7.2 Antiderivatives with Trigonometric Substitutions

I CAN FIND INTEGRALS BY USING TRIGONOMETRIC SUBSTITUTION

VIDEO LINKS:

a) http://bit.ly/2wnw3Az

Recall the following trigonometric derivatives:

If $f(x) = \sin u$ then $\frac{dy}{dx} = \cos u \frac{du}{dx}$ If $f(x) = \cos u$ then $\frac{dy}{dx} = -\sin u \frac{du}{dx}$

If $f(x) = \tan u$ then $\frac{dy}{dx} = \sec^2 u \frac{du}{dx}$ If $f(x) = \cot u$ then $\frac{dy}{dx} = -\csc^2 u \frac{du}{dx}$

If $f(x) = \sec u$ then $\frac{dy}{dx} = \sec u \tan u \frac{du}{dx}$ If $f(x) = \csc u$ then $\frac{dy}{dx} = -\csc u \cot u \frac{du}{dx}$

Recall the following trigonometric Identities:

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Addition and Subtraction Identities

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$

Supplement Angle Identities

$\sin(\pi-\theta) = \sin\theta$ $\csc(\pi-\theta) = \csc\theta$

$\cos(\pi-\theta) = -\cos\theta$ $\sec(\pi-\theta) = -\sec\theta$

$\tan(\pi-\theta) = -\tan\theta$ $\cot(\pi-\theta) = -\cot\theta$

$\sin(\pi+\theta) = -\sin\theta$ $\csc(\pi+\theta) = -\csc\theta$

$\cos(\pi+\theta) = -\cos\theta$ $\sec(\pi+\theta) = -\sec\theta$

$\tan(\pi+\theta) = \tan\theta$ $\cot(\pi+\theta) = \cot\theta$

Note: We did not learn the following identities but it is possible you may see them in University

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

EX #1: Evaluate $\int \sin^3 x\,dx$

EX #2: Evaluate $\int \cot 7x\,dx$

EX #3: Find the indefinite integrals. In each case you can use a trigonometric identity to set up a substitution.

a) $\displaystyle\int \frac{dx}{\cos^2 2x}$

b) $\int \cot^2 3x dx$

c) $\int \cos^3 x dx$

EX #4: Evaluate $\int_0^{\frac{\pi}{3}} \tan x \sec^2 x dx$

7.2 Assignment: P342 #47-51 Odd, 55, 64

I CAN FIND INTEGRALS BY PARTS

VIDEO LINKS:

a) http://bit.ly/2YPVGpU

Integration by Parts

In this section you will study an important integration technique called **integration by parts.** This technique can be applied to a wide variety of functions and is particularly useful for integrands involving *products* of algebraic and transcendental functions. For instance, integration by parts works well with integrals such as

$$\int x \ln x \, dx, \quad \int x^2 e^x \, dx, \quad \text{and} \quad \int e^x \sin x \, dx.$$

Integration by parts is based on the formula for the derivative of a product

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= uv' + vu'$$

where both u and v are differentiable functions of x. If u' and v' are continuous, you can integrate both sides of this equation to obtain

THEOREM 8.1 INTEGRATION BY PARTS

If u and v are functions of x and have continuous derivatives, then

$$\int u \, dv = uv - \int v \, du.$$

This formula expresses the original integral in terms of another integral. Depending on the choices of u and dv, it may be easier to evaluate the second integral than the original one. Because the choices of u and dv are critical in the integration by parts process, the following guidelines are provided.

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Note that dv always includes the dx of the original integrand.

Choosing our function u

- A mnemonic device which is helpful for selecting u when using integration by parts is the LIATE principle of precedence for u:

  Logarithmic
  Inverse Trigonometric
  Algebraic
  Trigonometric
  Exponential

- If the integrand has several factors, we try to choose let u be the highest function on the LIATE list.

EX #1: Find $\int xe^x\,dx$.

EX #2: Find $\int x^2 \ln x\,dx$.

