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7.1 Characteristics of Exponential Functions

To define an exponential function, analyse its graph and solve problems involving exponential growth or decay.

EXPONENTIAL FUNCTION: An EXPONENTIAL FUNCTION is a function f(x) of the form $f(x)=c^x$ where the exponent is a variable and the base "c" is a constant number value where c > 0 and $c \neq 1$

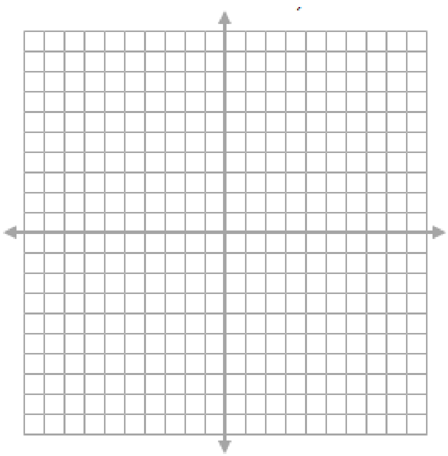
EXAMPLES: $y=2^x$ $\quad y=\left(\frac{1}{3}\right)^x$ $\quad y=(0.6)^x$ $\quad y=(2.8)^x$

INVESTIGATION:
Use a table of values to graph each exponential function and identify the following characteristics for each graph:

- Domain
- Range
- X intercept
- Y intercept
- Whether the graph represents an **INCREASING** or a **DECREASING** function
- The equation of the horizontal asymptote

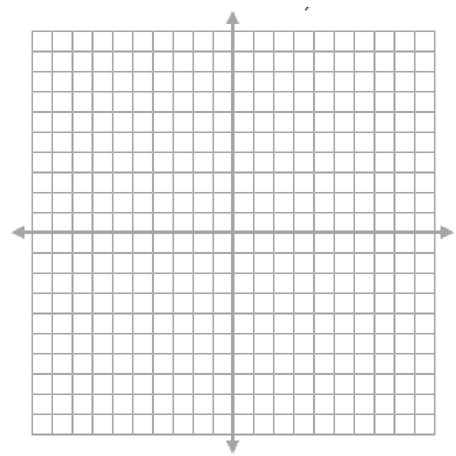
a) $y=3^x$

x	f(x)
-2	
-1	
0	
1	
2	



b) $y=\left(\frac{1}{3}\right)^x$

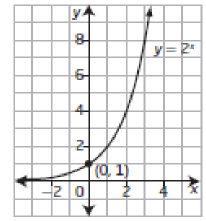
x	f(x)
-2	
-1	
0	
1	
2	



EXPONENTIAL FUNCTIONS:

CASE 1:

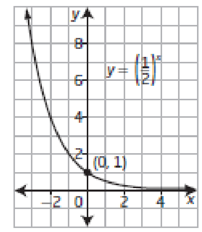
- When $c > 1$ in an exponential function of the form $f(x) = c^x$ the exponential function is increasing.
- This case is called **EXPONENTIAL GROWTH**



CASE 2:

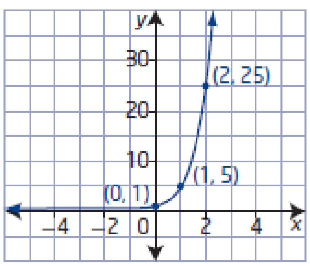
- When c is between 0 and 1 ($0 < c < 1$) in an exponential function of the form $f(x) = c^x$ the exponential function is decreasing.
- This case is called **EXPONENTIAL DECAY**

Exponential functions of the form $y = c^x$ have domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, no x-intercepts, and horizontal asymptote at $y = 0$. And y intercept at y = 1

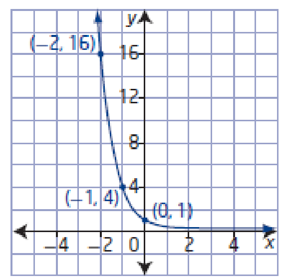


Example #1: What function of the form $y = c^x$ can be used to describe the following graphs?

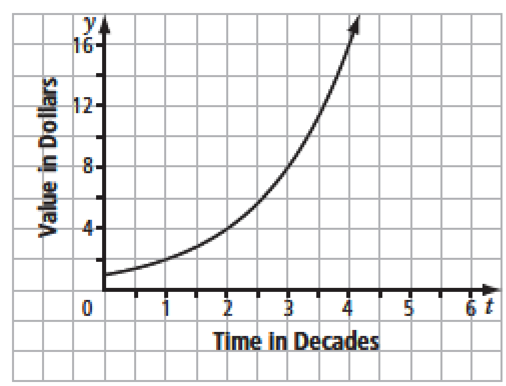
a)



b)



Example #2: Fong has just graduated from university and wants to begin her retirement planning. Her investment advisor suggests an investment that he expects to double in value every decade. The estimated growth rate is modelled by the exponential graph shown.



a) State the domain and range of the function. Explain the significance of the *y*-intercept.

b) Write an exponential equation that expresses the value of each dollar invested, after *t* decades.

