6.1 RATIONAL EXPRESSIONS

PC20

To simplify rational expressions and determine the non-permissible values.

RATIONAL EXPRESSIONS

- Are fractional expressions where both the numerator and denominator are polynomials

  EXAMPLES: $\frac{1}{x}$ $\frac{m}{m+1}$ $\frac{y^2}{y^2+2y+1}$ x^2-4

- Rational expressions are not allowed to have denominator values of zero – the expression is considered to be undefined at these values of the variable. Non-permissible values are values of the variable that make the expression undefined

EX #1: Determine the non-permissible values for each of the following.

a) $\frac{5t}{2s}$ b) $\frac{3x}{x(2x-3)}$ c) $\frac{5t}{4sr^2}$ d) $\frac{2p-1}{p^2-p-12}$

SIMPLIFYING RATIONAL EXPRESSIONS

1. Factor the numerator and the denominator of the expression
2. State the non-permissible values
3. Simplify by reducing any common factors between the numerator and denominator

EX #2: Simplify the following. State any non-permissible values/restrictions.

a) $\frac{9}{12}$ b) $\frac{m^3t}{m^2t^4}$ c) $\frac{x^2-1}{x^2+3x+2}$

d) $\frac{3x-6}{2x^2+x-10}$ e) $\frac{3x-6}{2-x}$

f) $\dfrac{x^2+2x-15}{x-3}$

g) $\dfrac{2y^2+y-10}{y^2+3y-10}$

h) $\dfrac{x^2-10x+24}{x^2-6x}$

i) $\dfrac{1-t}{t^2-1}$

j) $\dfrac{25-x^2}{x^2-3x-10}$

6.1 ASSIGNMENT

6.1 FA: P317 #1, 4, 5, 6, 8, 14, 15, 22, 26d, 29

6.1 ULA: P318 #7, 9, 10, 11, 13, 16, 18, 19, 21, 23, 24, 26abc

6.2 MULTIPLYING & DIVIDING RATIONAL EXPRESSIONS

PC20

To multiply and divide rational expressions.

REVIEW EX #1: Simplify the following:

a) $\frac{5}{8} x \frac{4}{15}$

b) $\frac{5}{3} \div \frac{1}{6}$

STEPS TO MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

1. Factor the numerators and denominators separately and completely (if they can be factored)

2. State the non-permissible values (values that make the denominator turn into zero)

3. If there is MULTIPLICATION OF RATIONAL EXPRESSIONS:
   - First simplify by reducing (cancelling out) common factors between any numerator and any denominator
   - Multiply the remaining numerators together and write that simplified polynomial in the final numerator. Multiply the remaining denominators together and write that simplified polynomial in the final denominator

4. If there is DIVISION OF RATIONAL EXPRESSIONS:
   - Rewrite the expression using multiplication (leave the first rational expression as it is, change the division sign to a multiplication sign and then flip the numerator and denominator of the rational expression that was originally after the division sign)
   - At this point you need to add the non-permissible values of the new factors in the denominator to your non-permissible value list in step 2
   - Using your new expression that contains multiplication signs, simplify by reducing (cancelling out) common factors between any numerator and any denominator
   - Multiply the remaining numerators together and write that simplified polynomial in the

EX #2 : Simplify the following. State the non-permissible values.

a) $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^5}{8}\right)$

b) $\frac{3x^2}{y^2} \div \frac{x}{y}$

c) $\left(\frac{a^2-a-12}{a^2-9}\right)\left(\frac{a^2-4a+3}{a^2-4a}\right)$

d) $\frac{x^2-25}{x^2-49} \bullet \frac{x^2-6x-7}{x^2+6x+5}$

e) $\frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$

f) $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \bullet \frac{2x-6}{x+4}$

g) $\dfrac{2m^2-7m-15}{2m^2-10m} \div \dfrac{4m^2-9}{6} \bullet (3-2m)$

h) $\dfrac{x^2-9}{y^3-y} \bullet \dfrac{y^2-y}{3-x}$

6.2 ASSIGNMENT

6.2 FA: P327 #1, 2, 4, 6, 8, 9, 15

6.2 ULA: P327 #10, 11, 12, 14, 16, 17, 18, 19, 23

6.3 ADDING & SUBTRACTING RATIONAL EXPRESSIONS

To add or subtract rational expressions.

