‘5.1 WORKING WITH RADICALS

PC20

To convert radicals in mixed form to entire form (and vice versa), to identify the restriction on the value for a variable in a radical expression and to add and subtract radicals.

REVIEW:

Terminology For Radicals:

The little number outside and to the left of the radical sign is called the INDEX.

$\sqrt[n]{x}$

The number/term inside the radical sign is called the RADICAND

- **MIXED RADICAL -** is the product of a monomial and a radical:

  Ex: $3\sqrt{5},\ \ 10\sqrt[4]{13},\ \ -2\sqrt[3]{9},\ \ 3x\sqrt{7xy}$

- **ENTIRE RADICAL -** is a radical with a coefficient of 1 or -1

  Ex: $\sqrt{122},\ \ -\sqrt[4]{32},\ \ \sqrt[3]{16x^4y^5}$

To the power 2 (Squared)	To the Power 3 (Cubed)	To the Power 4	To the Power 5
$2^2 = 4$ ∴ →	$2^3 = 8$ ∴ →	$2^4 = 16$ ∴ →	$2^5 = 32$ ∴ →
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$
$6^2 = 36$	$6^3 = 216$	$6^4 = 1296$	$6^5 = 7776$
$7^2 = 49$	$7^3 = 343$	$7^4 = 2401$	$7^5 = 16807$
$8^2 = 64$	$8^3 = 512$	$8^4 = 4096$	$8^5 = 32768$
$9^2 = 81$	$9^3 = 729$	$9^4 = 6561$	$9^5 = 59049$
$10^2 = 100$	$10^3 = 1000$	$10^4 = 10000$	$10^5 = 100000$
$11^2 = 121$	$11^3 = 1331$	$11^4 = 14641$	$11^5 = 161051$
$12^2 = 144$	$12^3 = 1728$	$12^4 = 20736$	$12^5 = 248832$
$13^2 = 169$	$13^3 = 2197$	$13^4 = 28561$	$13^5 = 371293$
$14^2 = 196$	$14^3 = 2794$	$14^4 = 38416$	$14^5 = 537824$
$15^2 = 225$	$15^3 = 3375$	$15^4 = 50625$	$15^5 = 759375$
$16^2 = 256$	$16^3 = 4096$	$16^4 = 65536$	$16^5 = 1048576$
$17^2 = 289$	$17^3 = 4913$	$17^4 = 83521$	$17^5 = 1419857$
$18^2 = 324$	$18^3 = 5832$	$18^4 = 104976$	$18^5 = 1889568$
$19^2 = 361$	$19^3 = 6859$	$19^4 = 130321$	$19^5 = 2476099$
$20^2 = 400$	$20^3 = 8000$	$20^4 = 160000$	$20^5 = 3200000$
$21^2 = 441$	$21^3 = 9261$	$21^4 = 194481$	$21^5 = 4084101$
$22^2 = 484$	$22^3 = 10648$	$22^4 = 234256$	$22^5 = 5153632$
$23^2 = 529$	$23^3 = 12167$	$23^4 = 279841$	$23^5 = 6436343$

EX #1: Express the following Entire Radicals as Mixed Radicals (you may wish to use the chart on the previous page).

a) $\sqrt{192}$

b) $\sqrt[3]{y^8}$

c) $\sqrt{48y^5}$

d) $5\sqrt[3]{-16}$

d) $3\sqrt[4]{162x^9y^{12}}$

- If the index is an even number, then the radicand must be ____________________
- If the index is an odd number, then the radicand can be ______________________

EX #2: Express each mixed radical in entire radical form. State the restrictions on the variable.

a) $5\sqrt{3}$

b) $2b\sqrt{b}$

c) $-4x^3\sqrt{7x^2}$

EX #3: Order the following five numbers in order from least to greatest without using a calculator.

$4(13)^{\frac{1}{2}},\ 8\sqrt{3},\ 14,\ \sqrt{202},\ 10\sqrt{2}$

In order to add or subtract radicals:

- The radicals must be _______ radicals.
- _________ radicals have the same ________________ and the same_______________.

