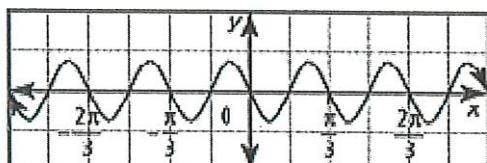


Name:
Date:

Pre – Calculus 30
Unit 5 Review Assignment
Graphs of Trigonometric Functions
Mrs. Boughen

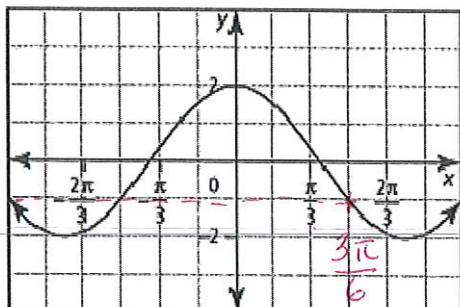
1. Determine the amplitude and period of each of the following trig functions.

a)



$$\text{period} = \frac{\pi}{3} \quad \text{amplitude} = \frac{3}{4}$$

b)



$$\text{period} = \frac{3\pi}{6} - (-\pi) = \frac{9\pi}{6} = \frac{3\pi}{2}$$

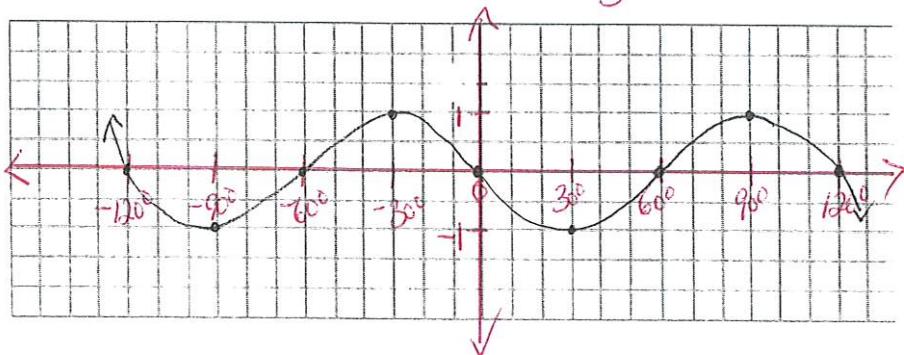
$$\text{amplitude} = 2 - (-2) = 2$$

3. Sketch one positive and one negative period of $y = -\sin 3x$ for $0^\circ \leq x \leq 360^\circ$. Clearly plot the key points and label the axes appropriately.

Amplitude: 1

Period: $\frac{360^\circ}{3} = 120^\circ$

Increments: $\frac{120^\circ}{4} = 30^\circ$



c) $y = 4 \sin 2x$

$$\text{period} = \frac{2\pi}{2} = \pi \quad \text{amplitude} = 4$$

d) $y = -3 \cos \frac{1}{5}x$

$$\text{period} = 2\pi \cdot \frac{1}{5} = 10\pi \quad \text{amplitude} = 3$$

2. Using the language of transformations, describe how each function's graph is related to the graph of $y = \cos x$.

a) $y = 2 \cos 4x$

vertical expansion scale factor 2
 horizontal compression s.f. $\frac{1}{4}$

b) $y = -\cos \frac{1}{5}x$

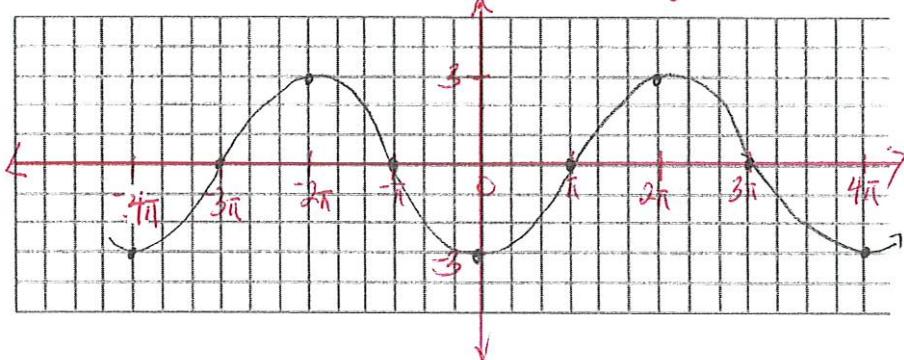
vertical reflection about the x-axis
 horizontal expansion s.f. 5.