REVIEW EX #1: Simplify: $\frac{5x}{3y}-\frac{7x}{4y}$

STEPS TO ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

1. Factor the numerators and denominators separately and completely (if they can be factored)
2. State the non-permissible values
3. Determine the Lowest Common Denominator (LCD).
4. Write each rational expression as an equivalent expression with a common denominator.
5. Add/subtract the numerators (keep the denominator the same)
6. Simplify the expression if possible

EX #2: Simplify (always state the restrictions when asked to simplify)

a) $\frac{7x+1}{2x}+\frac{5x-3}{2x}$

b) $\frac{2a}{b}-\frac{a-1}{b}$

c) $\frac{2x}{x+4}+\frac{8}{x+4}$

d) $\frac{x^2}{x-2}+\frac{3x}{x-2}-\frac{10}{x-2}$

e) $\frac{2x}{xy}+\frac{4}{x^2}$

f) $\frac{1}{2s}-\frac{s}{4t}+6$

g) $\frac{5}{y+1}-\frac{y-4}{y^2+y}$

h) $\frac{y^2-20}{y^2-4}+\frac{y-2}{y+2}$

i) $\frac{x-7}{x+3}-\frac{x^2+x-2}{x^2+6x+5}\div\frac{x^2+5x+6}{x^2-5x}$

6.3ASSIGNMENT

6.3 FA: P336 #1acde, 3, 5acf, 6, 7

6.3 ULA: P336 #2, 8, 9, 10, 15, 12, 14, 18,

6.4 Day 1: SOLVING RATIONAL EQUATIONS

PC20

To solve rational equations.

STEPS TO SOLVING RATIONAL EQUATIONS

Remember: Solving an equation means to find the value(s) of the variable that will make the equation true.

1. Completely Factor denominators
2. State the non-permissible values
3. Determine the Lowest Common Denominator (LCD).
4. Multiple each term on both sides of the equation by the LCD to eliminate all denominators.
5. Solve for the variable
6. Check your answers. There may be extraneous roots!

EX #1: Solve the following rational equations.

a) $\frac{x}{4}-\frac{7}{x}=3$

b) $\frac{2x}{x-4}=\frac{10}{x-4}$

c) $\frac{x}{x-2}+\frac{2}{x+2}=1$

d) $\frac{9}{y-3}-\frac{4}{y-6}=\frac{18}{y^2-9y+18}$

e) $\frac{3x}{x+2}-\frac{5}{x-3}=\frac{-25}{x^2-x-6}$

f) $\frac{4k-1}{k+2}-\frac{k+1}{k-2}=\frac{k^2-4k+24}{k^2-4}$

EX #2: The sum of two numbers is 10. The sum of their reciprocals is $\frac{10}{21}$. Determine the two numbers.

6.4 Day 1ASSIGNMENT

6.4 Day 1 FA: P348 #2, 3, 4, 6, 8, 9, 11
6.4 Day 1 ULA: P350 #21, 22, 23, 24, 27

6.4 Day 2: SOLVING RATIONAL EQUATIONS

PC20

To solve situational problems that are solved using rational equations.

EX #1: Mr. Buttons can eat a whole bag of cat food in 40 minutes. Madame Whiskers can eat the same bag of cat food in 50 minutes. How long would it take the 2 cats to eat the bag together?

EX #2: It takes Rhino 4 hours to paint a room. It takes Bumble 3 hours to paint the same room. How long will it take them if they work together?

