7.1 Day 1: Differential Equations & Initial Value Problems (30L)

I CAN SOLVE DIFFERENTIAL EQUATIONS AND INITIAL VALUE PROBLEMS

VIDEO LINKS:

a) http://bit.ly/2Bxsc6R   b) http://bit.ly/2SShYYH
c) http://bit.ly/2DMJZY2   d) http://bit.ly/2TYpdvm

DIFFERENTIAL EQUATIONS – Equations that contain derivatives (usually in Leibniz form $\frac{dy}{dx}$). The order of a differential equation is the order of the highest derivative involved in the equation.

EX: $\frac{dy}{dx} = x^2 + 4x + 1$ $\frac{dy}{dx} = 3\sin(x) + 5\cos(x)$

We want to **SOLVE** the Differential Equations (sometimes called Diffy Q's for short ☺)

- There are two types of solutions to diffy q's – the **GENERAL SOLUTION** contains a constant called C while the **PARTICULAR SOLUTION** is a specific answer where the value of C has been found.
- To solve a Diffy Q you need to take the Integral/Antiderivative of BOTH SIDES of the Diffy Q

STEPS TO SOLVING DIFFERENTIAL EQUATIONS

1. Gather the same variables on opposite sides of the equal sign.
2. Integrate both sides with respect to their appropriate variable.
3. Although both indefinite integrals will yield a generic constant, by convention we only include the "+c" on the side of the independent variable (in most cases the variable x or t)
4. If provided, use the initial condition to solve for c
5. Substitute the value of c back in and solve explicitly for the dependent variable (usually y)

EX #1: Find the GENERAL SOLUTION to the following differential equations

a) $\frac{dy}{dx} = \csc^2 x + 2x + 5$ b) $\frac{dy}{du} = (u^2 - 1)2u$

EX #2: Explicitly solve the following Initial Value problems.

a) $\frac{ds}{dt} = t^2 + 1$; t=0 and s = 1   b) $\frac{dy}{dx} = 1 + \cos x$; x = 0, y = 4

c) $y' = \sin(\pi t)$ given $y(2) = 2$

d) $y'' = 3x + 2$ given $y'(0) = 1$

EX #3: Find the particular solution to the equation $\frac{dy}{dx} = e^{2x} - 3x$, whose graph passes through the point $\left(1, \frac{1}{2}\right)$.

SEPARABLE DIFFERENTIAL EQUATIONS

- Separable differential equations have the form $\dfrac{dy}{dx} = f(x)g(y)$ and can be rewritten in the form

  $g(y)dy = f(x)dx$

  - This means that you are given an equation where it is possible to collect the function in terms of x on one side (along with the dx) and the function in terms of y and dy on the other side

EXPLICIT/IMPLICIT SOLUTION

- An **EXPLICIT** solution is any solution that is in the form $y = f(x)$ (Note: this means that the only place that y shows up is on one side of the equal sign and y is only raised to the power one)
- A solution that is NOT in explicit form is said to be in **IMPLICIT** form

EX #4: Solve $\dfrac{dy}{dx} = 6xy^2$ given $y(1) = \dfrac{1}{9}$

EX #5: Solve $\frac{dy}{dx} = \frac{-2x}{y}$ given $y(1) = -1$

EX #6: Solve $\frac{dy}{dx} = x^4(y-2)$ given $y(0) = 0$

EX #7: Application: At time $t=0$, a car traveling with velocity $96\,{}^{ft}\!/_{sec}$ begins to slow down with constant deceleration $a = -12\,{}^{ft}\!/_{sec^2}$ Find the velocity $v(t)$ at time t and the distance traveled before the car comes to a halt.

7.1 Day 1 Assignment P331 #1-23 Odd (Bottom of page)
P 361 1-9 Odd (bottom of page)
AP Free Response Question Below

Note: The FULL answer key for the following AP question with extra information (scoring guidelines and AP commentary) can be found in the Chapter 7 section of my website

2012 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

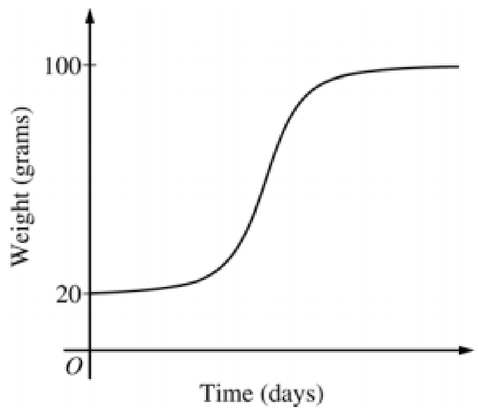
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

