### 3.1 QUADRATIC FUNCTIONS IN VERTEX FORM

To determine the coordinates of the vertex, the domain and range, the axis of symmetry, the $x$ and $y$ intercepts and the direction of opening of the graph of $f(x)=a(x-p)^{\mathbf{2}}+q$ without the use of technology.

EX \#1: Using a table of values, sketch $y=x^{2}$


## REVIEW:

- A QUADRATIC FUNCTION is a function of degree two: $y=x^{2}, y=2 x^{2}-5 x+1, y=2(x-3)^{2}-3, y=(x+1)^{2}$
- The graph of a quadratic function is in the shape of a $\qquad$
- The $\qquad$ of a parabola is the lowest point in a parabola that opens upwards or the highest point of a parabola that opens downwards.
- If the parabola opens upwards then there is a $\qquad$ value. If the parabola opens down there is a
$\qquad$ value. The $\qquad$ coordinate of the vertex defines the max or min value.
- The $\qquad$ is a line through the vertex that divides the graph of the quadratic function into two congruent halves. The $\qquad$ of the vertex defines the equation of the axis of symmetry.
- A quadratic function is written in STANDARD FORM when it is written in the form $\qquad$ and it is written in VERTEX FORM when it is in the form $\qquad$

EX \#2: State the vertex, the max or min value and the equation of the axis of symmetry for the following:


## REVIEW ACTIVITY:

1. Given the most basic parabola $y=x^{2}$, rewrite in vertex form, $y=a(x-p)^{2}+q$ and identify the values of $a, p$ and $q$
2. Graph this parabola using your graphing calculator. Does it matter if you enter it in standard form or vertex form?
3. Leaving the first parabola on your screen, graph the following two parabola's in the next two graphing spots.

| EQUATION | $Y_{2}=x^{2}+2$ | $Y_{3}=x^{2}-1$ |
| :--- | :--- | :--- |
| Which variable changed from step 1? <br> What is the new value of this variable? |  |  |
| How did the graph change from step 1 |  |  |

4. Delete the equations you have in $y_{2}$ and $y_{3}$ and use the following equations instead

| EQUATION | $Y_{2}=(x+2)^{2}$ | $Y_{3}=(x-1)^{2}$ |
| :--- | :--- | :--- |
| Which variable changed from step 1? <br> What is the new value of this variable? |  |  |
| How did the graph change from step 1 |  |  |

5. Delete the equations you have in $y_{2}$ and $y_{3}$ and use the following equations instead

| EQUATION | $\mathrm{Y}_{2}=2 \mathrm{x}^{2}$ | $\mathrm{Y}_{3}=5 \mathrm{x}^{2}$ | $y_{4}=\frac{1}{2} x^{2}$ | $y_{5}=\frac{1}{5} x^{2}$ | $y_{5}=-\frac{1}{5} x^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Which variable changed from step 1? What <br> is the new value of this variable? |  |  |  |  |  |
| How did the graph change from step 1 |  |  |  |  |  |

REVIEW: In VERTEX FORM $y=a(x-p)^{2}+q$, the following is true:

- The base or parent function is $y=x^{2}$ and is transformed/changed when $\mathrm{a}, \mathrm{p}$ and q are applied
- The coordinates of the vertex are at ( $\qquad$ , _)
- The equation of the axis of symmetry is at $\qquad$
- If the value of " $a$ " is negative, the graph will open $\qquad$ and have a $\qquad$ value at $\qquad$
- If the value of " $a$ " is positive, the graph will open $\qquad$ and have a $\qquad$ value at $\qquad$
- The parabola will be of average width if $\qquad$
- The parabola will be narrower if $\qquad$
- The parabola will be wider if $\qquad$
- The value of " $p$ " moves the parabola $\qquad$
- The value of " $q$ " moves the parabola $\qquad$
- The domain of a parabola (with arrowheads) will be $\qquad$ in set notation and $\qquad$ in interval notation
- The range of a parabola where $a>0$ (with arrowheads) will be $\qquad$ in set notation and $\qquad$ in interval notation while the range of a parabola where a $<0$ (with arrowheads) will be
$\qquad$ in set notation and $\qquad$ in interval notation.

