PC 30

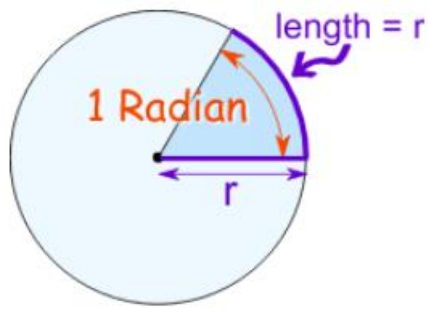
4.1 Day 1: Angles & Angle Measure

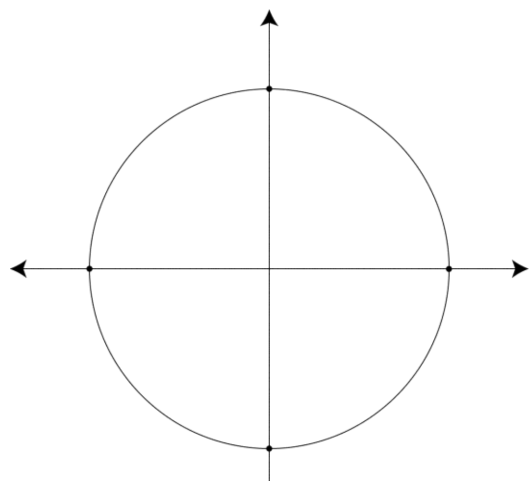
To convert degrees to radians and radians to degrees and determine the measures of anglese that are coterminal with a given angle in standard position.

https://www.youtube.com/watch?v=ifBhTdsTMuE

Angles can be measured in ***degrees*** or ***radians.*** Angle measures without units are considered to be in radians.

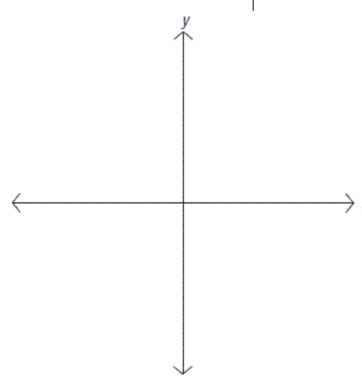
Radian: One radian is the measure of the central angle subtended in a circle by an arc equal in length to the radius of the circle.
(1 radian ≈ ________________ degrees)



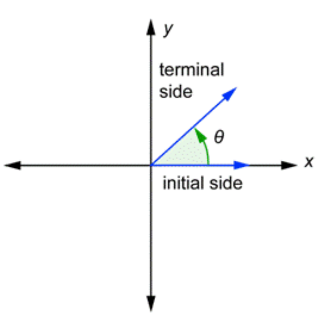


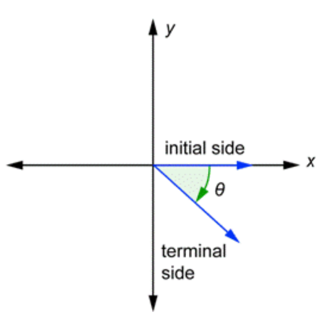
- One full rotation of a circle is ________ degrees or ______ radians
- One half rotation of a circle is ________ degrees or ______ radians
- To convert degrees to radians, since $\frac{2\pi}{360^\circ} = \frac{\pi}{180^\circ} = 1$, multiply by $\frac{\pi}{180}$
- To convert radians to degrees, since $\frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} = 1$, multiply by $\frac{180}{\pi}$

Angles in Standard Position



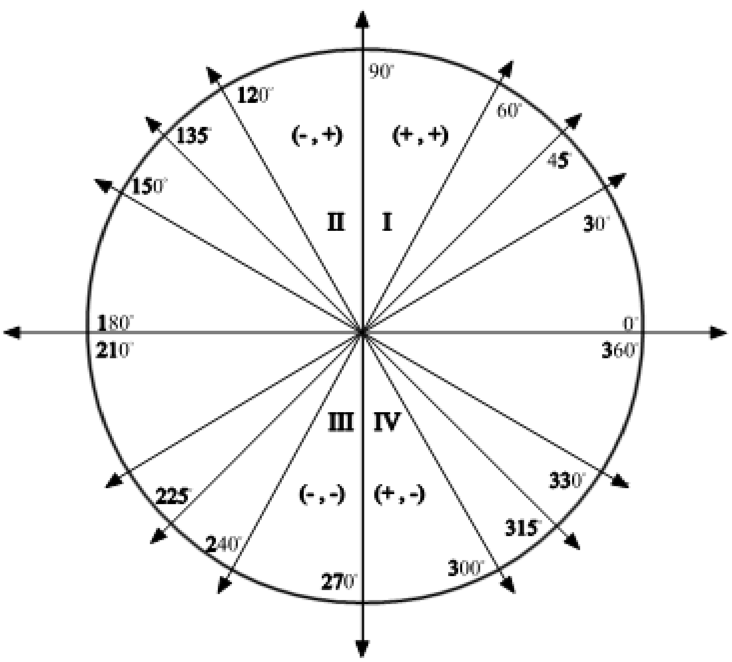
- vertex is at ______________________
- initial arm lies on the _____________ x-axis
- positive angle rotates ______________________
- negative angle rotates ______________________