4. Sketch one positive and one negative period of $y = -3\cos \frac{1}{2}x$, in radians. Clearly plot the key points and label the axes appropriately.

Amplitude: 3

$$\text{Period: } 2\pi \div \frac{1}{2} = 4\pi$$

$$\text{Increments: } \frac{4\pi}{4} = \pi$$



5. Given the following characteristics, write the equation of the sine function in the form $y = a \sin b(x - c) + d$, $a > 0$

- a) phase shift of $\frac{\pi}{2}$, period of $\frac{\pi}{2}$, vertical displacement of 5 up, and amplitude of 3

$$c = \frac{\pi}{2}, b = 2\pi \div \frac{\pi}{2} = 4, d = 5, a = 3 \quad \therefore y = 3\sin 4(x - \frac{\pi}{2}) + 5$$

- b) period of 120° , phase shift of 50° left, amplitude of $\frac{1}{2}$, and vertical displacement of 4 down

$$b = \frac{360^\circ}{120^\circ} = 3, c = -50^\circ, a = \frac{1}{2}, d = -4 \quad \therefore y = \frac{1}{2}\sin 3(x + 50^\circ) - 4$$

- c) period of 8π and phase shift of $\frac{\pi}{2}$ right

$$b = \frac{2\pi}{8\pi} = \frac{1}{4}, c = \frac{\pi}{2} \quad \therefore y = \sin \frac{1}{4}(x - \frac{\pi}{2})$$

6. Sketch **one positive period** of the following graphs carefully.

a) $y = 4\sin(x + \pi) + 1$

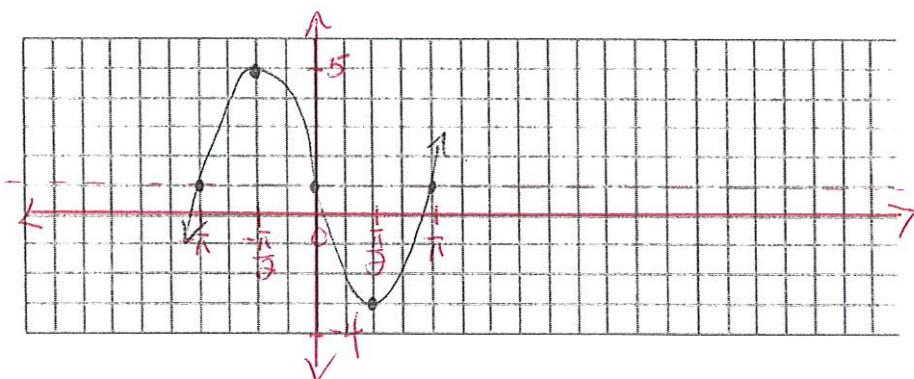
Amplitude: 4

Period: 2π

Increments: $\frac{2\pi}{4} = \frac{\pi}{2}$

Phase Shift: π left

Vertical Displacement: 1 up



b) $y = 2\cos 3\left(x - \frac{\pi}{2}\right) + 1$

Amplitude: 2

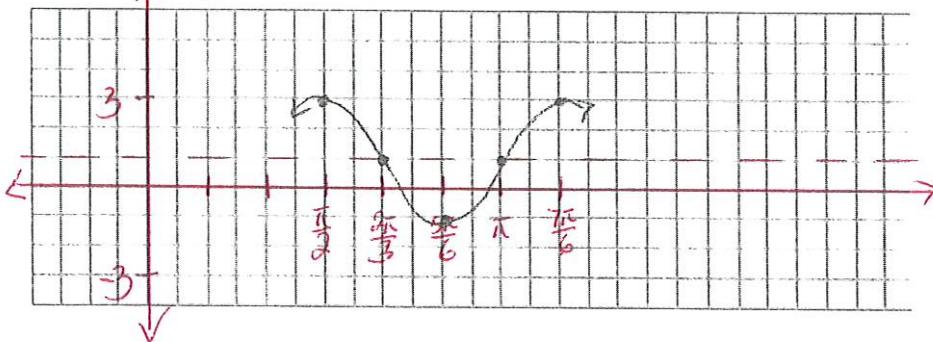
Period: $\frac{2\pi}{3}$

Increments: $\frac{2\pi}{3} \div 4 = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase Shift: $\frac{\pi}{2}$ right