EX \#3: Complete the following chart and sketch the last three functions using the transformations of $a, p$ and $q$

|  | $y=3 x^{2}$ | $y=2 x^{2}+5$ | $y=(x-5)^{2}$ | $y=-3(x+3)^{2}-2$ | $y=\frac{1}{2}(x-3)^{2}-2$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Value of a |  |  |  |  |  |
| Value of p |  |  |  |  |  |
| Value of q |  |  |  |  |  |
| Direction of <br> opening |  |  |  |  |  |
| Width |  |  |  |  |  |
| Vertex |  |  |  |  |  |
| Max/Min |  |  |  |  |  |
| Axis of <br> Symmetry |  |  |  |  |  |
| Domain |  |  |  |  |  |
| Range |  |  |  |  |  |
| Y intercept |  |  |  |  |  |
| Reflection of the <br> y intercept |  |  |  |  |  |





EX \#4: Determine the number of $x$ intercepts of each quadratic function by visualizing the graph.

|  | VERTEX | DIRECTION OF <br> OPENING | VISUALIZE \& SKETCH <br> THE GRAPH | NUMBER OF X <br> INTERCEPTS |
| :--- | :--- | :--- | :--- | :--- |
| $f(x)=\frac{1}{2}(x-2)^{2}-4$ |  |  |  |  |
| $f(x)=3(x+5)^{2}$ |  |  |  |  |
| $f(x)=-(x+3)^{2}-5$ |  |  |  |  |

EX \#5: Sketch the following graphs. Determine the vertex, the direction of opening, the $y$ intercept \& its reflection, the $x$ intercepts, the domain and range, the axis of symmetry and the max or min value. Create a table of values to help determine the shape of the graph.
a) $y=2(x+1)^{2}-8$


| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

b) $y=-(x-3)^{2}+8$


| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

### 3.1 FA\#1: P157 \#1ad, 2bc, 4ac (make a table), 6, 7, EXTRA QUESTION 1 BELOW 3.1 ULA\#1:P161 \#19

1. For each of the functions on the right, state the:

- values of $a, p, q$
- vertex
- $x$-intercepts
- $y$-intercept
- reflection point of the $y$-intercept
- domain \& range
- max/min value
$\begin{array}{ll}\text { a) } y=\frac{1}{3}(x+2)^{2}-3 & \text { d) } y=0.5(x-3)^{2}+1 \\ \text { b) } y=-3(x+1)^{2}-3 & \text { e) } y=(x+2)^{2}+1 \\ \text { c) } y=-\frac{1}{2}(x-3)^{2}+2 & \text { f) } y=-(x-3)^{2}+1\end{array}$
- sketch using a table of values


### 3.1 DAY 2 QUADRATIC FUNCTIONS IN VERTEX FORM

To graph quadratic functions in the form $f(x)=a(x-p)^{\mathbf{2}}+q$ using transformations.

EX \#1: Sketch the graph of $y=3(x+2)^{2}-4$ using transformations.

- STEP 1: Describe what the numerical change to "a" is compared to its parent PARENT FUNCTION $y=x^{2}$. $\qquad$
- How will this change alter the graph of the PARENT FUNCTION $y=x^{2}$ ? $\qquad$
- Will this change affect the $x$ or the $y$ value of the ordered pairs of the parent function? $\qquad$

Let's compare the table of ordered pairs between the parent function and the Step 1 Transformed table (which is the parent function with just the value of " $a$ " changed - we won't worry about the values of " $p$ " and " $q$ " yet)


- STEP 2: Describe how " $p$ " and $q$ " have changed compared to the parent function $y=x^{2}$ $\qquad$
- How will this change alter the graph of the PARENT FUNCTION $y=x^{2}$ ? $\qquad$
- How will $p$ and $q$ affect where the parent function moves to? Describe how $x$ moves and how $y$ moves. $\qquad$
- Fill in the Step 2 table by moving the $x$ and $y$ values by the appropriate amounts

STEP 2:

If the values of p and/or q changes, decide where and how they adjust the table

| STEP 2 of TRANSFORMED FUNCTION <br> $\mathbf{y}=\mathbf{=}(\mathbf{x}$ <br> Describe what changes \& how: <br> $\mathbf{2}$ |  |  |
| :--- | :--- | :--- |
| $\mathbf{x}$ | $\mathbf{y}$ | Final Ordered <br> Pair |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



- Sketch the graph using the Step 2 table above.