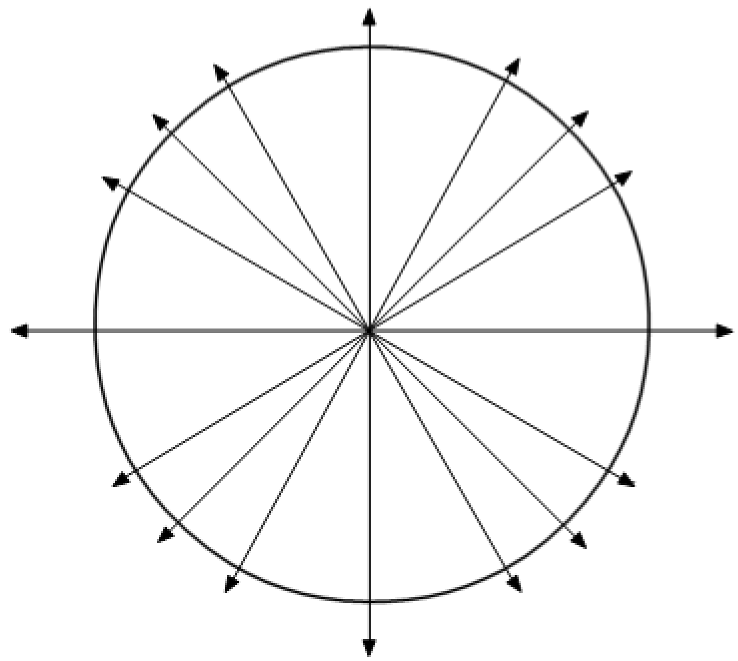


Example #1:

a) Convert each of the following angle Measures to radian measure and label each angle using Radians.



b) Label this diagram in degree measure as if it contained negative angles. What would each of these new measures be in radian measure?



Example #2: Convert each degree measure to radians (in exact form as a reduced fraction) and each radian measure to degrees. Draw each angle in standard position.

a) 140^o = ______ radians

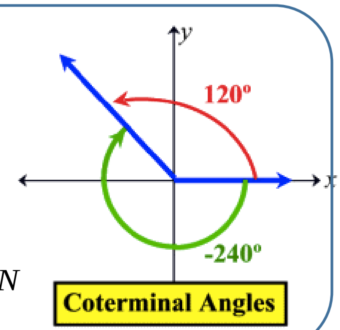
b) -260^o = _______ radians

c) $\frac{7\pi}{6}$ = _________ degrees

d) -1.2 radians = _________ degrees

Coterminal Angles

- are angles in standard position with the same terminal arm.
- may be measured in degrees or radians.



Coterminal Angle Formula

$\theta \pm (360°)n, n \in N$ **or** $\theta \pm (2\pi)n, n \in N$ **or (less exact)** $\theta \pm (6.28)n, n \in N$

Example #3: Determine one positive and one negative angle that is coterminal with each angle. Draw the angles.

a) $40°$ b) $-210°$ c) $\frac{\pi}{3}$

Example #4: Write an expression for all possible angles coterminal with each given angle. Identify the angles that are coterminal that satisfy $-360° \le \theta < 360°$ or $-2\pi \le \theta < 2\pi$

a) $110°$ b) $-500°$ c) $\frac{9\pi}{4}$

4.1 Day 1 ASSIGNMENT

4.1 Day 1FA: P175 #1, 2cdf, 3def, 4, 5acd, 6bde, 7ad, 9ab

4.1 Day 1 ULA: P175 #11abd, EXTENSION: 18, 19, 27

PC 30

4.1 Day 2: Arc Length of a Circle

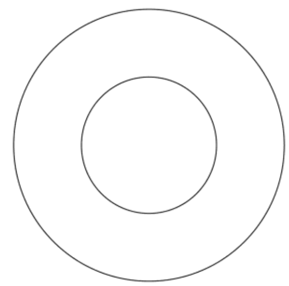
To calculate the measure of the arc length of a given circle as well as its central angle.