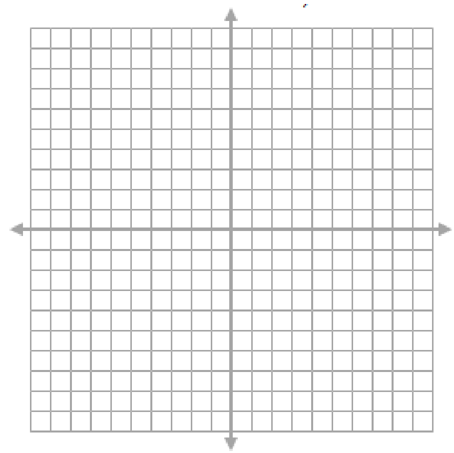
c) What is the value of a dollar invested for ten years?

d) How long will it take for $1 in this investment to be worth $8?

e) Use a table of values to determine how long it will take for $1 in this investment to be worth $26.

Example #3:

A radioactive substance has a half-life of 1 day (the substance decays to half the original amount after 1 day). Plot the mass of the substance remaining if the original amount was 1 gram.



a.) Determine the domain and range.

b.) Write the exponential decay model (equation) that relates the mass of the substance.

c.) Determine approximately how many days it would take for the substance to decay to 1/50th of its original mass.

7.1 ASSIGNMENT

7.1 FA: P342 #1, 2, 3, 4, 5bd

7.1 ULA: P243 #6-11

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7.2 Transformations of Exponential Functions

To apply transformations, stretches and reflections to the graph of exponential functions and use those transformations to model a situation.

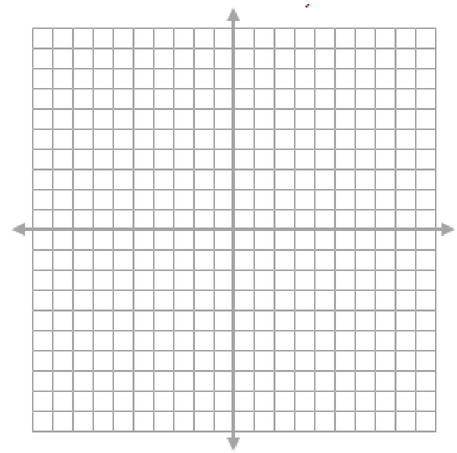
RECALL from Chapter 1 the following transformation model: y=f(x) $\longrightarrow$ $y = af\left(b\left(x-h\right)\right)+k$

Where:
- *a* affects the **vertical** stretch/compression, reflection in x axis (if negative)
- *b* affects the **horizontal** stretch/compression, reflection in y axis (if negative)
- *h* affects the **horizontal** translation (+ moves to the left, - moves to the right)
- *k* affects the **vertical** translation (+ moves up, - moves down)

We can apply the transformations to exponential functions:

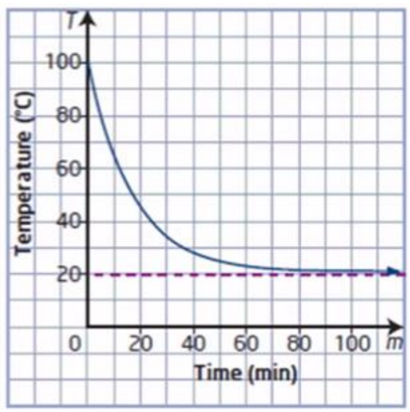
$f(x) = c^x \rightarrow f(x) = a(c)^{b(x-h)} + k$ where the mapping will be $(x, y) \rightarrow \left(\left(\frac{1}{b}x + h\right), (ay+k)\right)$

Example #1: Transform the graph of $y = 4^x$ to the graph of $y = \frac{1}{2}(4)^{-2(x+5)} - 5$. Describe the mapping, the effects on the domain, range, equation of horizontal asymptote and intercepts. Graph.



Example #2:

A cup of water is heated to 100 °C and then allowed to cool in a room with an air temperature of 20 °C. The temperature, *T*, in degrees Celsius, is measured every minute as a function of time, *m*, in minutes, and these points are plotted on a coordinate grid. It is found that the temperature of the water decreases exponentially at a rate of 25% every 5 min. A smooth curve is drawn through the points, resulting in the graph shown.