EX \#2: Sketch the graph of $f(x)=-\frac{1}{2}(x-1)^{2}+3$ using transformations.


STEP 1:
If the value of "a" changes, decide where how and where that adjusts the table

## STEP 2:

If the values of p and/or q changes, decide where and how they adjust the table

|  | STEP 2 of TRANSFORMED FUNCTION$y=\quad(x \quad)^{2}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | Describe what changes \& how: |  |  |
| changes, decide where | $\mathbf{x}$ | y | Final Ordered Pair |
| table. |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

### 3.1 ASSIGNMENIT: :2(No Graphing Calculator)

### 3.1 FA\#2: P157 \#3 \& Extra Questions 1 \& 2 Below <br> 3.1 ULA\#2:P158 \#10, 24

1. Complete the following chart

|  | $\boldsymbol{a}=$ | $\boldsymbol{p}=$ | $\boldsymbol{q}=$ | direction | width | vertex | max/min | axis of <br> symmetry | domain | range |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y=-4 x^{2}$ |  |  |  |  |  |  |  |  |  |  |
| $y=\frac{1}{5} x^{2}+7$ |  |  |  |  |  |  |  |  |  |  |
| $f(x)=\frac{4}{3}(x-2)^{2}-7$ |  |  |  |  |  |  |  |  |  |  |
| $f(x)=2(x-7)^{2}-10$ |  |  |  |  |  |  |  |  |  |  |
| $f(x)=-\frac{3}{4}(x+1)^{2}+5.7$ |  |  |  |  |  |  |  |  |  |  |
| $y=-12\left(x-\frac{2}{3}\right)^{2}+6$ |  |  |  |  |  |  |  |  |  |  |
| $y=\frac{6}{5} x^{2}-3$ |  |  |  |  |  |  |  |  |  |  |

2. Graph the following functions using transformations. Make sure to state transformations, the vertex and show the new tables of values. It is imperative that you use graph paper and a ruler!!
a) $y=2 x^{2}$
b) $y=-3(x+4)^{2}+2$
c) $y=\frac{1}{2}(x-1)^{2}-3$
d) $f(x)=-4(x+2)^{2}$
e) $f(x)=-\frac{1}{3}(x+1)^{2}+1$
f) $f(x)=\frac{4}{3}(x-2)^{2}-7$

### 3.1 DAY 3: QUADRATIC FUNCTIONS IN VERTEX FORM

To write quadratic functions in vertex form given a graph or situation and to solve situational questions.

EX \#1: Determine the quadratic function in vertex form for the following graphs.
a)

b)


EX \#2: Suppose a parabolic archway has a width of 280 cm and a height of 216 cm at its highest point above the floor.
a) Write a quadratic function in vertex form that models the shape of this archway.
b) Determine the height of the archway at a point that is 50 cm from its outer edge.
c) Is there more than one answer for part " $a$ ". Why?

EX \#3: Determine a quadratic function with the following characteristics: a minimum of 12 at $x=-4$ and $y$-intercept of 60.