EX #4: Simplify the following:

a) $2\sqrt{7}+13\sqrt{7}$

b) $\sqrt{24}-\sqrt{6}$

c) $\sqrt{20x}-3\sqrt{45x}, \quad x \geq 0$

d) $2\sqrt{27}+2\sqrt{75}$

e) $5\sqrt{8}-3\sqrt{18}+\sqrt{3}$

f) $3\sqrt{32a}-4\sqrt{162a}, \quad a \geq 0$

g) $10\sqrt[3]{24}+3\sqrt[3]{81}-6\sqrt[3]{3}$

EX #5: Identify the restrictions on the values for the variables.

a) $-5\sqrt{2a}$

b) $2a\sqrt{x-4}$

c) $\sqrt[3]{8r}$

5.1 ASSIGNMENT

5.1 FA: P278 # 1, 2, 3, 4, 5, 6 (no calculator), 8, 9, 10, 11, 15, 19, 20

NOTE: if you are struggling with questions 1-6 there are extra practice questions on my weebly.

5.1 ULA: P279 12, 13, 14, 16, 17, 18, 21, 22

5.2 DAY 1: MULTIPLYING RADICALS

To multiply radical expressions.

In order to multiply radicals:

- **It is easiest to multiply Radicals if they are in simplest form BEFORE multiplying**
- If they have the same index, multiply the ______________________ and multiply the _________________
- Leave your answer in simplest form
- State the restrictions for the variables (if the index is even, the radicand must be positive)

EX #1: Multiply the following. Always leave answers in SIMPLEST FORM and state any restrictions.

a) $\left(2\sqrt{7}\right)\left(4\sqrt{75}\right)$

b) $\left(-3\sqrt{2}\right)\left(4\sqrt{6}\right)$

c) $8\sqrt{3}\left(5\sqrt{5}-4\sqrt{3}\right)$

d) $-2\sqrt[3]{11}\left(4\sqrt[3]{2}-3\sqrt[3]{3}\right)$

e) $\left(4\sqrt{2}+3\right)\left(\sqrt{7}-5\sqrt{14}\right)$

f) $\left(8\sqrt{2}-5\right)\left(9\sqrt{5}+6\sqrt{10}\right)$

g) $-2\sqrt{11c}\left(4\sqrt{2c^3}-3\sqrt{3}\right)$

h) $\left(\sqrt{8}-\sqrt{3}\right)^2$

i) $\left(3\sqrt{a}-2\right)\left(4\sqrt{a}+5\right)$

j) $\left(2\sqrt{3}-5\right)\left(2\sqrt{3}+5\right)$

k) $\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)$

l) $\left(\sqrt{x}-2\sqrt{y}\right)\left(\sqrt{x}+2\sqrt{y}\right)$

THE CONJUGATE OF A BINOMIAL EXPRESSION:

- The CONJUGATE of a binomial expression is another binomial expression with the opposite middle sign. The conjugate of (a + b) is (a – b)
- The product of a binomial and its conjugate is a difference of squares. The answer to this product will be a binomial as the middle term will be eliminated (there will be two identical terms but opposite in sign)

EX #2: Identify the conjugate of each binomial. Multiply each binomial by its conjugate.

a) Binomial: $\left(2\sqrt{6}+5\sqrt{3}\right)$ Conjugate: ___________________________

Product:

b) Binomial: $\left(2-\sqrt{50}\right)$ Conjugate: ___________________________

Product:

c) Binomial: $\left(-5\sqrt{2}-8\right)$ Conjugate: ___________________________

Product:

5.2 ASSIGNMENT #1

5.2 FA#1: P289 # 1, 3, 4, 5, 9, 12

5.2 ULA#1: P292 # 21, 22, 27

5.2 DAY 2: DIVIDING RADICALS

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To simplify radical expressions involving division.

In order to divide radicals:

- YOU MAY OR MAY NOT WANT TO SIMPLIFY YOUR RADICALS BEFORE YOU DIVIDE
- If they have the same index, divide or reduce the ______________________ and divide or reduce the _________________
- State the restrictions for the variables (if the index is even, the radicand must be positive)
- By convention we are usually expected to leave our final answer in a form where there is NO radical left in the denominator. We need to RATIONALIZE THE DENOMINATOR if we end up with a radical in the denominator.
- Fully simplify your answer

EX #1: Simplify the following. Always leave answers in SIMPLEST FORM and state any restrictions.