DEVELOPMENT:

- All arcs that subtend angle θ have the same central angle, but have different arc lengths depending on the radius of the circle. The ***arc length*** is proportional to the radius.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}} \qquad \frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{sector angle}}{2\pi}$$

- Consider the 2 concentric circles to the right with center at O. The radius of the smaller circle is 1 and the radius of the larger circle is r. Let *x* represent the arc length of the smaller circle and *a* is the arc length of the larger circle.



Using proportions:

$$\frac{a}{x} = \frac{r}{1} \qquad \frac{x}{2\pi r} = \frac{\theta}{2\pi}$$

$$a = xr \qquad x = \frac{\theta\left(2\pi(1)\right)}{2\pi} \quad \text{since radius} = 1$$

$$x = \theta$$

So $a = \theta r$

Arc Length Formula: $a = \theta_R r$

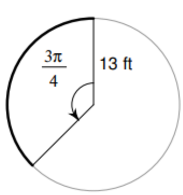
Where: a = the arc length in the same units as the radius

r = the radius in the same units as the arc length

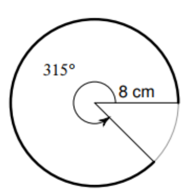
θ_R = the measure of the central angle in RADIANS

Example #1: Determine the arc length:

a)



b)

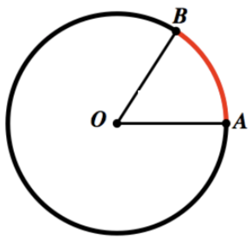


c) In a circle where r = 7cm & $\theta = 1.7$

Example #2: Determine the measure of the following central angles:

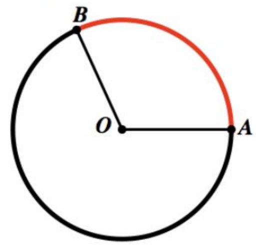
a) $OA = 4.16$ cm

Length $\overset{\frown}{AB} = 4.16$ cm



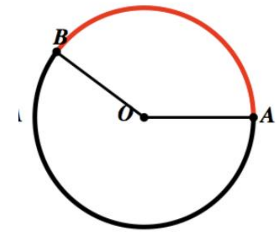
b) $OA = 4.16$ cm

Length $\overset{\frown}{AB} = 8.31$ cm



c) $OA = 4.16$ cm

Length $\overset{\frown}{AB} = 10.39$ cm



Example #3: An electric winch is used to pull a boat out of the water onto a trailer. The winch winds the cable around a circular drum of diameter 5 inches. Approximately how many times will the winch have to rotate in order to roll in 5 feet of cable?

4.1 Day 2 ASSIGNMENT

4.1 Day 1FA: P175 #12, 13, 14,

4.1 Day 1 ULA: P175 #15, 16, 17 & the question below

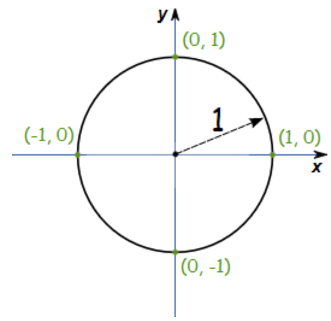
1. Assuming that the Earth is a sphere of radius 4000 miles, find the distance between the following two cities whose latitudes are given. You may assume that these cities are on the same basic longitude.
   Cincinnati, Ohio (39º 8’ 46” N) and Huntsville, Alabama (34º 43’ 41” N)

4.2: The Unit Circle

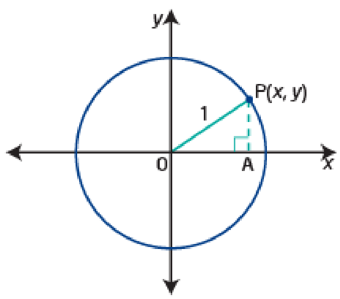
To determine the equation of the unit circle and coordinates of points on the unit circle.