### 3.1 ASSIGNMENI \#B(No Graphing Calculator)

### 3.1 FA\#3: P158 \#8abc, 9abd, 21 \& at least 2 of the following:13, 16, 17, 18 <br> 3.1 ULA\#3:P157 \#5, 20, 22

## 3.2 : QUADRATIC FUNCTIONS IN STANDARD FORM

To determine the coordinates of the vertex, the domain and range, the axis of symmetry, the $x$ and $y$ intercepts and the direction of opening of the graph of a function in standard form $\mathbf{y}=\mathbf{a x} \mathbf{a}^{\mathbf{2}}+\mathbf{b x}+\mathbf{c}$

A quadratic in standard form is $y=a x^{2}+b x+c$ or $f(x)=a x^{2}+b x+c \quad$ where $a, b$ and $c$ are real numbers and $a \neq 0$

- $\quad a$ determines the width of the parabola and whether the parabola opens upwards or downwards (the same as it did for vertex form)
- $\quad b$ INFLUENES the position of the graph (vertex)
- $\quad c$ determines the $y$ intercept of the graph
- In Foundations 20, we found the $x$ value of the vertex by calculating the $x$ intercepts and finding the middle $x$ value of those intercepts. We will now develop a formula that we can use
- $\quad \therefore$ the $x$-coordinate of the vertex can be calculated by using the formula

- The $y$ coordinate of the vertex can be found by substituting the above answer into the original function
- The formula to find the vertex is: $(-$,

EX \#1: Using the graph, determine the vertex, direction of opening, axis of symmetry, $\mathrm{max} / \mathrm{min}$ value, domain, range, $x$ intercept(s) andy intercept

Find the vertex using the formula


EX \#2: Without looking at a graph, determine the same information as in example 1 for the following:
a) $y=x^{2}+6 x+5$
b) $y=-x^{2}+2 x+3$

## REVIEW:

To find the $y$ intercept in standard form, use the value of $\qquad$ . To find the x intercepts in standard form you must set the value of $\qquad$ to $\qquad$ , then solve the resulting equation by $\qquad$ or $\qquad$

## EX \#3:

A diver jumps from a $3-\mathrm{m}$ springboard with an initial vertical velocity of $6.8 \mathrm{~m} / \mathrm{s}$. Her height, $h$, in metres, above the water $t$ seconds after leaving the diving board can be modelled by the function $h(t)=-4.9 t^{2}+6.8 t+3$.
a) Graph the function by finding the vertex, $x$ intercept(s), $y$ intercept and it's reflection.

b) What does the $y$-intercept represent?
c) What maximum height does the diver reach? When does she reach that height?
d) How long does it take before the diver hits the water?
e) What domain and range are appropriate in this situation?
f) What is the height of the diver 0.6 s after leaving the board?

EX \#4: At a children's music festival, the organizers are roping off a rectangular area for stroller parking. There is 160 m of rope available to create the perimeter.
a) Write a quadratic function in standard form to represent the area for the stroller parking.
b) What are the coordinates of the vertex? What does the vertex represent in this situation?
c) Sketch the graph for the function you determined in part a).
d) Determine the domain and range for this situation.
e) Identify any assumptions you made.


> 3.2 ASSIGNMENH \#3(No Graphing Calculator)

## 3.3: COMPLETING THE SQUARE

To change the form of a quadratic function from Standard Form, $y=a x^{2}+b x+c$, to Vertex Graphing Form, $y=a(x-p)^{2}+q\left(\right.$ note that it is sometimes called $y=a(x-h)^{2}+k$

EX \#1: Rewrite the following quadratic function from vertex form to standard form

$$
y=-2(x+2)^{2}-5
$$

What form do you feel is more helpful in be able to quickly sketch the graph?
Why?

EX \#2 : Review! Factor the following perfect trinomial squares:
a) $x^{2}+6 x+9$
b) $x^{2}+12 x+36$
c) 3 . $x^{2}-4 x+4$
d) $x^{2}+14 x+49$

What is the relationship between the middle and the last term?

e) $2 x^{2}-16 x+32$
f) $5 x^{2}+10 x+5$
g) $-x^{2}-8 x-16=$
h) $-3 x^{2}+30 x-75$

EX \#3: How could you use what you learned about the relationship between the middle and last term to test to see if $x^{2}+6 x-7$ is a perfect trinomial square?

EX \#3: Rewrite the function $y=x^{2}+6 x-y$ to VERTEX form(From the form $y=a x^{2}+b x+c$ to the form $\left.y=a(x-p)^{2}+q\right)$ What is the vertex of this function?