a) $\dfrac{\sqrt{56}}{\sqrt{7}}$

b) $\dfrac{12\sqrt{18}}{3\sqrt{2}}$

c) $\dfrac{4\sqrt[4]{2^{12}}}{\sqrt[4]{2^{5}}}$

d) $\dfrac{30\sqrt[3]{144t^2}}{36\sqrt[3]{16t}}$, $t \geq 0$

e) $\left(\dfrac{3}{2}\sqrt[3]{12}\right)\left(-\dfrac{8}{9}\sqrt[3]{4}\right)\left(\sqrt[3]{2}\right)$

f) $\dfrac{\sqrt{24x^3}}{\sqrt{3x}}, x \geq 0$

g) $\dfrac{15\sqrt{45x^4}-9\sqrt{20x^3}}{12\sqrt{5x^2}}$

RATIONALIZING THE DENOMINATOR:

- If you have a final answer that has a monomial in the denominator that contains a radical, multiply the numerator and the denominator of the fraction by the radical in the denominator and then simplify (you should no longer have a radical in the denominator)
- If you have a final answer that has a binomial in the denominator that contains a radical, multiply the numerator and the denominator of the fraction by the CONJUGATE of the binominal in the denominator and then simplify (you should no longer have a radical in the denominator)

EX #2: Simplify the following. Always leave answers in SIMPLEST FORM (without any radicals in the denominator) and state any restrictions

a) $\dfrac{8\sqrt{5}}{4\sqrt{2}}$

b) $\dfrac{7\sqrt{3}}{3\sqrt{x}}$

c) $\dfrac{15y^4\sqrt{30x^2}}{5\sqrt{5y^4x^3}}$

d) $\dfrac{6}{3+\sqrt{2}}$

e) $\dfrac{5\sqrt{2}}{3\sqrt{2}-\sqrt{3}}$

f) $\dfrac{6-3\sqrt{3}}{\sqrt{3}}$

g) $\dfrac{4+\sqrt{2}}{\sqrt{3}+5\sqrt{2}}$

h) $\dfrac{4\sqrt{11}}{y\sqrt[3]{6}}$

5.2 ASSIGNMENT #2

5.2 FA#1: P289 # 6, 7, 8, 10, 11

5.2 ULA#1: P292 # 13, 14, 15, 16, 17, 19, 20, 23, 24, 26, 31

5.3 SOLVING RADICAL EQUATIONS

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To solve RADICAL EQUATIONS (equations where the variable you are solving for is IN THE RADICAND)

STEPS:

1. State your restriction(s)
2. Isolate one radical (if there are two you should begin by getting the most complicated one by itself)
3. Square both sides (if there is a binomial on one side you must square that binomial and get a trinomial!)
4. If there is still a radical in your equation, isolate it and repeat step 2 again
5. Solve for the variable
6. Verify your answers

EX #1: Solve and state the restrictions on $\sqrt{2x-5}=5$

You MUST check your answers to make sure that they work! IF your answers do not work when you check, then that value is called an EXTRANEOUS ROOT and you need to cross it out and not include it in your solution set or final answer.

EX #2: Solve and state the restrictions on $3\sqrt{3x+2}-2=4$

EX #3: Solve and state the restrictions on $n-\sqrt{5-n}=-7$

EX #4: Solve and state the restrictions on the following:

a) $\sqrt{x+2}-\sqrt{3x-5}=-1$

b) $\sqrt{z+5}=\sqrt{2z-1}$

c) $7+\sqrt{3x}=\sqrt{5x+4}+5$

EX #4: What is the speed, in metres per second, of a 0.4-kg football that has 28.8 J of kinetic energy? Use the kinetic energy formula, $E_k=\frac{1}{2}mv^2$, where E_k represents the kinetic energy, in joules; *m* represents mass, in kilograms; and *v* represents speed, in metres per second.

5.3 ASSIGNMENT

5.3 FA: P300 #3bc, 4, 5ac, 7abc, 8ad, 9, 10, 15
5.3 ULA: P300 # 12, 13, 14, 16, 17, 18, 20, 23, 26

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

Section 5.1

- https://goo.gl/Sj7ze3
- https://goo.gl/JFPKGu
- https://goo.gl/Gu8MFc

Section 5.2

- https://goo.gl/tveBXf (multiplying radicals)
- https://goo.gl/qiWsNz (dividing radicals)
- https://goo.gl/N18pWZ (multiplying & dividing radicals)

Section 5.3

- https://goo.gl/RZWSTt (with one radical)
- https://goo.gl/c1im8z (with two radicals)
- https://goo.gl/Sq5sTp (with one or two radicals)