Unit Circle is a circle:

- Whose centre is at the origin
- Has a radius of 1 unit
- Has an equation of



Example #1: Find the equation of the unit circle



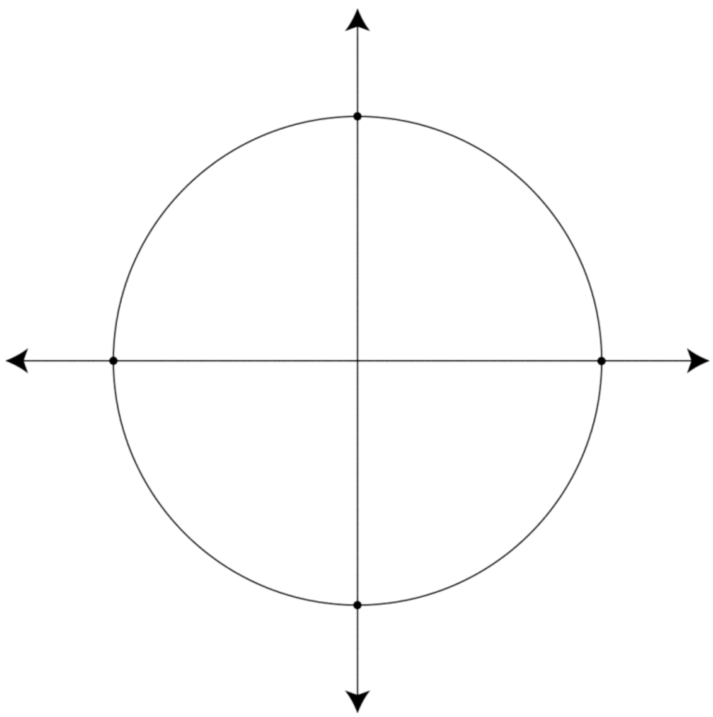
Example #2: Find the equation of a circle whose centre is at the origin and whose radius is 7.

Example #3: Determine the coordinates for all points on the unit circle where the x coordinate is $\frac{3}{10}$

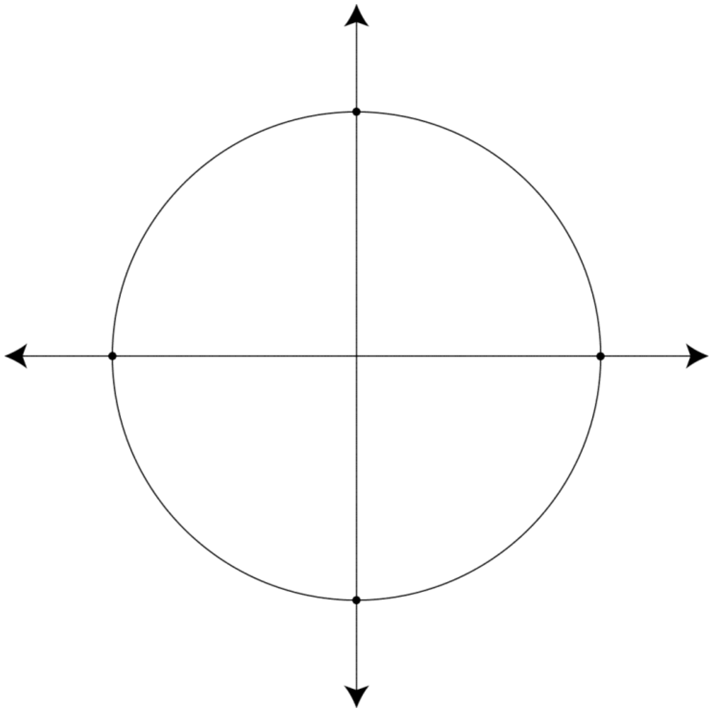
Example #4: Determine the coordinates for all points on the unit circle where the y coordinate is $-\frac{5}{13}$ in QIII

In the Unit circle $a = \theta_R r$ becomes $a = \theta_R$ since r = 1. That means that the central angle AND its subtended arc have the same numerical value

