CALC 30: QUIZ OUTCOME 7: DAYS 1-5

Name: Key **Date:** _____

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For full marks, show all work in the space provided.

1. Differentiate the following.

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a) $y = 3\log_3(x^2 - 1)$

$\frac{dy}{dx} = \frac{3 \cdot 1}{x^2-1} \log_3 e (2x)$

$= \frac{6x}{x^2-1} \log_3 e$

or

$\frac{6x \log_3 e}{x^2-1}$

b) $y = \ln(2x^3 + 5)^3$

$\frac{dy}{dx} = \frac{1}{(2x^3+5)^3} \cdot 3(2x^3+5)^2(6x^2)$

$= \frac{18x^2\cancel{(2x^3+5)^2}}{(2x^3+5)^{\cancel{3}}}$

$= \frac{18x^2}{2x^3+5}$

c) $f(x) = \frac{\ln x}{x^3}$ quotient rule

$f'(x) = x^3 \cdot \frac{1}{x}(1) + \ln x(3x^2)$

$= x^2 + 3x^2 \ln x$

$= x^2(1 + 3\ln x)$ factored form

d) $y = 3^{2x-7}$

$\frac{dy}{dx} = 3^{2x} \ln 3 (2)$

$= 2 \cdot 3^{2x} \ln 3$

$= 2(\ln 3)(3^{2x})$

e) $y = e^{3\ln x} = \cancel{e^{\ln}} x^3 = x^3$

$\frac{dy}{dx} = 3x^2$

f) $y = x^6 e^{3x}$ product rule

$\frac{dy}{dx} = x^6 e^{3x}(3) + 6x^5 e^{3x}$

$= 3x^6 e^{3x} + 6x^5 e^{3x}$

$= 3x^5 e^{3x}(x+2)$

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2. Find the slope of the tangent line to the graph of $f(x) = \ln\left(\frac{x}{x-1}\right)$ at x = 2.

$f'(x) = \frac{\cancel{x-1}}{x}\left[\frac{(x-1)(1) - x(1)}{(x-1)^{\cancel{2}}}\right]$

$= \frac{x-1-x}{x(x-1)} = \frac{-1}{x(x-1)}$

$f'(2) = \frac{-1}{2(2-1)} = \frac{-1}{2(1)} = \frac{-1}{2}$ is the slope of the tangent line at $x=2$

3. Evaluate the following limits.

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a) $\lim_{x\to 0}\frac{\sin 3x}{5x}$

$= \frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{x}$ 1pt

$= \frac{3}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}$

$= \frac{3}{5}\cdot 1$ 1pt

$= \frac{3}{5}$

b) $\lim_{\theta\to 0}\frac{2\tan^2\theta}{\theta^2}$

$= \lim_{\theta\to 0} 2\left(\frac{\tan\theta}{\theta}\right)^2$

$= \lim_{\theta\to 0} 2\left(\frac{\sin\theta}{\cos\theta\cdot\theta}\right)^2$

$= 2\underbrace{\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)^2}\lim_{\theta\to 0}\left(\frac{1}{\cos\theta}\right)^2$

$= 2\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)\lim_{\theta\to 0}\left(\frac{1}{\cos\theta}\right)^2$

$= 2(1)(1)\cdot\left(\frac{1}{1}\right)^2 = 2$

c) $\lim_{x\to 0}\frac{1-\cos 2x}{x}$

note: $\theta = 2x$

$\lim_{x\to 0} 2\frac{(1-\cos 2x)}{2x}$

$2\underbrace{\lim_{x\to 0}\frac{1-\cos 2x}{2x}}$

$2\cdot 0$

0

4. Find the derivative of each of the following functions.

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a) $f(x) = \sin(3x)$

$f'(x) = \cos 3x\,(3)$

$= 3\cos 3x$ 1pt

b) $f(x) = \sqrt{\cos x} = (\cos x)^{1/2}$ chain rule

$f'(x) = \frac{1}{2}(\cos x)^{-1/2}\cdot(-\sin x)(1)$ 1pt

$= \frac{-\sin x}{2\sqrt{\cos x}}$ 1pt

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c) $f(x) = \sqrt{x+1}\,\sin(5x^2)$ product rule + chain rule

$= (x+1)^{1/2}\sin 5x^2$

$f'(x) = (x+1)^{1/2}\cos 5x^2(10x) + \frac{1}{2}(x+1)^{-1/2}(1)\sin 5x^2$

$= 10x(x+1)^{1/2}\cos 5x^2 + \frac{1}{2}(x+1)^{-1/2}\sin 5x^2$

Factored form: $= \frac{1}{2}(x+1)^{-1/2}\left[20(x+1)\cos 5x^2 + \sin 5x^2\right]$

$\frac{20(x+1)\cos 5x^2 + \sin 5x^2}{2\sqrt{x+1}}$

d) $f(x) = \cos^2 5x^3 = (\cos 5x^3)^2$ chain rule

$f'(x) = 2(\cos 5x^3)(-\sin 5x^3)(15x^2)$ 1pt

$= -30x^2\cos 5x^3\sin 5x^3$ 1pt