EX #3: A train is to travel 160 km between the two cities. If the average speed is decreased by 15 km/h, the trip will take 0.5 hours longer. What is the average speed of the train?

6.4 Day 2 ASSIGNMENT

6.4 Day 1 FA: P348 #5, 7, 10, 12, 13, 14, 16,

6.4 Day 1 ULA: P350 #15, 17, 18, 19, 20, 26

7.4 Day 1: RECIPROCAL OF LINEAR FUNCTIONS

To graph the reciprocal of a linear function

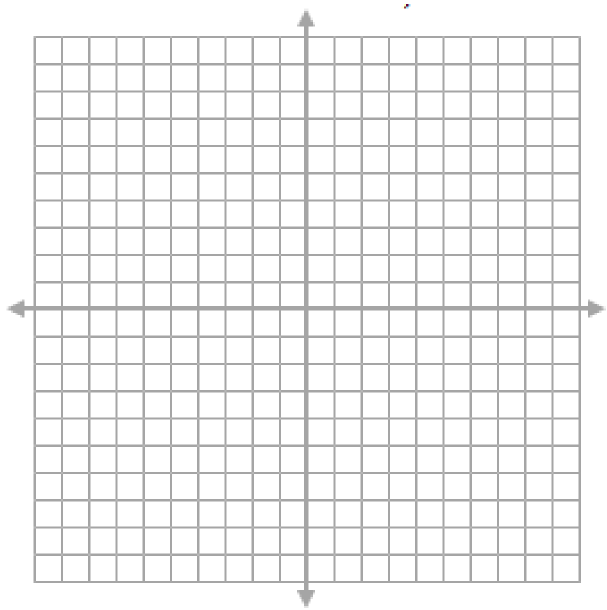
RECIPROCAL FUNCTIONS:

- If f(x) is a function, then it's reciprocal is $\frac{1}{f(x)}$ provided that $f(x) \neq 0$
- The product of a function and it's reciprocal is _______ because $f(x) \bullet \frac{1}{f(x)} =$

EX #1: Sketch the graphs of $y = x$ and $y = \frac{1}{x}$ on the same set of axes.

$y = x$	
x	***y***
-10	
-2	
-1	
-1/2	
-1/10	
0	
1/10	
½	
1	
2	
10	

$y = \frac{1}{x}$	
x	***y***
-10	
-2	
-1	
-1/2	
-1/10	
0	
1/10	
½	
1	
2	
10	



Characteristic	$y = x$	$y = \frac{1}{x}$
Domain		
Range		
End Behaviour	As *x* becomes a very "large negative value", *y* becomes a As *x* becomes a very "large positive value", *y* becomes a	As *x* becomes a very "large negative value", *y* becomes a As *x* becomes a very "large positive value", *y* becomes a
Behaviour near the non-permissible value		
Invariant Points of $y = x$ **and** $y = \frac{1}{x}$	Invariant points are points that remain unchanged between an original function and any transformation of that function (including taking its reciprocal)	

ASYMPTOTE

- a line that the graph gets infinitely close to but doesn't touch or cross
- an asymptote is drawn as a dashed line on the graph itself
- graphs can have vertical and/or horizontal asymptote(s)