## STEPS FOR MS. CARIGNAN'S METHOD OF WRITING IN VERTEX FORM WHEN a = 1

1. Move the value of " $c$ " to the left side of the function.
2. When the value of $\mathrm{a}=1$, calculate the value of $\left(\frac{b}{2}\right)^{2}$ and add this value to both sides of the function.
3. Simplify the left side of the function and factor the right side of the function. Note that the function will now always factor into a perfect square of the form
$\left(x \pm \frac{\square}{\square}\right)^{2}$
4. Move the constant on the left side to the right side.

EX \#4: Rewrite the following in vertex form.
a) $y=x^{2}+8 x-5$
b) $y=x^{2}+9 x-1$

Do you think this parabola would have a maximum or a minimum?

What is the max/min of this parabola?

Do you think this parabola would have a maximum or a minimum?

What is the max/min of this parabola?

EX \#4: Rewrite the following in Vertex form:
a) $y=2 x^{2}-16 x+11$
b) $y=-3 x^{2}-27 x+13$

## STEPS FOR MS. CARIGNAN'S METHOD OF WRITING IN VERTEX FORM WHEN a $=1$

1. Move the value of " $c$ " to the left side of the function.
2. Factor out the value of "a" on the right side (even if it doesn't factor out evenly you must factor it out! To factor it out of "b" when it isn't divisible by "a" you will turn the term into $\frac{b}{a} x$ ) Leave a short space inside the brackets on the right end.
3. Calculate the value of $\left(\frac{b}{2}\right)^{2}$ and add this value to THE RIGHT SIDE side of the function only (add this value where you left the space inside the brackets)
4. Now calculate the value of $a \bullet\left(\frac{b}{2}\right)^{2}$ and add this value to the LEFT side the function.
5. Simplify the left side of the function and factor the right side of the function. Note that the function will now always factor into a perfect square of the form
$a\left(x \pm \frac{\square}{\square}\right)^{2}$
6. Move the constant on the left side to the right side.
d) $y=-5 x^{2}-8 x$

EX \#5: Complete the following table:

|  | $y=2 x^{2}-20 x$ | $y=-3 x^{2}-18 \mathrm{x}-24$ | $y=\frac{1}{3} x^{2}+2 x-9$ | $\mathrm{y}=\mathrm{x}^{2}+5 \mathrm{x}-3$ |
| :--- | :--- | :--- | :--- | :--- |
| Vertex |  |  |  |  |
| Axis of Symmetry |  |  |  |  |
| Direction of <br> Opening |  |  |  |  |
| Max/Min |  |  |  |  |
| Domain |  |  |  |  |
| Range |  |  |  |  |
| Y intercept |  |  |  |  |

### 3.3 ASSIGNMENH: \#1 (No Graphing Calculatior)

## 3.3 \#1 FA: The following Questions (do them first) followed by P193 \#5d, 7e

1. Rewrite the following functions in Vertex Form by Completing the Square.
a) $y=x^{2}-4 x-5$
b) $y=x^{2}+6 x-16$
c) $y=x^{2}-8 x+18$
d) $y=x^{2}+10 x$
e) $y=x^{2}+9 x$
f) $y=x^{2}+3 x-10$
g) $y=-x^{2}+4 x+12$
h) $y=3 x^{2}+12 x-15$
i) $y=2 x^{2}+5 x-3$
j) $y=\frac{1}{3} x^{2}+2 x-4$
k) $y=\frac{1}{2} x^{2}+x-8$
2. Find each of the following answers for each question in \#1 above:
a) Vertex
b) Axis of Symmetry
c) Direction of Opening
d) $\operatorname{Max} / \mathrm{Min}$
e) Yintercept
d) Domain
e) Range