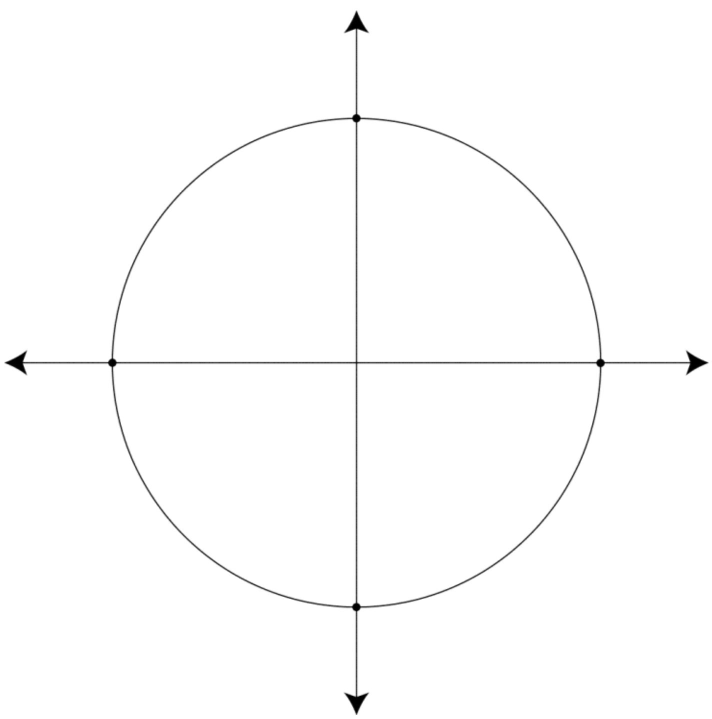
Example #5: On the following unit circle, find all multiples of $\frac{\pi}{4}$ where $0 \le \theta \le 2\pi$. Use special triangles to calculate the exact value of the point of intersection between the terminal arm of each multiple and the unit circle.