SOLUTION TO AP QUESTION (EXTRA DETAILS ON MY WEBSITE)

(a) $\left.\frac{dB}{dt}\right\vert_{B=40} = \frac{1}{5}(60) = 12$ $\left.\frac{dB}{dt}\right\vert_{B=70} = \frac{1}{5}(30) = 6$ Because $\left.\frac{dB}{dt}\right\vert_{B=40} > \left.\frac{dB}{dt}\right\vert_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.	$2: \begin{cases} 1: \text{uses } \frac{dB}{dt} \\ 1: \text{answer with reason} \end{cases}$
(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$ Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.	$2: \begin{cases} 1: \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1: \text{explanation} \end{cases}$
(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$ $\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$ $-\ln\vert 100 - B\vert = \frac{1}{5}t + C$ Because $20 \le B < 100$, $\vert 100 - B\vert = 100 - B$. $-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$ $100 - B = 80e^{-t/5}$ $B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$	$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } B \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables

AP Calculus 30 & 30L

7.1 Day 2: Slope Fields (30L)

I CAN CREATE SLOPE FIELDS

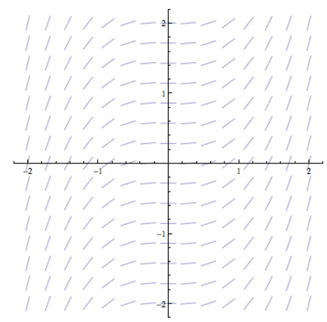
VIDEO LINKS:

a) http://bit.ly/2N9AG8X b) http://bit.ly/2X2ruYt c) http://bit.ly/2SASRKQ

SLOPE FIELDS are graphs of the **SHAPE** of the function that are produced using the differential equation

- The graph of a slope field is comprised of mini tangent lines at each point on the Cartesian plane. When we look at a slope field, we should look at the trend of how the slopes are curving
- Slope Fields give us an idea of what the curve would look like depending on different values of c (different initial values).
- Within it's diagram of mini tangent lines, it shows different pathways that the curve would follow if we were to consider different translations of the antiderivative for different values of c

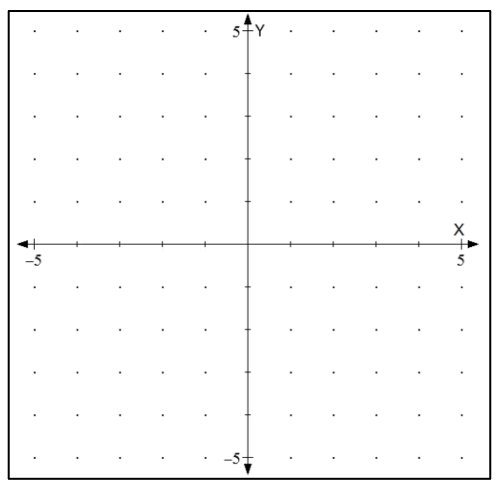
- Here is an example of a slope field
  - What equation do you think the path of the various curves resemble? ____________
  - The slope field of the above equation would graph the individual SLOPES of the above equation. Slope is rate of change which we call____________ in Calculus.



EX #1: Given that $\frac{dy}{dx} = x + 1$, fill in the following table and sketch the slope field and solve the differential equation.

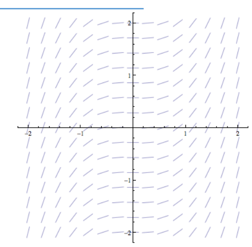
https://www.desmos.com/calculator/frl7pma2qt

x	$\frac{dy}{dx}$
-3	
-2	
-1	
0	
1	
2	
3	



Go back and look at the slope field sketch from the top of the page.

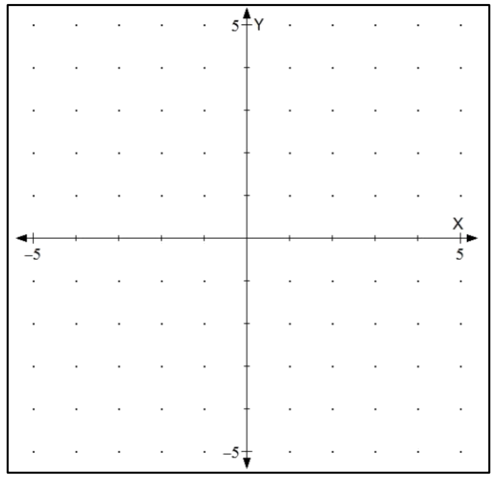
- The pathway that the slope fields form would be an equation of the form y = ______________
- The slope field equation that was used to sketch this graph would be of the form $\frac{dy}{dx} =$ ___________



Informally – it is useful to think of a differential equation as a set of instructions that tells the solutions which "direction" to go at each point (x, y). If we draw each "direction" as a tiny slope (x, y), we then have a collection of slopes known as a slope field (or direction field) that gives us a visual representation of the family of solutions to the differential equations.