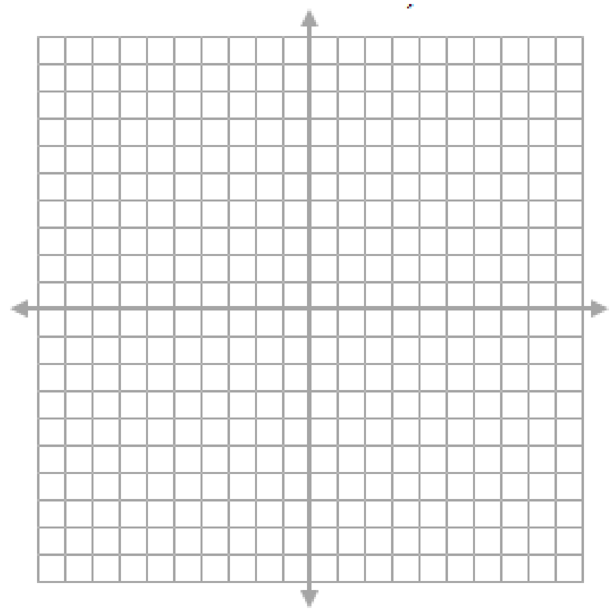
Vertical Asymptotes

- occur at the value(s) of x that are the non-permissible values of the domain of the rational function
- The graph will have a vertical asymptote at all non-permissible value locations (if the original function was linear the reciprocal graph will only have one vertical asymptote)
- Each vertical asymptote will have an equation where x = (a non-permissible value)

Horizontal Asymptote

- occurs when y= 0 because $\frac{1}{x} \neq 0$
- Functions will only have one horizontal asymptote
- The equation of the horizontal asymptote will always be y=0 for the reciprocal of all polynomial functions

EX #2: **Graph y = 2x + 5 and its reciprocal**



a) Write the reciprocal function:

b) Find the non-permissible values. Use these to determine the equation(s) of the vertical asymptote(s) of the reciprocal function. Sketch the asymptotes onto the graph (use a different colour and used dashed lines)

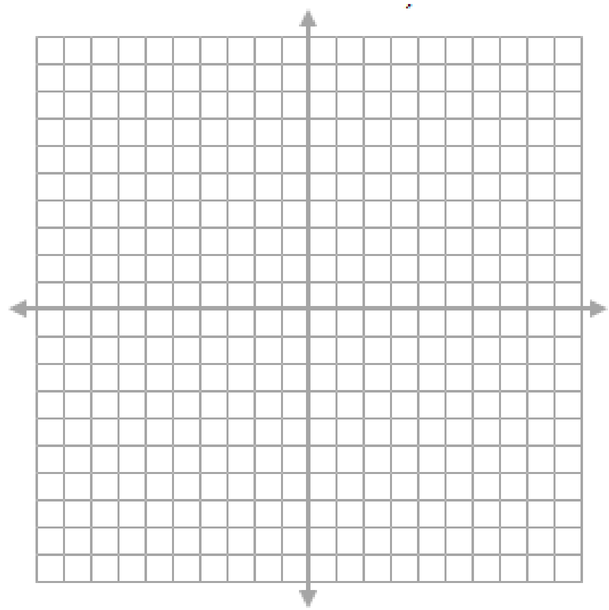
c) Find the x and y intercept of the reciprocal function. Sketch them on the graph.

d) The inverse of a linear graph will have two separate and distinct portions to its graph. These two portions will be in opposite sections formed by the asymptotes. You need to have at least three points in each "section". Find additional points in each section so that you have 3 in each. Plot these points.

e) Join your points in each section so that they form a smooth curve that approaches the asymptotes as it continues to grow in both directions.

f) Using a different colour, sketch the original linear function. Are there any invariant points? What are they?

EX #3: Graph $f(x) = 3x - 9$ and its reciprocal.



a) Find the zeros (x-intercept) of the f(x).(original function)

b) Determine its reciprocal function $y = \frac{1}{f(x)}$.

c) Determine the equation of the vertical asymptote of the reciprocal function. How are the zeros (x-intercept) of the original function related to the non-permissible values?

d) Sketch the graph of the reciprocal function using the same steps as example 2.

e) State the following:

- Domain:
- Range:
- End Behavior:
- Behavior near the non-permissible value (NPV):

7.4 Day 1 ASSIGNMENT

7.4 Day 1 FA: P403 # 1ab, 2ab, 5ab, 6a, 7cd (also state the domain, the range, the end behavior and the behavior near the NPV