Vertical Displacement: 1 up

$* \frac{3\pi}{6}$



c) $y = -\sin \frac{1}{2}\left(x + \frac{\pi}{4}\right) - 2$

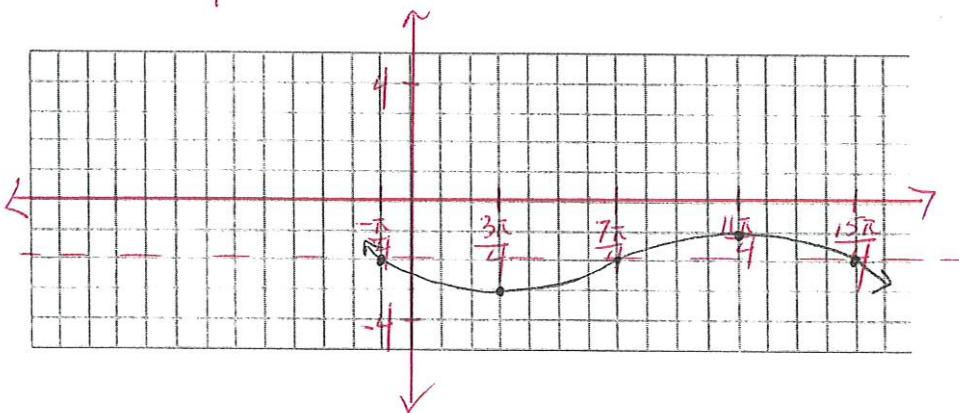
Amplitude: 1

Period: $2\pi \div \frac{1}{2} = 4\pi$

Increments: $\frac{4\pi}{4} = \pi$ or $\frac{4\pi}{4}$

Phase Shift: $\frac{\pi}{4}$ left

Vertical Displacement: 2 down



d) $y = -\frac{1}{3} \sin\left(2x + \frac{\pi}{4}\right)$ ** Be careful!! $y = -\frac{1}{3} \sin 2(x + \frac{\pi}{8})$

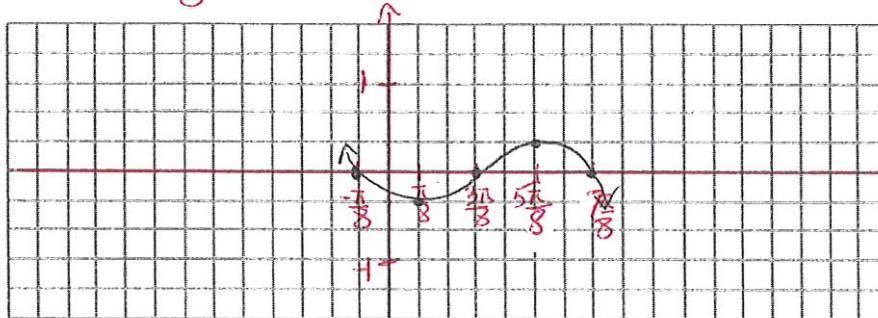
Amplitude: $\frac{1}{3}$

Period: $2\pi \div 2 = \pi$

Increments: $\frac{\pi}{4} * \frac{2\pi}{8}$

Phase Shift: $\frac{\pi}{8}$ left

Vertical Displacement: 0



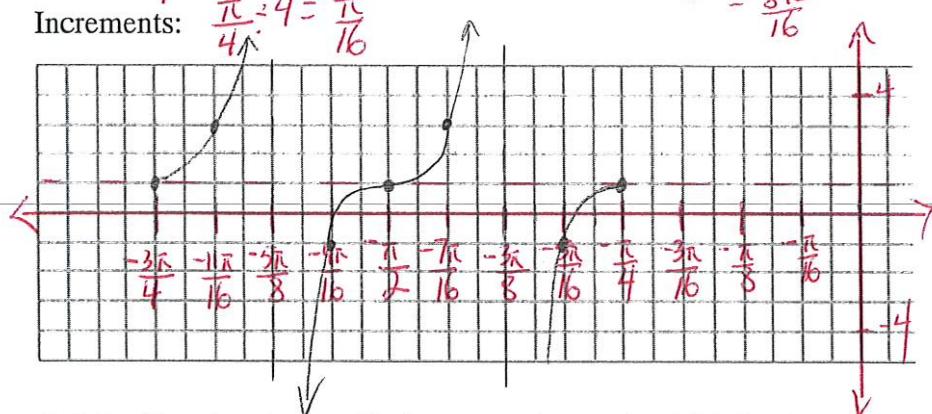
7. Graph $y = 2 \tan 4\left(x + \frac{\pi}{2}\right) + 1$ for one positive and one negative period.