EX #2: Given that $\frac{dy}{dx} = x + y$, fill in the following table and sketch the slope field.

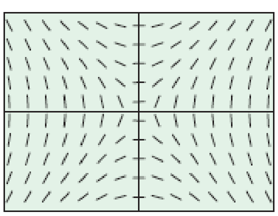
x	y	$\frac{dy}{dx}$
-3		
-2		
-1		
0		
1		
2		
3		

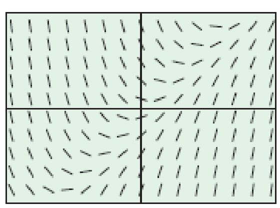


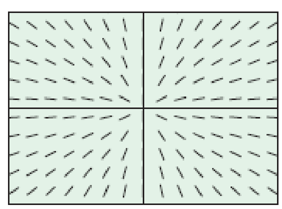
b) Use the slope field to sketch the graph of the particular solution through the point (2, 0)

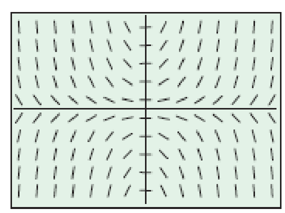
EX #3: Use slope analysis to match each of the following differential equations with one of the slope fields (a) through (d). (Do not use your graphing calculator.)

1. $\frac{dy}{dx} = x - y$ **2.** $\frac{dy}{dx} = xy$ **3.** $\frac{dy}{dx} = \frac{x}{y}$ **4.** $\frac{dy}{dx} = \frac{y}{x}$


(a)


(b)


(c)

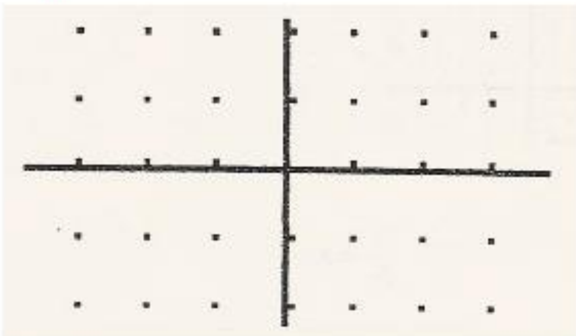

(d)

Note: on my website is the link to the Desmos Slope Field Generator Program that you can use to CHECK your work (Short link at http://bit.ly/2TTVW58)

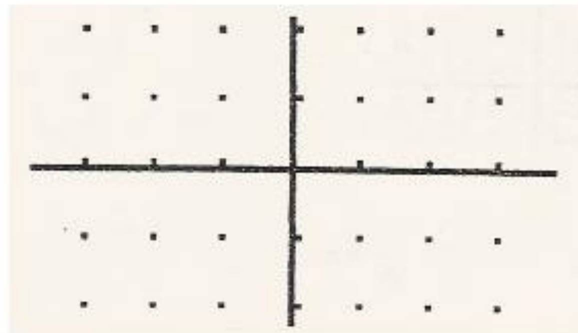
7.1 Day 2 Assignment FOLLOWING PAGES 7 - 10

Draw a slope field for each of the following differential equations. Each tick mark is one unit.