EX #3: Evaluate $\int_0^1 \arcsin x \, dx$

EX #5: Find $\int \sec^3 x \, dx.$

(there's an extra trick here! Plus you need to know that $\int \sec x dx = \ln|\sec x + \tan x|$ **)**

Some integrals require repeated use of the integration by parts formula.

EX #4: Find $\int x^2 \sin x \, dx$.

EX #5: Use the tabular method to find the following integral (the tic tac toe method from Stand and Deliver!)

$\int x^3 e^x dx$

- To use this method, choose *u* as the variable who, if repeated derivatives were applied, would end up with a derivative of zero.
  U = __________________ dv = _________________
- Fill in the following table starting with the middle column. At the top of the column, write *u*. Under it, write the derivative. Under that, write the second derivative. Continue until you get to a derivative of zero.
- Next, fill in the first column. Place a plus sign at the top, followed by a minus sign, followed by a plus sign etc
- In the last column, write *dv* at the top. Under it, write its integral. Under that, write the next integral. Continue until you are at the row with the derivative of zero.
- Connect row one of columns one and two to row two of column three
- Connect row two of columns one and two to row three of column three
- Continue until everything is connected except the top right spot and bottom left spot
- The answer for the integral will be connection 1 + connection 2 + connection 3 +…+ connection "n" + C

Alternate Signs	*u and Its Derivatives*	*v′ and Its Antiderivatives*

EX #6:

Find $\int x^2 \sin 4x \, dx$.

Solution Begin as usual by letting $u = x^2$ and $dv = v'\,dx = \sin 4x \, dx$. Next, create a table consisting of three columns, as shown.

Alternate Signs	*u and Its Derivatives*	*v′ and Its Antiderivatives*

SUMMARY OF COMMON INTEGRALS USING INTEGRATION BY PARTS

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \qquad \int x^n \sin ax\,dx, \qquad \text{or} \qquad \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}\,dx$, $\sin ax\,dx$, or $\cos ax\,dx$.

2. For integrals of the form

$$\int x^n \ln x\,dx, \qquad \int x^n \arcsin ax\,dx, \qquad \text{or} \qquad \int x^n \arctan ax\,dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\,dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \qquad \text{or} \qquad \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\,dx$.

7.3 Assignment: P350 #1-9 Odd, 17, 21, 23, 25, 27

7.5 Integration with Partial Fractions

I CAN FIND INTEGRALS BY USING THE METHOD OF PARTIAL FRACTIONS

VIDEO LINKS:

a) Part 1: http://bit.ly/2MmfTCa

In pre-calculus you learned how to combine functions such as

$$\frac{1}{x-2} + \frac{-1}{x+3} = \frac{5}{(x-2)(x+3)}.$$

The method of partial fractions requires you to reverse this process

$$\frac{5}{(x-2)(x+3)} = \frac{?}{x-2} + \frac{?}{x+3}$$

Partial Fraction Decomposition with Distinct Linear Denominators

If $f(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials with the degree of P less than the degree of Q, and if $Q(x)$ can be written as a product of distinct linear factors, then $f(x)$ can be written as a sum of rational functions with distinct linear denominators.

EX #1: Write the partial fraction decomposition for $\frac{1}{x^2 - 5x + 6}$.

EX #2: Write the function $f(x) = \dfrac{x - 13}{2x^2 - 7x + 3}$ as a sum of rational functions with linear denominators.

EX #3: Integrate $\displaystyle\int \frac{x-13}{2x^2-7x+3}\,dx$ using partial fractions

EX #4: Find $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx.$ using partial fractions

EX #5: Find $\int \frac{3x^4 + 1}{x^2 - 1}\, dx.$

EX #6: Find the general solution to $\dfrac{dy}{dx} = \dfrac{6x^2 - 8x - 4}{(x^2 - 4)(x - 1)}$

7.5 Assignment: P373 #1-13 Odd, 19,21