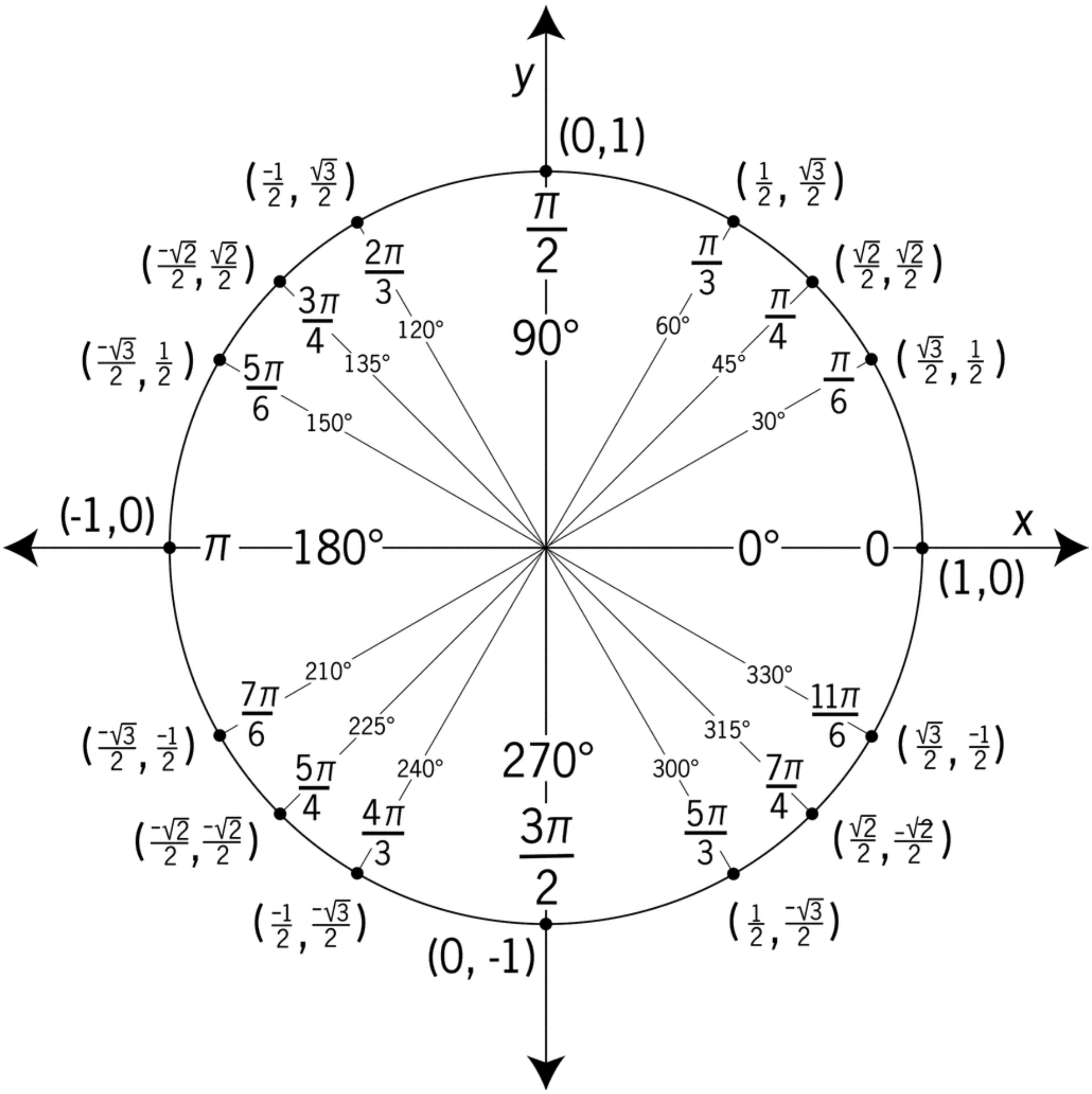


Example #6: On the following unit circle, find all multiples of $\frac{\pi}{3}$ where $0 \le \theta \le 2\pi$. Use special triangles to calculate the exact value of the point of intersection between the terminal arm of each multiple and the unit circle.



Example #7: On the following unit circle, find all multiples of $\frac{\pi}{6}$ where $0 \le \theta \le 2\pi$. Use special triangles to calculate the exact value of the point of intersection between the terminal arm of each multiple and the unit circle.





4.2 ASSIGNMENT

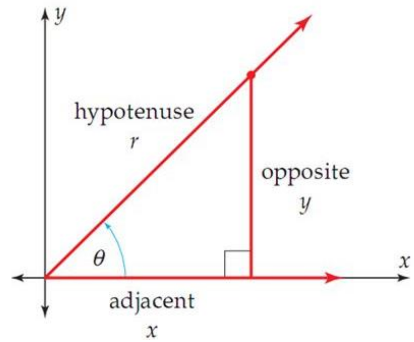
4.2 FA: P186 #1cd, 2, 3, 4, 5, 6,

4.2 ULA: P186 #9, 11, 12, 13, 19, C3

PC 30

4.3 Day 1 : Trigonometric Ratios

To develop the six trigonometric ratios for any angle given a point on the terminal arm of the angle in the unit circle.



- In Pre-Calculus 20 you learned how to graph angles on the coordinate plane and how to calculate the three primary trigonometric ratios of any angle ϴ with the aide of reference angles
- sinϴ = cosϴ = tanϴ =

THE RECIPROCAL TRIGONOMETRIC RATIOS:

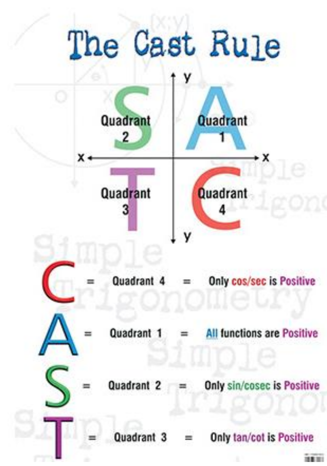
- Each primary trigonometric ratio has a corresponding reciprocal ratio
- The reciprocal of the SINE ratio is the COSECANT RATIO: $\csc\theta = \frac{hyp}{opp}$

  The reciprocal of the COSINE ratio is the SECANT ratio: $\sec\theta = \frac{hyp}{opp}$

  The reciprocal of the TANGENT ratio is the COTANGENT ratio: $\cot\theta = \frac{adj}{opp}$
- On the coordinate plane and within any circle, the following is true:
  - The hypotenuse has a length of r (in the unit circle r = 1)
  - The opposite side has a length of y
  - The adjacent side has a length of x
- Therefore , we can use the following formula's for the six Trigonometric ratios. The CAST rule for Sin, Cos and Tan also applies to their reciprocal ratios

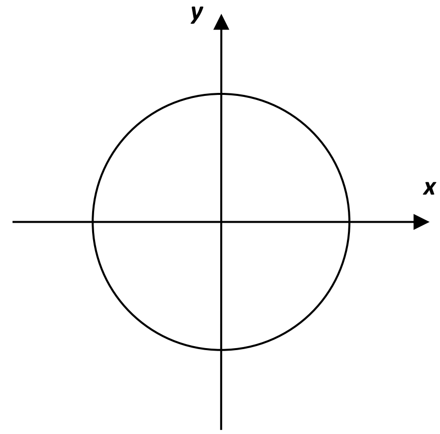
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$



Example #1:

The point $B\left(-\frac{1}{3},\frac{2\sqrt{2}}{3}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle Θ in standard position. Draw a diagram to model the situation and find the values of the six trigonometric ratios for Θ in exact form.