1. $\frac{dy}{dx} = x + 2$

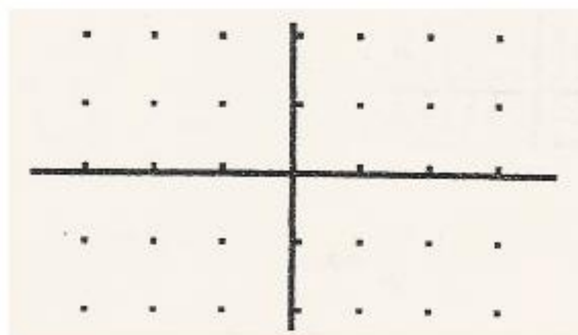


2. $\frac{dy}{dx} = 2y$



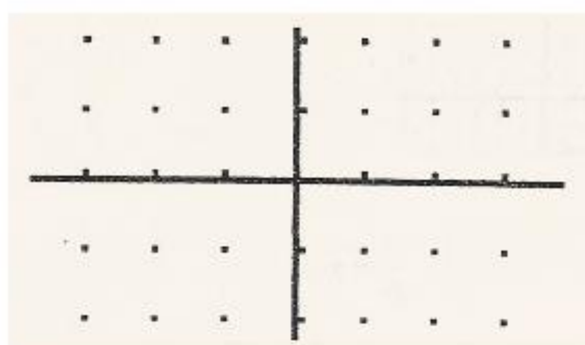
3. $\frac{dy}{dx} = y - x$

4. $\frac{dy}{dx} = 2x$



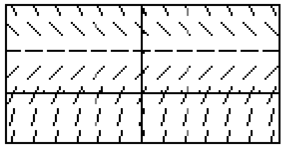
5. $\frac{dy}{dx} = y - 1$

6. $\frac{dy}{dx} = -\frac{y}{x}$

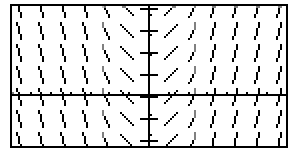


Match the slope fields with their differential equations.

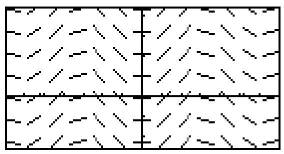
(A)



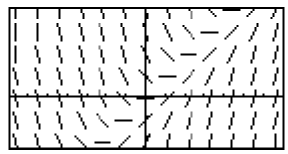
(B)



(C)



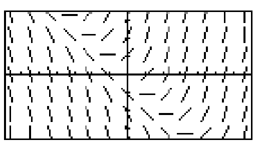
(D)



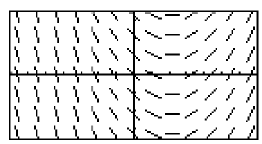
7. $\dfrac{dy}{dx} = \sin x$ 8. $\dfrac{dy}{dx} = x - y$ 9. $\dfrac{dy}{dx} = 2 - y$ 10. $\dfrac{dy}{dx} = x$

Match the slope fields with their differential equations.

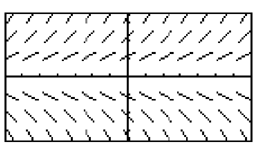
(A)



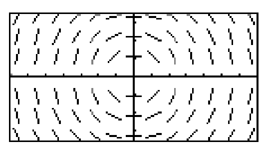
(B)



(C)

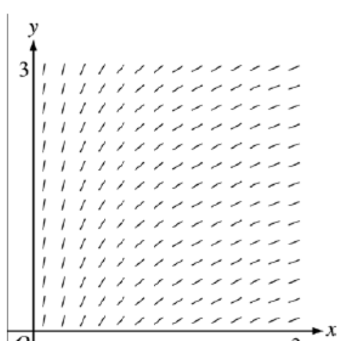


(D)



11. $\dfrac{dy}{dx} = 0.5x - 1$ 12. $\dfrac{dy}{dx} = 0.5y$ 13. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ 14. $\dfrac{dy}{dx} = x + y$

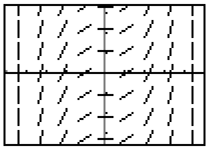
15.



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

16.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

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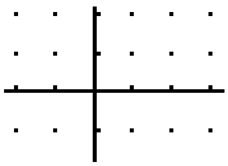
17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

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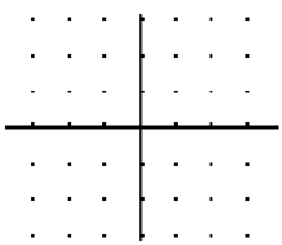
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(D) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(E) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

19.

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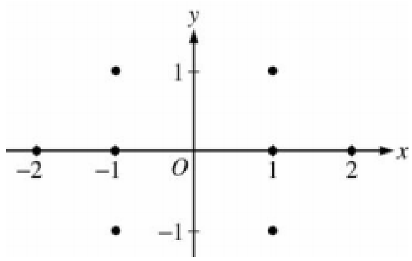
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

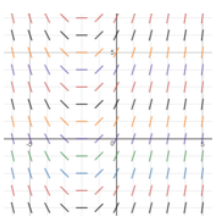
(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated. **(Note: Use the axes provided in the pink exam booklet.)**



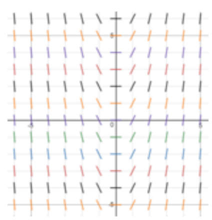
(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

SOLUTIONS:

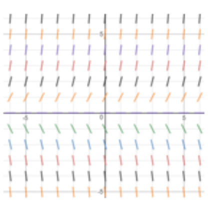
1. $\frac{dy}{dx} = x + 2$



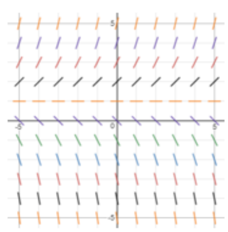
4. $\frac{dy}{dx} = 2x$



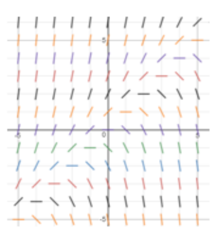
2. $\frac{dy}{dx} = 2y$



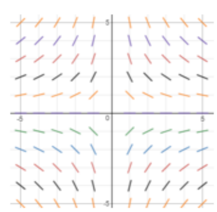
5. $\frac{dy}{dx} = y - 1$



3. $\frac{dy}{dx} = y - x$



6. $\frac{dy}{dx} = -\frac{y}{x}$



7. C
8. D
9. A
10. B
11. B
12. C
13. D
14. A
15. E
16. D

17. (B) Tangent line: $y - 1 = \frac{1}{2}(x-1)$

$f(1.2) \approx 1.1$

(C) $y = e^{\frac{x^2-1}{4}}$

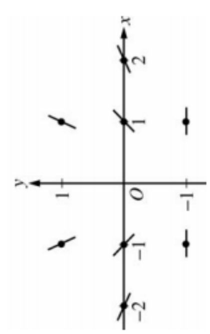
$f(1.2) = 1.116$

(D) The estimate from part (b) was an underestimate. Since the graph of $y = e^{\frac{x^2-1}{4}}$ is concave up, the tangent line found in part (b) lies below the curve.

18. (C) $y = \sqrt{x^2+1}$

(E) $y = -\sqrt{x^2+1}$

AP SOLUTION

(a) 	2 : sign of slope at each point and relative steepness of slope lines in rows and columns
(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$ $\ln\lvert 1+y\rvert = \ln\lvert x\rvert + K$ $\lvert 1+y\rvert = e^{\ln\lvert x\rvert + K}$ $1 + y = C\lvert x\rvert$ $2 = C$ $1 + y = 2\lvert x\rvert$ $y = 2\lvert x\rvert - 1$ and $x < 0$ or $y = -2x - 1$ and $x < 0$	7 : { 6 : { 1 : separates variables 2 : antiderivatives 1 : constant of integration 1 : uses initial condition 1 : solves for y } Note: max 3/6 [1-2-0-0-0] if no constant of integration Note: 0/6 if no separation of variables 1 : domain }

7.2 Day 1: Antidifferentiation by Substitution (30 & 30L)

I CAN FIND THE INTEGRAL OF FUNCTIONS USING "u" SUBSTITUTION

VIDEO LINKS:
a) **https://bit.ly/2L1M44A** b) **https://bit.ly/2LDBeCG**

When we are taking the integral of functions where the chain rule and/or the product rule was originally used in the derivative process, we can use *u* substation to help make taking the integral easier. The following formula's will be used (more to come! ☺:

Integration Formulas
(*u* represents a function)

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$	2. $\int e^u du = e^u + C$
3. $\int b^u du = \frac{1}{\ln b} b^u + C$	4. $\int \frac{1}{u} du = \ln\lvert u\rvert + C$
5. $\int \sin u du = -\cos u + C$	6. $\int \cos u du = \sin u + C$

Ex #1: Evaluate $\int (4x+3)^9 dx$.

Ex #2: Evaluate $\int x^2 (x^3 - 5)^5 dx$

Ex #3: Evaluate $\int (x+1)\sqrt[3]{x^2+2x+3}\,dx$

Ex #4: Evaluate $\int \frac{\sin\sqrt{5x+3}}{\sqrt{5x+3}}\,dx$

Ex #5: Evaluate $\int \frac{x^4-1}{x^5-5x}dx$

STEPS TO USING *U* SUBSTITUTION:

1. Choose your *u*. This will be the most complex function within the integral.
   - If u is a polynomial function is raised to an outside power, do NOT include the power in *u.*
   - If *u* is the inside function of a trig function, or the power of an exponential function, include any and all powers of that *u*
2. Find the derivative of *u*, $\frac{du}{dx}$
3. Cross multiply your answer in step 2 to solve for du
4. Compare your original integral to the integral that would contain *u* and du. Often there will be a constant term missing from the du part of the integral. If you are missing a constant, you can add that constant to your original integral as long as you multiply by 1 over that constant on the outside of the original integral.
5. Once you have all the components of *u* and du in your original integral, rewrite your integral change everything to *u* and du, making sure to add any power outside of *u*.
6. Use your basic integration rules to find the integral of step 5 (don't forget the C!)
7. Substitute *u* back to the expression that *u* originally was originally
8. You can quickly check your work by taking the derivative of your answer and checking to see that it matches your original integral

7.2 Day 1 Assignment FOLLOWING QUESTIONS

1. $\int e^{\sin x}\cos x dx$; let $u=\sin x$
2. $\int x\left(2x^2+5\right)^8 dx$; let $u=2x^2+5$
3. $\int x^2\cos 5x^3 dx$; let $u=5x^3$
4. $\int \frac{1}{10x+7}dx$; let $u=10x+7$
5. $\int \sqrt{5x-9}dx$; let $u=5x-9$
6. $\int \frac{\sin(\ln x)}{x}dx$; Let $u=\ln x$.