Period: $\frac{\pi}{4}$

Phase Shift: $\frac{\pi}{2}$ left

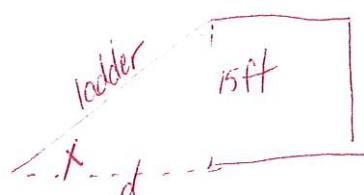
Vertical Displacement: 1 up

Increments: $\frac{\pi}{4} : 4 = \frac{\pi}{16}$



8. Mrs. Boughen has to climb up onto her roof, which is 15 ft. high. She sets the base of the ladder at a distance of d ft. away from the house to lean against the side of the house. The ladder makes an angle, x , with the ground.

a) Draw a diagram that models this situation.



b) Write a tan function that would model the slope of the ladder.

$$\tan x = \frac{15}{d}$$

$$\therefore d = \frac{15}{\tan x}$$

c) How far away from the house is the ladder when it makes an angle of:

i) 45°

$$d = 15 \tan 45^\circ \\ = 15 \text{ ft}$$

ii) 60°

$$d = 15 \tan 60^\circ \\ = 8.7 \text{ ft}$$

d) What is the slope of the ladder when it is 4 feet away from the house?

i) $\tan x = \frac{15}{4}$

$x = 65^\circ$ \therefore slope of ladder = $\frac{15}{7}$

ii) $\tan x = \frac{15}{0}$

= undefined

\therefore the slope is undefined.

9. For the given graph determine:

a) the vertical displacement $i + -9 = -4$ 4 down

b) the amplitude $\frac{1-(-9)}{2} = 5 \therefore \text{amplitude} = 5$

c) the period $\frac{2\pi}{3} \therefore b = 2\pi \div \frac{2\pi}{3} = \frac{2\pi \cdot 3}{2\pi} = 3$

d) its equation in the form $y = a \cos b(x - c) + d$

$$y = 5 \cos 3(x) - 4 \Rightarrow y = 5 \cos 3x - 4$$

e) the maximum and minimum value of y

max: $y = 1$

min: $y = -9$

10. The partial graphs of the functions $y = 4\sin 2(x + 45^\circ)$ and the line $y = 3$ are shown. Determine the solutions to the equation $4\sin 2(x + 45^\circ) = 3$ over the interval $0^\circ \leq x \leq 360^\circ$. Express your answers to the nearest degree.

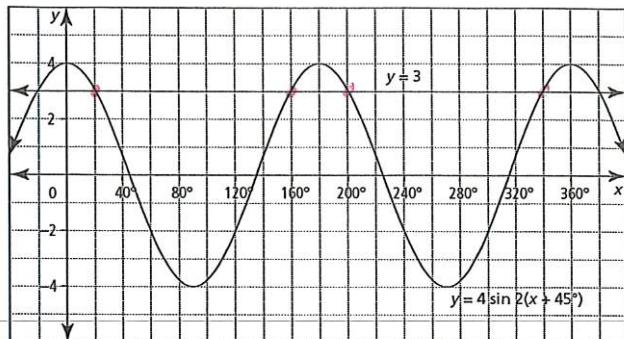
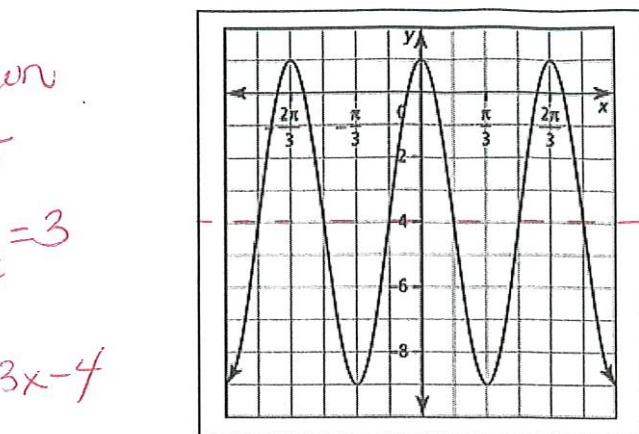
$x = 20^\circ$