Example #2:

Using your knowledge of the unit circle and/or special triangles, determine the exact value for each of the following:

a) $\cos\frac{5\pi}{6}$

b) $\sec 315°$

c) $\sin\left(-\frac{4\pi}{3}\right)$

d) $\cot 270°$

IN THE UNIT CIRCLE THE FOLLOWING PROPERTY IS TRUE:

- Given that r = 1, any point (x, y) on the circle can also be named (cosΘ, sinΘ)
- The formula $\tan\theta=\frac{y}{x}$ can also be written as $\tan\theta=\frac{\sin\theta}{\cos\theta}$

Example #3: The point (-7, 4) lies on the terminal arm of angle ϴ in standard position. What is the exact value of each trigonometric ratio for ϴ? Does this situation have a unit circle?

Sometimes we cannot use the unit circle to find the exact trig value but we can find an approximate answer.

Example #4: Sketch the following angle. Determine the reference angle. Use the reference angle and the CAST rule along with your calculator to find the approximate value for the following trigonometric ratios. Give answers to four decimal places.

a) $\tan\frac{7\pi}{5}$ (*in radians!)

b) $\cos 260°$ (* in degrees)

Example #5: Use your calculator to find the approximate answer to the following (always to four decimals)

a) sin 4.2

b) $\cos\frac{8\pi}{3}$

c) $\csc(88°)$

d) $\cot(-70°)$

Example #6: Without the use of a calculator, determine if each trig function will be positive or negative

a) $\sin 580°$

b) $\cot 340°$

c) $\sec\frac{2\pi}{3}$

4.3 Day 1 ASSIGNMENT

4.3 Day 1FA: P201 #1, 2, 3, 6, 8,

4.3 Day 1 ULA: P201 #9, 13, 16,

4.3 Day 2 : Finding Angles Given Their Ratios

To find the measure of an angle given its trig ratios.

STEPS:

1. If the trig ratio is from our special triangles and on the unit circle, refer to the unit circle to find out the corresponding angle measure
2. If the trig ratio is not on the unit circle, follow these steps:
   - Find the absolute value of your given ratio (use the positive version of the ratio). Use this value along with the appropriate inverse trig function key ($\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$) to find the size of the REFERENCE ANGLE. Note that the following terms may also be used: $\sin^{-1}$ is the same as arcsin, $\cos^{-1}$ is the same as arccos and $\tan^{-1}$ is the same as arctan.
   - Use the CAST rule and the original sign (- or +) of the original ratio to determine which quadrants the reference triangle may belong to. Sketch all possible situations. In each situation, find the actual value of the angle by determining the size of the angle between the positive x axis and the hypotenuse of the reference triangle.

Example #1:

If $\sin\theta = 0.5$, find θ in the domain $0^o \le \theta < 360^o$.

Example #2:_Determine the measure of all angles that satisfy each of the following. Use diagrams to help you find all possible answers.

a) $\cos\theta = -0.843$ in the domain $0^\circ \le \theta < 360^\circ$. Give your answer to the nearest tenth of a degree.

b) $\sin\theta = 0$ in the domain $0^\circ \le \theta \le 180^\circ$. Give exact answers.

c) $\tan\theta = \sqrt{3}$, $-180° \le \theta \le 180°$

c) cot θ = -2.777 in the domain $-\pi \le \theta < \pi$. Give approximate answer to the nearest tenth.

d) $\sec\theta = -\frac{2}{\sqrt{2}}$ in the domain $-2\pi \le \theta < \pi$. Give exact answers.

Example #3: The point (-5, -12) lies on the terminal arm of an angle θ in standard position. What is the exact value of each trig ratio for θ?