## 3.3 \#2 ULA: P193 8bc, 9, 10, 12, 16

## Answers to \#1:

1a) $y=(x-2)^{2}-9$
b) $y=(x+3)^{2}-25$
c) $y=(x-4)^{2}+2$
d) $y=(x+5)^{2}-25$
e) $y=\left(x+\frac{9}{2}\right)^{2}-\frac{81}{4}$
f) $y=\left(x+\frac{3}{2}\right)^{2}-\frac{49}{4}$
g) $y=-(x-2)^{2}+16$
h) $y=3(x+2)^{2}-27$
i) $y=2\left(x+\frac{5}{4}\right)^{2}-\frac{49}{8}$
j) $y=\frac{1}{3}(x+3)^{2}-7$
k) $y=\frac{1}{2}(x+1)^{2}-\frac{17}{2}$

## 3.3: MAX/MIN WORD PROBLEMS

## To solve situational questions involving maximums and minimums of quadratic functions.

EX \#1: At a children's music festival, the organizers are roping off a rectangular area for stroller parking. There is 160 m of rope available to create the perimeter.
a) Write a quadratic function in standard form to represent the area for the stroller parking.
b) What is the maximum area that can be roped off? What are the dimensions of the rectangular with the maximum area? What two methods can you use to find the maximum? Use both methods!

EX \#2: Two numbers have a sum of 29 and a product that is a maximum. Determine the two numbers and the maximum product.

## 

## 3.3 \#2 FA: The following Questions (do first) followed by P 195 \#18

1. Solve the following problems algebraically using the vertex formula and completing the square. Then, check your answer on the graphing calculator.
a) What is the maximum product that two numbers can have if their sum is 100 ? What are the two numbers?
b) One number is 10 larger than another. What is the smallest possible value for the sum of their squares? What are the two numbers?
2. What is the maximum rectangular area that can be enclosed with 100 m of fencing material? What are the dimensions of the rectangle?
3. A farmer wishes to fence in a rectangular pen using his barn as one of the sides of the rectangle. If the farmer has 40 $m$ of fencing, what is the largest area that can be enclosed? What are the dimensions of the rectangle?
4. The managers of a business are examining costs. It is more cost-effective for them to produce more items. However, if too many items are produced, their costs will rise because of factors such as storage and overstock. Suppose that they model the cost, $C$, of producing $n$ thousand items with the function: $C(n)=75 n^{2}-1800 n+60000$. Determine the number of items that will minimize their costs.
5. A gymnast is jumping on a trampoline. His height, $h$, in metres, above the floor on each jump is roughly approximated by the function $h(t)=-5 t^{2}+10 t+4$, where $t$ represents the time, in seconds, since he left the trampoline. Determine his maximum height on each jump.
6. Sandra is practicing at an archery club. The height, $h$, in feet, of the arrow on one of her shots can be modelled as a function of time, $t$ in seconds, since it was fired using the function $h(t)=-16 t^{2}+10 t+4$. What is the maximum height of the arrow, in feet, and when does it reach that height?

## Answers:

1. a) Max product is 2500 when the numbers are 50 and 50 b) Min sum is 50 when the numbers are -5 and 5
2. Max area is $625 \mathrm{~m}^{2}$ when each side of the rectangle is 25 m .
3. Max area is $200 \mathrm{~m}^{2}$ when the rectangle is 10 m by 20 m .
4. 12000 items
5. 9 m
6. Max height is 5.56 ft after 0.31 seconds being shot

### 3.3 ULA: P195 \#19, 20, 22, 24, 25, 28, 31

## VIDEO LINKS THAT MAY AIDE IN UNDERSTANDING

## Section 3.1

Review of SET Notation Vs Interval Notation: https://goo.gl/XAvxjV
(Note: On the Set Notation part of this video, we typically also add $x \in R$ to the end just before the curly bracket - ie we would prefer the first answer to look like $\{x \mid x<3, x \in R\}$ )

- https://goo.gl/WZQhjf
- https://goo.gl/dnT31E
- https://goo.gl/okcN4a
- https://goo.gl/wPVSGA
- https://goo.gl/Yvdyga


## Section 3.2

Review of Window/Box Method for Factoring: https://goo.gl/B53bR4

- https://goo.gl/o7gN28
- https://goo.gl/xbqZE6
- https://goo.gl/Urv7nG
- https://goo.gl/re5H2G


## Section 3.3

- https://goo.gl/5vrNxg
- https://goo.gl/ctteSJ