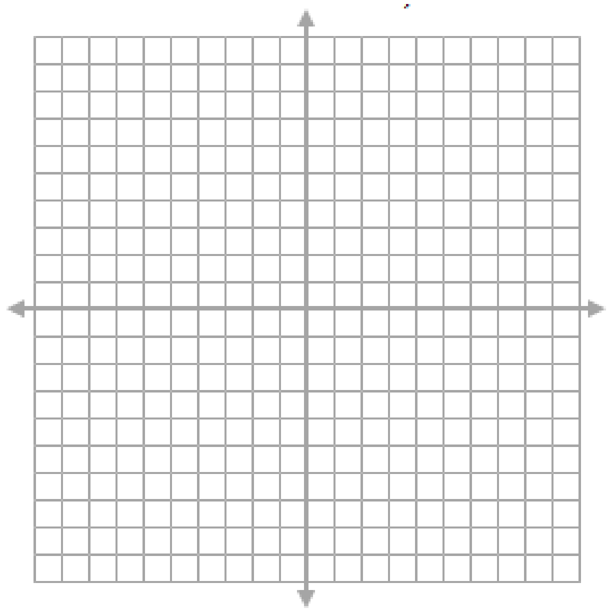
7.4 Day 1 ULA: P404 # 11, 12, 17

7.4 Day 2: RECIPROCAL OF QUADRATIC FUNCTIONS

To graph the reciprocal of a quadratic function

EX #1: Given $f(x) = x^2 - 4$

a) What is the reciprocal function of *f(x)*?



b) State the non-permissible values of x and the equation(s) of the vertical asymptotes of the reciprocal function.

c) State the x-intercept(s) and y-intercept of the reciprocal function.

d) State the following:

- Domain:
- Range:
- End Behavior:
- Behavior near the non-permissible value(s) (NPV):

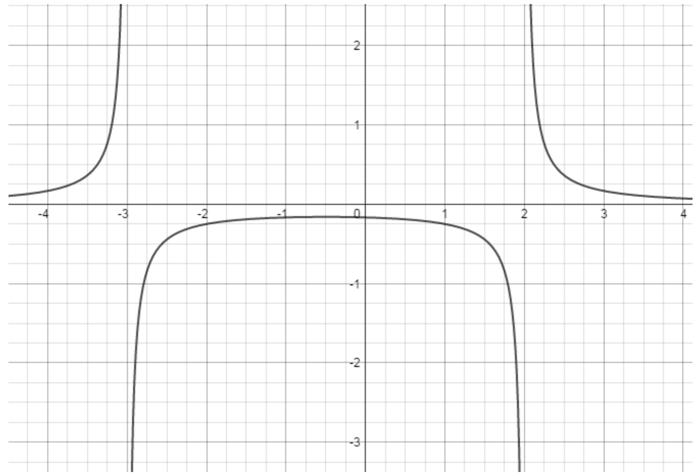
e) State the invariant points. (Let y = ±1 and solve for x)

f) Graph the function using all the points you have (add points if you need)

g) Using a different colour, graph f(x).

h) What is the graphical relationship between f(x) and $\frac{1}{f(x)}$?

EX #2: Given the graph of $y = \dfrac{1}{f(x)}$ (reciprocal function), match it with the graph of f(x) (original function).
Hint: Determine the equations of the vertical asymptotes



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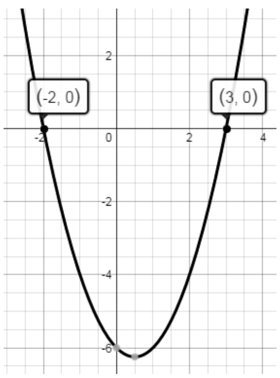

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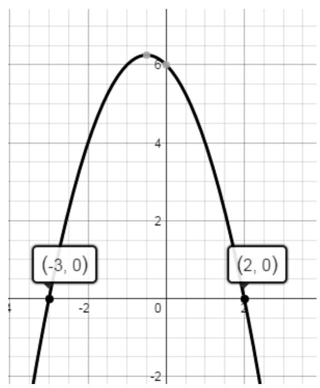

