each root using x =# or by listing all roots in a solution set {#,…. }
   - Note that some teachers or textbooks prefer the solution set method.

EX #1: Solve each of the following.

a) $10x^2 + x - 3 = 0$

b) $2x^2 + 8x = 42$ (Verify your solution)

c) $5x^2 = 10x$

EX #2: Determine the roots for the following:

a) $2x^2 - 9x = 5$

b) $-6(3x - 5)(2x + 7) = 0$

c) $2x^2 + x + 2 = 0$

d) $2x(x - 6) + 3x = 2x - 9$

e) $\frac{x^2}{2} + \frac{7}{6}x = 1$ (Verify your solutions)

EX #2: Without factoring, determine if d – 5 is one of the factors of $-\frac{3}{10}x^2 + \frac{11}{10}x + 2 = 0$

EX #3: A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modelled by this formla: $h = 1 + 20t - 5t^2$

a) Determine the height of the football after 2 s.

b) When is the football 16 m high?

EX #4: When twice a number is subtracted from the square of the number, the result is 99. Determine the number.

EX #5: A rectangular garden has dimensions 5 m by 7 m. When both dimensions are increased by the same length, the area of the garden increased by 45 m^2. Determine the dimensions of the larger garden.

4.2 ASSIGNMENT #2(NO Graphing Calculator)

4.2 FA#2: P 229 #7, 8, 9, 10, 18ad, 14, 15, 16, 23

4.2 ULA #2 P 229 # 13, 17, 19, 21, 22, 24, 25, 28, 29, 30, 31, 32

4.3 SOLVING QUADRATICS BY COMPLETING THE SQUARE

PC20

To solve quadratics by completing the square and using the square root method.

STEPS to solving equations by completing the square and USING THE SQUARE ROOT METHOD:

1. Complete the square (if not already in the form of a completed square).
   - Keep in mind that in these questions the y has already been changed to zero and so there will be no "y"
   - Do NOT move the constant back to the right hand side at the end
   - The form we want to achieve looks like either $\# = (x - \#)^2$ or $(x - \#)^2 = \#$
     - NOTE: the above using the correct variables actually looks like $-q = (x - p)^2$
2. Take the square root of both sides. Technically we need to take the $\pm$ of both sides but mathematically it works out needing to only take the $\pm$ of one side
   - WHY DO YOU THINK THIS IS TRUE??
3. Solve both resulting equations (once using the + sign, once using the – sign)
4. ALWAYS CHECK your answers. Watch out for EXTRANEOUS roots (an answer not satisfying the restrictions on the variable)
5. Write your answer by either listing each root using x =# or by listing all roots in a solution set {#,…. }

REVIEW EX #1: Simplify the following:

a) $\sqrt{75}$

b) $-\sqrt{98}$

$\sqrt{48}$

EX #2: Solve each equation using the square root method. Leave your answers in exact form and decimal form (where appropriate) . Verify the solution(s).

a) $(x - 4)^2 = 12$

b) $2x^2 - 1 = 5$

c) $x^2 + 6x + 16 = 0$

d) $x^2 - 10x = 3$

e) $3x^2 + 12 = 0$

f) $8h^2 - 100 = 6h^2$

g) $9(x - 2)^2 = 27$

h) $2x^2 = 12x - 3$

i) $-2x^2 - 3x + 7 = 0$

EX #3: A football is kicked vertically. The approximate height of the football, h metres after t seconds is modelled by the formula $h(t) = 1 + 20t - 5t^2$. When will the football hit the ground? Give the answer in both exact and decimal form.

EX #4: Write a quadratic equation in standard form that has roots $3+\sqrt{5}$ and $3-\sqrt{5}$

4.3 ASSIGNMENT #2(NO Graphing Calculator)

4.3 FA: P 240 #4bcd, 5bcef, 6bcde, 7abde, 9, 10, 15

4.3 ULA: P 240 #8, 11, 12, 13, 14, 18, 19,

4.4 SOLVING QUADRATICS USING THE QUADRATIC FORMULA

PC20

To solve quadratic equations using the quadratic formula.

GO OVER QUESTION 15 from p 240

STEPS to solving equations using the QUADRATIC FORMULA.