4.3 Day 2 ASSIGNMENT

4.3 Day 1FA: P201 #7, 10, 11, 12
4.3 Day 1 ULA: P201 #5ac, 13, 15, 19

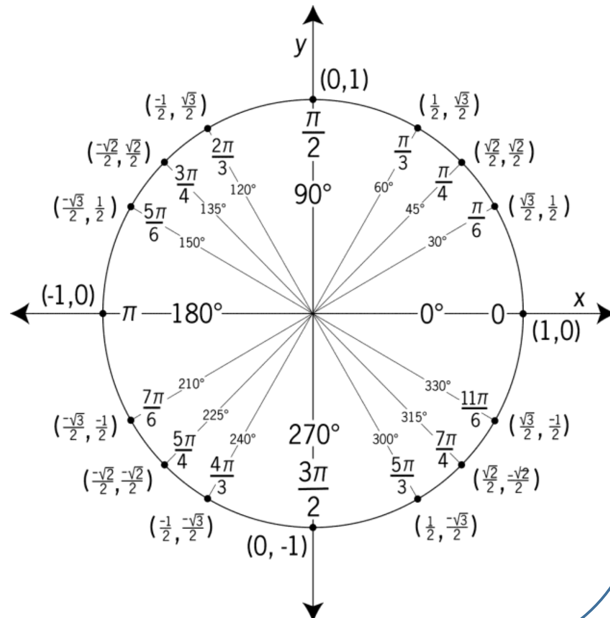
PC 30

4.4 Day 1 : Solving Trigonometric Equations

To find the solution to trigonometric equations in radians and degrees.

BACKGROUND INFO ON SOLVING TRIG EQUATIONS:

1. What does it mean to solve an equation?
   - It means to find the value of the variable that makes the equation ______. In this chapter, the variable is often called _____
2. We solve an equation by ___________________ the variable. We have solved several types of equations in the past including linear equations, quadratic equations, cubic equations, quartic equations, absolute value equations and radical equations.
3. In many cases, we need to factor to solve.
   a. Factor $2x^2 - 5x + 3 = 0$



4. In order to solve trig equations we need to:
   - Use factoring skills
   - Use the unit circle/special triangles and/or our calculator
   - Use the CAST rule

Example #1: Solve $\sin\theta = \frac{\sqrt{2}}{2}$ for $0 \le \theta \le 2\pi$

Hint: Since, in the unit circle, $\sin\theta = x$,look on the unit circle for places where the value of x is $\frac{\sqrt{2}}{2}$. Use this information to draw all the appropriate reference angles in the correct quadrant(s) to find all values of ϴ within the given domain.

IF NO DOMAIN IS GIVEN:

- You will have an infinite number of solutions because there are an infinite number of coterminal angles
- Rather than giving a specific list of solutions we will create a *General Solution* for all values of the variable. This general solution will take the specific answers for Θ for $0 \le \theta \le 360°$ or $0 \le \theta \le 2\pi$ and then add multiples of 2π or $360°$

Example #2:

a) Solve cscx = 2 for all values of x measured in degrees

b) Solve $\cos^2 x - \frac{1}{2}\cos x = 0$ for $0 \le x \le 2\pi$

c) Solve $2\cos^2\theta = 1 - \cos\theta, \quad 0 \ge \theta \ge 360°$

d) Solve $\tan^3 x = 3\tan x$ for $0 \le x \le 2\pi$

e) Solve $5\sin\theta + 2 = 1 + 3\sin\theta$ for all values of θ in radians.

4.4 Day 1 ASSIGNMENT

4.3 Day 1FA: P 211 #2, 3, 5

4.3 Day 1 ULA: P211 #9, 10, 12, 17

4.4 Day 2 : Solving Trigonometric Equations

To find the solution to trigonometric equations in radians and degrees where the solution is not on the unit circle.

Example #1:

a) Solve $\tan^2\theta - 2\tan\theta - 8 = 0, \quad (0°, 360°]$

b) $\sin^2 x - 3\sin x + 2 = 0$ where the domain is all real numbers measured in radians

4.4 Day 2 ASSIGNMENT

4.3 Day 1FA: P 211 #4, 5, 7

4.3 Day 1 ULA: P211 #15, 16, 18, 20, 22

LIST OF VIDEOS THAT MAY AIDE IN UNDERSTANDING

Section 4.1

https://goo.gl/Hy4ZXL

https://goo.gl/3hijX5

Section 4.2

https://goo.gl/dj0KW3

https://goo.gl/5sxdup

Section 4.3

https://goo.gl/ZLndZB

https://goo.gl/rCDXiA

Section 4.4

https://goo.gl/nYfbfG

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