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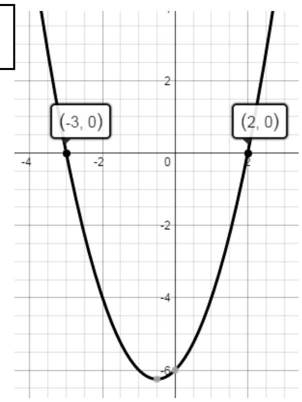


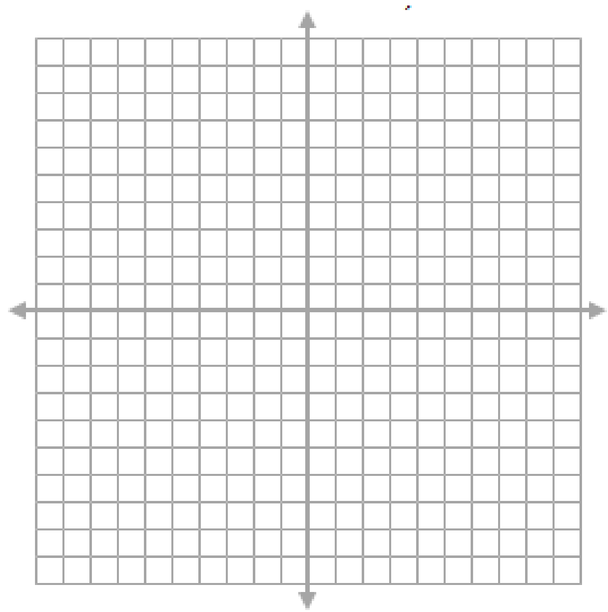
EX #: Given $f(x) = -x^2 - x + 6$

a) What is the reciprocal function?

b) What are the asymptote(s)?

c) State the x and y intercept(s)?

d) Sketch the graph.



e) State the following:

- Domain:
- Range:
- End Behavior:
- Behavior near the non-permissible value(s) (NPV):

STEPS TO FINDING & GRAPHING RECIPROCAL FUNCTIONS

- Given an f(x) is a function where $f(x) \neq 0$, then it's reciprocal is $\dfrac{1}{f(x)}$
- Factor the denominator to find the vertical asymptotes. The horizontal asymptote is at y = 0. Sketch the asymptotes using dashed lines
- Find any intercept(s) and plot them
- Find test points in each "zone" so that you have at least three points to draw each section (invariant points are usually helpful to find). Connect your dots in a smooth curve so that they approach (but don't cross or touch) the asymptotes.
- **Domain**: will be all reals except for the non-permissible values – you need to use set notation

  Range : will be all reals except for y = 0

  End Behavior: as $y \to 0,\ |x| \to \infty$

  Behavior near NPV Points: as $x \to \text{each NP Value},\ |y| \to \infty$ (You need to describe the behavior

7.4 Day 2 ASSIGNMENT

7.4 Day 1 FA: P403 # 2cd, 5cd, 6bc, 8bc (also state the domain, the range, the end behavior and the behavior near the NPV), & #9

7.4 Day 1 ULA: P404 # 8d, 10, 15, 18, 19,

VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

Section 6.1

- https://goo.gl/PTN9pn
- https://goo.gl/hGGSVL
- https://goo.gl/UzXgqT

Section 6.2

- https://goo.gl/iouZvD
- https://goo.gl/9tc2Ye

Section 6.3

- https://goo.gl/jQvzWW
- https://goo.gl/kCkhto

Section 6.4

- https://goo.gl/iicjVx
- https://goo.gl/w7PB1U
- https://goo.gl/vLvVEo

Section 7.4

- https://goo.gl/nyYkm3
- https://goo.gl/Wrc7Ue