1. Make sure your quadratic equation is in standard form **$ax^2 + bx + c = 0$**

2. To find the roots use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3.. Write the solution by either listing each root using x =# or by listing all roots in a solution set {#,…. }

EX #1: Solve the following using the quadratic formula. Write your answer in exact form.

a) $x^2 + 4x - 1 = 0$

b) $x^2 - x + 4 = 0$

c) $x^2 + 6x + 9 = 0$

d) $2x = 3(x - 1)(x + 1)$

e) $\frac{2}{3}x^2+1=\frac{5}{6}x$

EX #2: The surface area of a cylinder is 250 cm^2. The height of the cylinder is 7 cm. What is the radius of the cylinder to the nearest thousandth of a centimetre?

QUESTIONS:

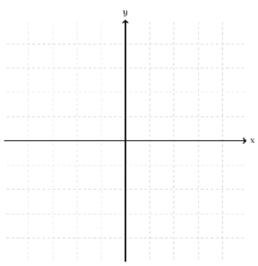
- **When we are using the quadratic formula, how can we tell early on that there will be no solution?**
- **When we are using the quadratic formula, how can we tell early on that there will be two solutions that are the same?**

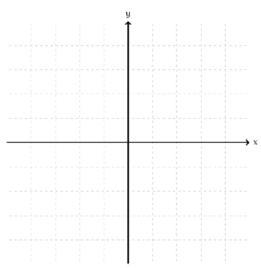
We can use THE DISCRIMINANT to predict how many roots a quadratic equation has without actually solving the equation. The discriminant is the portion of the quadratic formula under the root sign: $D = b^2 - 4ac$

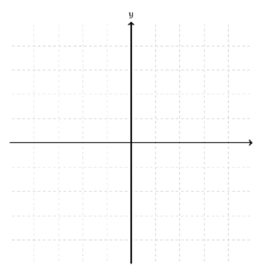
VALUE OF THE DISCRIMINANT $D = b^2 - 4ac$	EXAMPLE OF A NUMBER THAT FITS THIS DESCRIPTION	HOW WE DESCRIBE THE "NATURE OF THE ROOTS"
$D < 0$ (The value of the discriminant D is a negative number)		The solution will have "NO REAL ROOTS"
$D > 0$, D is a perfect square (The value of the discriminant D is positive and a perfect square number)		The solution will have "2 REAL RATIONAL ROOTS"
$D > 0$, D is not a perfect square number (The value of the discriminant D is positive but NOT a perfect square number		The solution will have "2 REAL IRRATIONAL ROOTS"
$D = 0$ (The value of the discriminant D is zero)		The solution will have "1 REAL ROOT/A DOUBLE ROOT"

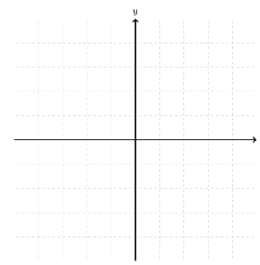
Draw a sketch of a parabola where:

a) $D < 0$ b) $D > 0$ & a perfect square c) $D > 0$ &not a perfect square d) $D = 0$









EX #2: Use the discriminant to determine the nature of the roots for each quadratic equation. Check by graphing (you may use technology)

a) $x^2 - 5x + 4 = 0$

b) $2x^2 - 8x = -9$

EX #2: Without solving, determine whether the equation $5x^2 - 8x + 6 = 0$ has one, two or no real roots.

EX #2: **a)** Determine the values of k for which $2x^2 + 7x + k = 0$ has no real roots.

b) Use one value of k to write an equation that has no real roots.

QUESTION: What type of discriminant will a question have if it is factorable?

4.4 ASSIGNMENT #2 (Graphing Calculator Allowed to Check)

4.4 FA: P 254 #1cdef, 2bcde, 3, 4ac, 5bce, 9

4.4 ULA: P 254 #910, 11, 12, 13, 15, 17, 18, 20

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

Section 4.1

- https://goo.gl/rTQLpe
- https://goo.gl/BwrMZC
- https://goo.gl/RRGiFx

Section 4.2 DAY 1

- https://goo.gl/rPyxf9
- https://goo.gl/myq473

Section 4.2 DAY 2

- https://goo.gl/BpHJvQ
- https://goo.gl/oNr5wq
- https://goo.gl/ZaJU6L

Section 4.3

- https://goo.gl/W1V5sU
- https://goo.gl/7qhRiw

Section 4.4

- https://goo.gl/zAwV89
- https://goo.gl/D6T2JB
- https://goo.gl/dsQuFD