7. $\int e^{6x}dx$
8. $\int \cos 4x dx$
9. $\int \frac{1}{3x+8}dx$
10. $\int (6x-11)^8 dx$
11. $\int x\left(x^2-6\right)^{11} dx$
12. $\int x^2\sin x^3 dx$
13. $\int 3^{2x+1}dx$
14. $\int x\sqrt{2x^2-5}dx$
15. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$
16. $\int \sqrt{x+2}dx$
17. $\int \frac{5}{(x-2)^3}dx$
18. $\int (3-t)^4 dt$

19. $\int \frac{x}{x^2-4}dx$ 20. $\int \frac{3x+1}{\sqrt{3x^2+2x+1}}dx$ 21. $\int \sin\theta e^{\cos\theta}d\theta$ 22. $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}}dx$

23. $\int (3x+7)^{4.2}dx$ 24. $\int \sqrt[4]{5-2x}dx$ 25. $\int \sin\frac{1}{2}xdx$ 26. $\int (r+2)^{20}dr$

27. $\int \frac{x}{\left(x^2-1\right)^{11}}dx$ 28. $\int \sin xe^{\cos x}dx$ 29. $\int (\sin 2t)^3\cos 2tdt$ 30. $\int \sin x\sqrt{\cos x}dx$

31. $\int x^2\sqrt[4]{x^3+4}dx$ 32. $\int \frac{x+1}{x^2+2x-5}dx$ 33. $\int e^x\left(e^x+1\right)^4dx$ 34. $\int \frac{(\ln x)^4}{x}dx$

35. $\int \cos^6 x\sin xdx$ 36. $\int \frac{x+5}{x^2+10x-23}dx$ 37. $\int e^{4x}\cos\left(e^{4x}\right)dx$ 38. $\int x^4\cos x^5dx$

39. $\int x^3\sin x^4\left(\cos^5 x^4\right)dx$ 40. $\int x^2e^{-x^3}dx$ 41. $\int e^{-\sin x}\cos xdx$ 42. $\int x\cos\left(x^2-1\right)dx$

43. $\int \frac{\sqrt{\ln x}}{x}dx$ 44. $\int \sin x\cos xdx$ 45. $\int \frac{\cos x}{\sin x}dx$ 46. $\int \frac{e^{1/x}}{x^2}dx$

47. $\int \frac{x}{\left(x^2+1\right)^{5/2}}dx$ 48. $\int 3x^2\cos x^3dx$ 49. $\int \left(x^3-5\right)e^{x^4-20x}dx$ 50. $\int (\cos 2t)e^{\sin 2t}dt$

51. $\int (1+\sin x)^4\cos xdx$ 52. $\int x^{1/3}\sqrt{x^{4/3}+1}dx$ 53. $\int \frac{\cos x}{1+\sin x}dx$ 54. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx$

SOLUTIONS:

1. $e^{\sin x}+C$ **2.** $\frac{1}{36}\left(2x^2+5\right)^9+C$ **3.** $\frac{1}{15}\sin 5x^3+C$ **4.** $\frac{1}{10}\ln|10x+7|+C$ **5.** $\frac{2}{15}(5x-9)^{3/2}+C$

6. $-\cos(\ln x)+C$ **7.** $\frac{1}{6}e^{6x}+C$ **8.** $\frac{1}{4}\sin 4x+C$ **9.** $\frac{1}{3}\ln|3x+8|+C$ **10.** $\frac{1}{54}(6x-11)^9+C$

11. $\frac{1}{24}\left(x^2-6\right)^{12}+C$ **12.** $-\frac{1}{3}\cos x^3+C$ **13.** $\frac{3^{2x+1}}{2\ln 3}+C$ **14.** $\frac{1}{6}\left(2x^2-5\right)^{3/2}+C$ **15.** $-2\cos\sqrt{x}+C$