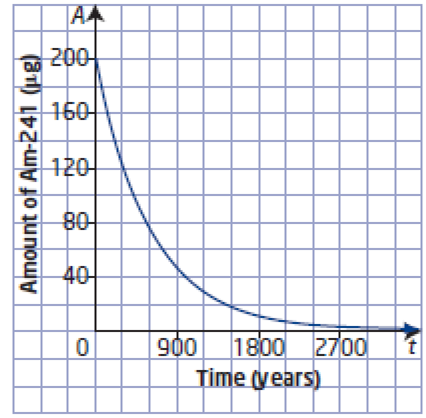


a) What is the transformed exponential function in the form $y = a(c)^{b(x-h)} + k$ that can be used to represent this situation?

b) Describe how each of the parameters in the transformed function relates to the information provided.

Example #3: The radioactive element americium (Am) is used in household smoke detectors.Am-241 has a half life of approximately 432 years. The average smoke detector contains 200 μg of Am-241.

a) What is the transformed exponential functions that models the graph showing the radioactive decay of 200 μg of Am-241?



b) Identify how each of the parameters of the function relates to the transformed graph.

7.2 ASSIGNMENT

7.2 FA: P354 #1-4, 6, 9, 11

7.2 ULA: P354 #7, 12, 13, 14

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7.3 Solving Exponential Equations

To solve exponential equations algebraically and to apply to real life situations.

REVIEW: In order to solve exponential equations:

- you will need to remember your exponent rules
- remember that you need **THE SAME BASE** to use these rules

1. $b^0 = 1$
2. $b^x \cdot b^y = b^{x+y}$
3. $\dfrac{b^x}{b^y} = b^{x-y}$
4. $\left(b^x\right)^y = b^{xy}$
5. $b^{-x} = \dfrac{1}{b^x}\left(- \right)$
6. $\left(\dfrac{b}{a}\right)^{-x} = \left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$
7. $(ab)^x = a^x b^x$
8. $\sqrt[x]{b} = b^{\frac{1}{x}}$
9. $\sqrt[y]{b^x} = \left(\sqrt[y]{b}\right)^x = b^{\frac{x}{y}}$

Example #1: Rewrite each expression as a power with a base of 3

a) 27

b) 9^2

c) $27^{\frac{1}{3}}\left(\sqrt[3]{81}\right)^2$

d) $\left(\frac{\sqrt{3}}{81}\right)^{-3}$

Example #2: Rewrite with a common base

a) $\left(4^x\right)^{2+x}\left(32^x\right)^{-x}$

b) $\frac{9^x\left(27^{x-3}\right)}{243^{x+1}}$

It is often helpful to rewrite exponential expressions using a different base since there is an additional exponent law to note:

If $c^x = c^y$ then $x = y$ (for $c \neq -1, 0, 1$)

Example #2: Solve the following exponential equations:

a) $2^{4x} = 4^{x+3}$

b) $9^{4x} = 27^{x-1}$

c) $8^{x-2} = \left(\frac{1}{4}\right)^{x+3}$

d) $8(8)^x = 2$

Example #3: Solve the equation $3^x = 12$

NOTE: If we can't obtain a common base we can solve by trial and error.

COMPOUND INTEREST: The formula for compound interest is **$A = P(1+ i)^n$** where the following is true:

A = Amount of money at the end of the investment

P = Principal amount deposited (or invested)

i = interest rate *per compounding period* (as a decimal – found using the formula $\dfrac{\text{Annual/Yearly Interest Rate}}{\text{Number of Compounding Periods per Year}}$)

n = number of compounding periods

Example #4: Determine how long $1000 needs to be invested in an account that earns 8.3% compounded semi-annually before it increases in value to $1490.

Example #4: A colony of ants grows by 50% every hour. After how many hours will it take the colony to reach 151875 ants? Solve algebraically.

7.3 ASSIGNMENT

7.3 FA: P364 #1-4, 5(no check), 9, 11ab PLUS THE QUESTIONS BELOW:

7.3 ULA: P364 #6ab, 7ab, 8, 11c, 12, 13, 14

EXTRA FA QUESTIONS:

a) $2^x = 32$ b) $4^{x-2} = 8^4$ c) $2^{x-5} = 4$ d) $4^{1-x^2} = 8^x$

e) $2^{x^2} = \left(16^{x-1}\right)\left(2^x\right)$ f) $4^{x-1} = \left(\dfrac{1}{2}\right)^{4x-1}$ g) $9^{2x} = \sqrt{27}$ h) $13^{x^2-4} = 1$

Answers: a) {5} b) {8} c) {7} d) {½, -2} e) {1, 4} f) {½} g) {3/8} h) {2, -2}

LIST OF VIDEOS THAT MAY AIDE IN UNDERSTANDING

Section 7.1

- https://goo.gl/L2MZxY
- https://goo.gl/ZqbzFS

Section 7.2

- https://goo.gl/1Ykvyr
- https://goo.gl/pSktXV

Section 7.3

- https://goo.gl/R5SkBZ
- https://goo.gl/zfTLPB