$\{20^\circ, 160^\circ, 200^\circ, 340^\circ\}$

$x = 160^\circ$

$x = 200^\circ$

$x = 340^\circ$

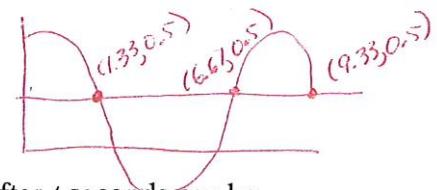


11. Solve the equation graphically: $\sin\left(\frac{\pi}{4}(x-6)\right) = 0.5, 0 \leq x \leq 3\pi$

Graph $y_1 = \sin\left(\frac{\pi}{4}(x-6)\right)$

$y_2 = 0.5$

$\{1.33, 6.67, 9.33\}$



12. The height, h , in meters, above the ground of a rider on a Ferris wheel after t seconds can be modeled by the sine function $h(t) = 12 \sin\left(\left(\frac{\pi}{45}\right)(t-30)\right) + 15$. Graph the function using graphing technology to determine the following information.

a) Determine the maximum and minimum heights of the rider above the ground.

max = 27 m

min = 3 m

b) Determine the time required for the Ferris wheel to complete one revolution.

period = $2\pi \div \frac{\pi}{45}$

= $2\pi \cdot \frac{45}{\pi}$

= 90 seconds

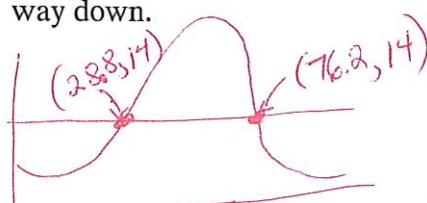
d) Determine the height of the rider above the ground after 45 seconds.

value: $x = 45$

$y = 25.4$

\therefore The rider is 25.4 m above ground.

e) Determine how long it takes for the rider to be 14 m in the air on the way up AND on the way down.

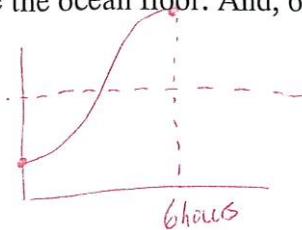


\therefore It takes 2.88 seconds on the way up and 7.62 seconds on the way down to reach 14 m.

13. Each day, the tide continuously goes in and out, raising and lowering a boat sinusoidally in the harbor. At low tide, at a time of 0 hours, the boat is only 2 feet above the ocean floor. And, 6 hours later, at peak high tide, the boat is 40 feet above the ocean floor.

a) What is the period of the tide?

\therefore The period is 12 hours



b) Write a cosine function that describes the boat's height above the ocean floor as it relates to time.

$$d = \frac{40+2}{2} = 21$$

$$b = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$a = \frac{40-2}{2} = 19$$

$$c = 0$$

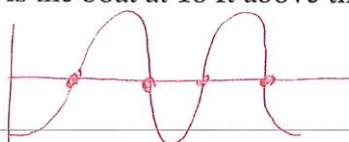
$$\therefore h(t) = 19 \cos \frac{\pi}{6} t + 21$$

c) How high above the ocean floor is the boat at 24 hours?

value: $x = 24$
 $y = ?$

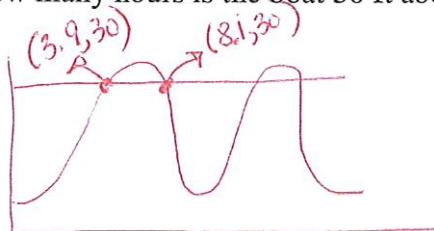
\therefore The boat will be 2ft above the ocean floor.

d) When is the boat at 18 ft above the ocean floor within the day?



At 2.7 hours, 9.3 hours, 14.7 hours, and
21.3 hours

e) For how many hours is the boat 30 ft above the ocean floor?



$$\therefore 8.1 - 3.9 = 4.2 \text{ hours} \quad \times 2$$

\therefore It is 30ft. above the ocean floor for 8.4 hours within the day.