16. $\frac{2}{3}(x+2)^{3/2}+C$ **17.** $-\frac{5}{2}(x-2)^{-2}+C$ **18.** $-\frac{1}{5}(3-t)^5+C$ **19.** $\frac{1}{2}\ln\left|x^2-4\right|+C$

20. $\sqrt{3x^2+2x+1}+C$ **21.** $-e^{\cos\theta}+C$ **22.** $2e^{\sqrt{x-1}}+C$ **23.** $\frac{5}{78}(3x+7)^{5.2}+C$ **24.** $-\frac{2}{5}(5-2x)^{5/4}+C$

25. $-2\cos\frac{1}{2}x+C$ **26.** $\frac{1}{21}(r+2)^{21}+C$ **27.** $-\frac{1}{20}\left(x^2-1\right)^{-10}+C$ **28.** $-e^{\cos x}+C$ **29.** $\frac{1}{8}\sin^4 2t+C$

30. $-\frac{2}{3}(\cos x)^{3/2}+C$ **31.** $\frac{4}{15}\left(x^3+4\right)^{5/4}+C$ **32.** $\frac{1}{2}\ln\left|x^2+2x-5\right|+C$ **33.** $\frac{1}{5}\left(e^x+1\right)^5+C$

34. $\frac{1}{5}(\ln x)^5+C$ **35.** $-\frac{1}{7}\cos^7 x+C$ **36.** $\frac{1}{2}\ln\left|x^2+10x-23\right|+C$ **37.** $\frac{1}{4}\sin\left(e^{4x}\right)+C$ **38.** $\frac{1}{5}\sin x^5+C$

39. $-\frac{1}{24}\cos^6 x^4+C$ **40.** $-\frac{1}{3}e^{-x^3}+C$ **41.** $-e^{-\sin x}+C$ **42.** $\frac{1}{2}\sin\left(x^2-1\right)+C$ **43.** $\frac{2}{3}(\ln x)^{3/2}+C$

44. $\frac{1}{2}\sin^2 x+C$, also $-\frac{1}{2}\cos^2 x+C$ **45.** $\ln|\sin x|+C$ **46.** $-e^{1/x}+C$ **47.** $-\frac{1}{3}\left(x^2+1\right)^{-3/2}+C$

48. $\sin x^3+C$ **49.** $\frac{1}{4}e^{x^4-20x}+C$ **50.** $\frac{1}{2}e^{\sin 2t}+C$ **51.** $\frac{1}{5}(1+\sin x)^5+C$ **52.** $\frac{1}{2}\left(x^{4/3}+1\right)^{3/2}+C$

53. $\ln|1+\sin x|+C$ **54.** $\ln\left(e^x-e^{-x}\right)+C$ **55.** $\tan^{-1}\left(e^x\right)+C$ **56.** $\frac{1}{5}\sin^{-1}(5x)+C$ **57.** $\frac{1}{3}\tan x^3+C$

58. $-\frac{1}{4}\cot^4 x+C$ **59.** $\frac{1}{6}\sec^3 2x+C$ **60.** $\frac{2}{5}\tan^{-1}\left(\frac{1}{5}x\right)+C$ **61.** $-\frac{1}{2}\cot^2 e^x+C$ **62.** $\tan^{-1}(\sin x)+C$

7.2 Day 2: Substitution & Definite Integrals (30 & 30L)

I CAN FIND THE INTEGRAL OF FUNCTIONS USING "u" SUBSTITUTION

VIDEO LINKS:
a) http://bit.ly/2Gyhxgn b) http://bit.ly/2DWroJc

Strategy for Solving Integrals using u-substitution

1. In the integrand, identify the function $u(x)$ [It is commonly an "inside" function or the function on the bottom of a fraction bar. When looking for $u(x)$ you are often also looking to find its derivative or modified derivative in the integrand BUT that isn't always the case.]
2. Next find du by taking the derivative of u, $\frac{du}{dx}$, and multiplying the dx to the other side. (You may also need to solve for x given $u(x)$.)
   a. If evaluating a definite integral, we must also change the limits of integration by substituting them into $u(x)$. We then do not return to the original integrand.
3. Perform a change of variables by replacing the function and its derivative with u and du.
4. Evaluate the integral with respect to the variable u
5. If solving an indefinite integral, replace u with the original substitution function $u(x)$

Ex #1: Evaluate the following indefinite integrals:

a) $\int 3x^2 \cos(x^3)dx$

b) $\int xe^{x^2+1}\,dx$

c) $\int \tan x dx$

d) $\int x^2 \cos(x^3)dx$

SUBSTITUTION METHOD FOR DEFINITE INTEGRALS:

If u(x) is differentiable on [a, b] and f(x) is integrable on the range of u(x) then

$$\int_{x=a}^{x=b} f\big(u(x)\big)u'(x)dx = \int_{u(a)}^{u(b)} f(u)du$$

Ex #2: Evaluate the following definite integrals:

a) $\int_0^2 x^2\sqrt{x^3+1}dx$

b) $\int_1^3 \frac{x}{x^2+1}dx$

c) $\int_0^{\pi} 2x\, cos\,(x^2)dx$

d) $\int_0^2 \sqrt{2x+1}\,dx$

Integrals of More Transcendental Functions

Derivatives	General Antiderivatives/Indefinite Integrals
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$ $\quad \int e^{kx+b}dx = \frac{1}{k}e^{kx+b} + c$
$\frac{d}{dx}b^x = \ln(b)b^x$	$\int b^x dx = \frac{1}{\ln b}b^x + c = \frac{b^x}{\ln b} + c$
$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}dx = \arctan x + c$ $\quad \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$ $\quad \int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\left(\frac{x}{a}\right) + c$

Ex #3: Evaluate the following:

a) $\int 4^x dx$

b) $\int 7^{-x} dx$

c) $\int \cos x \, 3^{sinx} \, dx$

d) $\int \frac{2}{9+x^2} dx$

e) $\int \frac{1}{1+4x^2} dx$

f) $\int \frac{dx}{\sqrt{9-x^2}}$

g) $\int \frac{dx}{(\tan^{-1} x)(1+x^2)}$

h) $\int_0^1 \frac{dx}{1+x^2}$

7.2 Day 2 Assignment P 342 #1-11 Odd, 17-33 Odd, 35-65 Odd

7.4: Exponential Growth & Decay (30 & 30L)

I CAN SOLVE EXPONENTIAL GROWTH & DECAY QUESTIONS USING CALCULUS

VIDEO LINKS:

a) http://bit.ly/2E5JPLR b) http://bit.ly/2GyqnuO

REMEMBER: An equation is **SEPARABLE** if it can be written in the form $\frac{dy}{dx} = f(y)g(x)$

- To SOLVE a separable equation, we need to:
  1. Separate the variables into the following form: $\frac{1}{f(y)}dy = g(x)dx$
  2. Anti-differentiate with respect to the newly isolated variable.

Ex #1: Solve for y if $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$ and y = 1 when x = *e*.

LAW OF EXPONENTIAL CHANGE

- Involves growth in which the rate of change is proportional to the amount present (Be sure you remember this phrase!)
  - ie: The more bacteria in the dish, the faster they multiply. The more radioactive material present, the faster it decays, the greater your bank account in a compound interest account, the faster it grows.
- The differential equation that describes this growth is $\frac{dy}{dt} = ky$, where k is called The Growth Constant (when positive) or The Decay Constant (when negative)
- This equation can be solved by separated the variables:

$$\frac{dy}{dt} = ky$$

EXPONENTIAL GROWTH AND DECAY

Occurs where a quantity (y) increased or decreases at a rate proportional to the amount present (ex: population, money, radioactive element decay, cooling temperature, bacteria)

$y = y_0 e^{kt}$ k = the growth or decay constant

y = the final amount

y_0 = the initial amount

t = time

Ex #2: Scientists who use carbon-14 dating use 5700 years for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

NOTE: For carbon dating, since we have a ratio of ½, the base formula used is: $\boxed{A = A_0\left(\frac{1}{2}\right)^{\frac{t}{5700}}}$

Ex #3: A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 h there are 10,000 bacteria. At the end of 5 h there are 40,000 bacteria. How many bacteria were present initially?

Ex #3: The charcoal from a tree killed in the volcanic eruption that formed Crater Lake in Oregon contained 44.5% of the carbon-14 found in living matter. About how old is Crater Lake?

- Suppose that A_0 dollars are invested at a fixed annual interest rate *r*. If interest is added to the account *k* times a year (the number of compounding periods), the amount of money present after *t* years is: $\boxed{A(t)=A_0\left(1+\frac{r}{k}\right)^{kt}}$
- If the interest is compounded continuously, the formula used is $\boxed{A(t)=A_0e^{rt}}$ where the number r is called the continuous interest rate.

Ex #4: Suppose you deposit $1200 in an account that pays 5.7% interest. How much will you have 11 years later if the interest is:

a) Compounded continuously

b) Compounded Quarterly

7.4 Assignment: (Please read section 7.4 in the textbook for additional formulas to know)
P 361 #21, 23, 25, 27, 31, 35, 47-52

Ch 7 Review: P 377 1-7 Odd, 11-17 odd, 25-31 odd, 35, 37, 39-42, 49, 51, 53, 55